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EEE 471

Question 1

In control theory & stability theory, root locus analysis is a graphical method for examining how the roots of a system change with variation of a certain system parameter, commonly a gain with a feedback system. This is a technique used as a stability criterion in the field of classical control theory. The root locus plots the poles of the closed loop transfer function in the complex s-plane as a function of a gain parameter.

A graphical method that uses a special protractor called a "spirule" was once used to determine angles and draw the root loci.

Question 2

A)

Initial layout for Routh's table

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	b_1	b_2	b_3
s^1	c_1	0	0
s^0	d_1	0	0

$$b_1 = \frac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} \qquad b_2 = \frac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3}$$

$$b_3 = \frac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$$

In that same sense;

$$C_1 = -\frac{\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} \qquad C_2 = -\frac{\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$$

$$d_1 = -\frac{\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$$

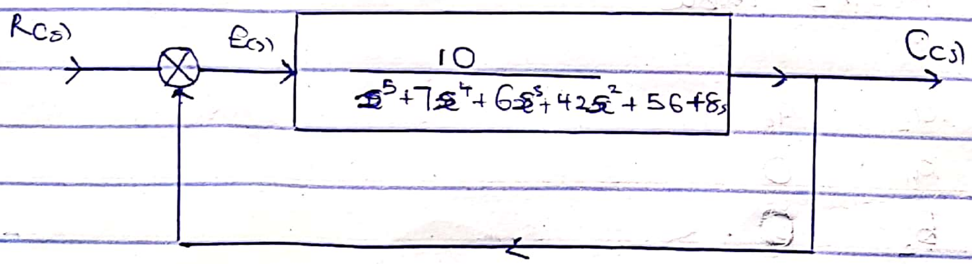
Therefore, standard equation will be

$$\boxed{b_k = -\frac{\det \begin{vmatrix} a_n & a_{n-2k} \\ a_{n-1} & a_{n-2k-1} \end{vmatrix}}{a_{n-1}}}$$

where $d(s) = a_n s^n + a_{n-1} s^{n-1} + \text{----} \; a_1 s + a_0$

Example.

In a case where a whole row becomes zero.



$$T(s) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 56 + 8s}$$

s^5	1	6	8
s^4	7	42	56
s^3	$\dfrac{-\begin{vmatrix}1 & 6\\ 7 & 42\end{vmatrix}}{7} = 0$	$\dfrac{-\begin{vmatrix}1 & 8\\ 7 & 56\end{vmatrix}}{7} = 0$	$\dfrac{-\begin{vmatrix}1 & 0\\ 7 & 0\end{vmatrix}}{7} = 0$ } Entire row is 0

When a row of zeros appears, we develop an auxillary polynomial from s^4. So we have:

$$P(s) = 7s^4 + 42s^2 + 56 \quad \text{--- (1)}$$

Differentiating (1), we'll have.

$$\frac{dP(s)}{ds} = 28s^3 + 84s \quad \text{--- (2)}$$

$\therefore$ We continue with auxillary equation.

s^3	28	84	0
s^2	$\dfrac{-\begin{vmatrix}7 & 42\\ 28 & 84\end{vmatrix}}{28} = 21$	$\dfrac{-\begin{vmatrix}7 & 56\\ 28 & 0\end{vmatrix}}{28} = 56$	$\dfrac{-\begin{vmatrix}7 & 0\\ 28 & 0\end{vmatrix}}{28} = 0$
s^1	$\dfrac{-\begin{vmatrix}28 & 84\\ 21 & 56\end{vmatrix}}{21} = 28/3$	$\dfrac{-\begin{vmatrix}28 & 0\\ 21 & 0\end{vmatrix}}{21} = 0$	0
s^0	$\dfrac{-\begin{vmatrix}21 & 56\\ \frac{28}{3} & 0\end{vmatrix}}{\frac{28}{3}} = 56.$	0	0

There is no sign change therefore the system is marginally stable.

B) To determine the poles on $j\omega$ axis.

Ans: When the entries from the row before the row of the zeros to the last row are looking at the even polynomials and there are no sign changes then all the poles there belong to the $j\omega$ axis.