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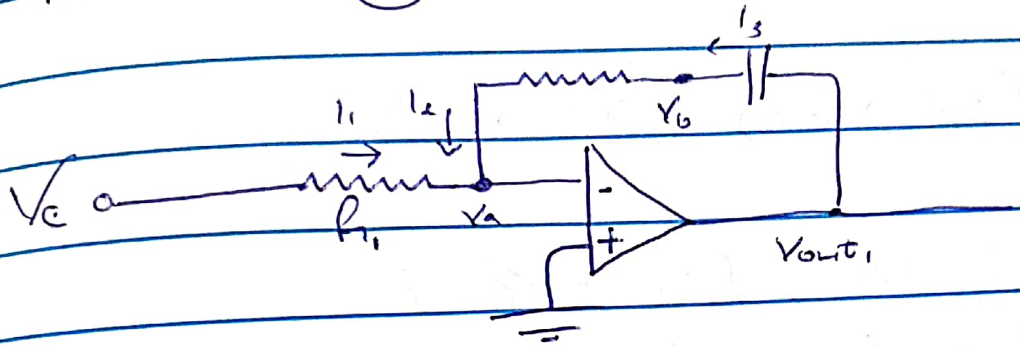
17/Eng04/079
Electrical and Electronics Engineering
Process control and Automation

1) Proportional - Integral controller



Taking KCL equations

$$I_1 + I_2 = 0 \quad \dots (a)$$

$$I_2 - I_3 = 0 \quad \dots (b)$$

$$I_1 = \frac{V_c - V_a}{R_1} \Rightarrow \frac{V_c}{R_1}$$

$$I_2 \Rightarrow \frac{V_b - V_a}{R_2} \Rightarrow \frac{V_b}{R_2}$$

$$I_3 = C\frac{d(V_{out_1} - V_b)}{dt}$$

Subing into equation (a) and (b)

$$\frac{V_c}{R_1} + \frac{V_b}{R_2} = 0 \Rightarrow V_b = -\frac{R_2}{R_1}V_c$$

$$\frac{V_b}{R_2} - C\frac{d(V_{out_1} - V_b)}{dt} = 0 \quad \dots (c)$$

Taking laplace transform of equ (c)

$$V_{b(s)}/R_2 - sC\left(V_{out_1}(s) - V_{b}(s)\right) = 0$$

$$V_{b(s)}/R_2 + sCV_{bs} = sC\,V_{out_1(s)}$$

$$V_{b(s)}\left(sC + 1/R_2\right) = sC\,V_{out_1(s)}$$

$$V_{b(s)} \Rightarrow -R_2/R_1\,V_{e(s)}$$

$$sC\,V_{out_1(s)} = -R_2/R_1\,V_{e(s)}\left\{sC + 1/R_2\right)$$

$$V_{out_1(s)} = \frac{-R_2}{sCR_1}V_{e(s)}\left\{sC + 1/R_2\right)$$

$$V_{out_1(s)} \Rightarrow -R_2/R_1\,V_{e(s)} - R_2/R_1 \cdot \frac{1}{sCR_2}V_{e(s)}$$

$V_{out_1} = -V_{out}$ ie Inverting amplifier

$$V_{out(s)} = R_2/R_1\,V_{e(s)} + R_2/R_1 \cdot \frac{1}{sCR_2}V_{e(s)}$$

Taking inverse laplace

$$V_{out} = R_2/R_1\,V_e + R_2/R_1\,\frac{1}{R_2}\int_0^t V_e(t)\,dt + V_0$$

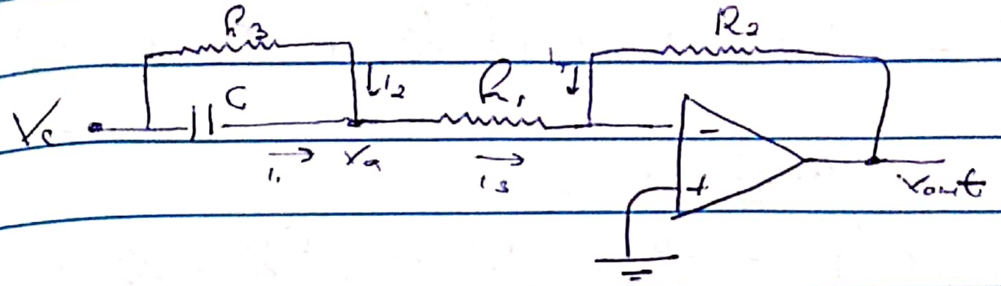
Expressed alternately

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e\,dt + V(0)$$

where $G_P = R_2/R_1$

$$G_I = 1/R_2C$$

2) Propotional Derivative controller.



