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Mechanical Engineering
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EEE561 (~~Control~~ Process Control & Automation)

PI Controller

$V_a = 0$

$I_1 + I_2 = 0$ ——— (1)

$I_3 - I_2 = 0$ ——— (2)

Current through the Capacitor

$$I_c = \frac{C\,dV_c}{dt}$$

$$I_1 = \frac{V_c - V_a}{R_1} \quad (V_a = 0)$$

$$= \frac{V_e}{R_1}$$

$$I_2 = \frac{V_o - V_a}{R_a} \quad (V_a = 0)$$

$$= \frac{V_o}{R_2}$$

$$I_3 = \frac{Cd}{dt}(V_{out_1} - V_o)$$

sub into equ(1) & equ(2)

$$\frac{V_c}{R_1} + \frac{V_b}{R_2} = 0 \quad \text{——— (1)}$$

$$\frac{Cd}{dt}(V_{out_1} - V_b) - \frac{V_b}{R_2} = 0 \quad \text{——— (2)}$$

from eq (1)

$$\frac{V_b}{R_2} = \frac{-V_e}{R_1}$$

$$\boxed{V_b = -\frac{R_2 V_e}{R_1}}$$

Taking Laplace transform of eq(2)

$$SC(V_{out,1} - V_b) - \frac{V_b(s)}{R_2} = 0$$

$$SC\,V_{out,1}(s) = SC\,V_b(s) + \frac{V_b(s)}{R_2}$$

$$SC\,V_{out,1}(s) = V_b(s)\left(SC + \frac{1}{R_2}\right)$$

recall ; $V_b = -\frac{R_2}{R_1} V_e$

$$\therefore SC\,V_{out,1}(s) = -\frac{R_2}{R_1} V_e(s)\left(SC + \frac{1}{R_2}\right)$$

$$V_{out,1}(s) = \frac{-R_2}{SCR_1} V_e(s)\left(SC + \frac{1}{R_2}\right)$$

$$V_{out,1}(s) = -\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)$$

from the Inverting Circuit

$$V_{out,1} = -V_{out}$$

$$\therefore V_{out}(s) - \left(-\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)\right)$$

~~from the Inverting~~

$$V_{out}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)$$

taking Inverse laplace

$$V_{out} = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{R_2}\int_0^t V_e(t)\,dt + V_{(0)}$$

$$\left(\text{where } \frac{1}{s} = \int_0^t dt + k\right)$$

$$\boxed{V_{out} = G_p V_e + G_p G_I \int_0^t V_e\,dt + V_{(0)}}$$

where $G_p = \frac{R_2}{R_1}$, $G_I = \frac{1}{R_2 C}$

PD Controller

$I_1 + I_2 = I_3$ ——— (1)

$I_3 + I_4 = 0$ ——— (2)

$$I_1 = \frac{V_c - V_a}{R_3}$$

$$I_2 = \frac{cd}{dt}(V_c - V_a)$$

$$I_3 = \frac{V_a - V_b}{R_1} \quad (V_b = 0)$$

$$= \frac{V_a}{R_1}$$

$$I_4 = \frac{V_{out1} - V_b}{R_2} \quad (V_b = 0)$$

$$= \frac{V_{out1}}{R_2}$$

$$R = \frac{R_1 R_3}{R_1 + R_3} \longleftarrow \text{Effective resistance}$$

sub Into eq(1) & eq (2)

$$\frac{V_e - V_a}{R_3} + \frac{cd}{dt}(V_e - V_a) = \frac{V_a}{R_1} \quad \text{———} *$$

$$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0 \quad \text{———} **$$

from equ **

$$\frac{V_a}{R_1} = -\frac{V_{out1}}{R_2}$$

$$\boxed{V_a = -\frac{R_1}{R_2} V_{out1}}$$

rearranging eq *

$$\frac{V_c - V_a}{R_3} + \frac{cd}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$$

Scanned with CamScanner

taking Laplace transform

$$\frac{V_e(s) - V_a(s)}{R_s} + SC(V_e(s) - V_a(s)) - \frac{V_a(s)}{R_1} = 0$$

(Initial conditions go to zero)

$$\frac{V_c(s)}{R_3} + SCV_e(s) = \frac{V_a(s)}{R_1} + \frac{V_a(s)}{R_3} + SCV_a(s)$$

$$V_c(s)\left(\frac{1}{R_3} + SC\right) = V_a(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + SC\right)$$

recall, $V_a = \frac{-R_1}{R_2} V_{out_1}$

$$V_e(s)\left(\frac{1}{R_3} + SC\right) = \frac{-R_1}{R_2} V_{out_1}(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + SC\right)$$

taking L.C.M

$$V_c(s)\left(\frac{1 + R_3SC}{\cancel{R_3}}\right) = -\frac{\cancel{R_1}}{R_2} V_{out_1}(s)\left(\frac{R_3 + R_1 + SCR_1R_3}{\cancel{R_1}\cancel{R_3}}\right)$$

$$V_e(s)(1 + SCR_3) = -\frac{V_{out_1}(s)}{R_2}(R_3 + R_1 + SCR_1R_3)$$

$$-V_{out_1}(s) = \frac{V_c(s)(1 + SCR_3)R_2}{(R_1 + R_3 + SCR_1R_3)}$$

$$-V_{out_1}(s) = \frac{V_e(s)(R_2 + SCR_2R_3)}{(R_1 + R_3 + SCR_1R_3)}$$

dividing num & denom. by $R_1 + R_3$

$$-V_{out_1}(s) = \frac{V_c(s)(R_2 + SCR_2R_3)/R_1 + R_3}{\frac{R_1 + R_3}{R_1 + R_3} + \frac{SCR_1 + R_3}{R_1 + R_3}}$$

recall, $R = \frac{R_1R_3}{R_1 + R_3}$

$$-V_{out_1}(s) = \frac{V_c(s)(R_2 + SCR_2R_3)/R_1 + R_3}{1 + SCR}$$

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CONTINUATION.

If $SCR << 1$

$$\therefore -(-V_{out}(s) + V(0)) = V_e \frac{(R_2 + SCR_2R_3)}{R_1+R_3}$$

from the Inverting circuit

$V_{out_1} - V_{out} + V_e$

$$\therefore -(-V_{out}(s) + V(0)) = V_e \frac{(R_2 + SCR_2R_3)}{R_1+R_3}$$

$$V_{out}(s) - V(0) = \frac{V_e(s) R_2}{R_1+R_3} + \frac{SCR_2R_3}{R_1+R_3} V_e(s).$$

$$V_{out}(s) = \frac{R_2}{R_1+R_3} V_e(s) + \frac{R_2}{R_1+R_3} R_3 CS V_e(s) + V(0)$$

taking Inverse laplace

$$V_{out} = \frac{R_2}{R_1+R_3} V_e + \frac{R_2}{R_1+R_3} R_3 C \frac{dV_e}{dt} + V_0$$

$$\boxed{V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_0}$$

where; $G_P = \frac{R_2}{R_1+R_3}$

$G_D = R_3C$

Scanned with CamScanner