$$I_1 + I_2 = I_3 \quad --(a)$$

$$I_3 + I_4 = 0 \quad --(b)$$

$$I_1 = V_e - V_a / R_3$$

$$I_2 = C \frac{d}{dt}(V_e - V_a)$$

$$I_3 = \frac{V_a - V_b}{R_1} \quad (i.e) \Rightarrow I_3 = V_a / R_1$$

$$I_4 = \frac{V_{out_1} - V_b}{R_2} \Rightarrow I_4 = \frac{V_{out}}{R_2}$$

Sub into eqn (a) and (b)

$$V_e - V_a / R_3 + \frac{C\, d(V_e - V_a)}{dt} = V_a / R_1 \quad --- (c)$$

$$V_a / R_1 + V_{out_1} / R_2 = 0 \quad --- (d)$$

$$\therefore V_a / R_1 = \frac{-V_{out_1}}{R_2}$$

$$= V_a = -R_1 / R_2 \; V_{out_1}$$

From equation (c)

$$\frac{V_e - V_a}{R_3} + \frac{C\, d(V_e - V_a)}{dt} - V_a / R_1 = 0$$

Taking laplace transform

$$\frac{V_{e(s)} - V_{a(s)}}{R_3} + sC(V_{e(s)} - V_{a(s)}) - V_a(s) / R_1 = 0$$

$$\frac{V_e(s)}{R_3} + sC V_e(s) = \frac{V_a(s)}{R_1} + \frac{V_a(s)}{R_3} + sC V_a(s)$$

$$V_e(s)\left(\frac{1}{R_3} + sC\right) = V_a(s)\left\{\frac{1}{R_1} + \frac{1}{R_3} + sC\right\}$$

But $V_a = -\frac{R_1}{R_2} V_{out_1}$

$$\therefore V_e(s)\left\{\frac{1}{R_3} + sC\right\} = -\frac{R_1}{R_2} V_{out_1}(s)\left\{\frac{1}{R_1} + \frac{1}{R_3} + sC\right\}$$

$$= V_e(s)\left\{\frac{1 + R_3 sC}{R_3}\right\} = -\frac{R_1}{R_2} V_{out}(s)\left\{\frac{R_3 + R_1 + sCR_1R_3}{R_1R_3}\right\}$$

$$\Rightarrow V_e(s)\{1 + sCR_3\} = -\frac{V_{out}}{R_2}\{R_3 + R_1 + sCR_1R_3\}$$

$$-V_{out_1}(s) = \frac{V_e(s)(1 + sCR_3)R_2}{R_1 + R_3 + sCR_1R_3}$$

Dividing through by $R_1 + R_3$ and also since $\frac{R_1R_3}{R_1+R_3} = R$ {equivalent Resistance}

$$-V_{out_1}(s) = V_e(s)\,\frac{\frac{R_2 + sCR_3R_2}{R_1 + R_3}}{\frac{R_1 + R_3}{R_1 + R_3} + \frac{sCR_1R_3}{R_1 + R_3}}$$

$$-V_{out_1}(s) = V_e(s) \cdot \frac{\frac{R_2 + sCR_3R_2}{R_1 + R_3}}{1 + sCR}$$

If $sCR \lll 1$

$$-V_{out_1}(s) = V_e(s)\,\frac{R_2 + sCR_3R_2}{R_1 + R_3}$$

$$V_{out} = -V_{out} + V_0$$

$$V_{out}(s) + V(0) = \frac{V_e(R_2 + sCR_2R_3)}{R_1 + R_3}$$

$$V_{out}(s) - V(s) = \frac{V_e(s) R_2}{R_1 + R_3} + \frac{sCR_2R_3 V_e(s)}{R_1 + R_3}$$

Taking inverse laplace

$$V_{out} = \frac{R_2}{R_1 + R_3} V_e + \frac{R_2}{R_1 + R_3} R_3 C \frac{dV_e}{dt} + V_0$$

$$V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_0$$

where,
$$G_P = \frac{R_2}{R_1 + R_3}$$

$$G_D = R_3 C$$