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MECHATRONICS ENGINEERING
EEE 441 [Servo Mechanism and Control Systems.]

1) Briefly explain the Root Locus Technique?

Answer

Root Locus technique in control systems and stability theory is a graphical method in which roots of the characteristic equation are plotted in s-plane for the different values of parameter ie it is a method (graphical) for examining how the roots of a system changes with variation of a certain system parameter, commonly a gain within a feedback system. It is a technique used to determine the stability of a system. The root locus plots the poles of the closed loop transfer function in the complex s-plane as a function of a gain parameter. The locus of the roots of the characteristic equation when gain is varied from zero to infinity is called ROOT LOCUS.

2) Describe the use of Routh Hurwitz to find the stability of a closed loop System when

a) Entire row is zero on the Routh Table

b) To determine the poles on the jw axis.

Answer(s)

The Routh criterion is a method for determining continuous system stability, for systems with an nth-order characteristic equation of the form

$$a_n S^n + a_{n-1} S^{n-1} + \cdots + a_1 S + a_0 = 0$$

This criterion is applied using a Routh table defined as follows:

s^n	a_n	a_{n-2}	a_{n-4}	$\cdots$
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	$\cdots$
s^{n-2}	b_1	b_2	b_3	$\cdots$
$\cdot$	c_1	c_2	c_3	$\cdots$
$\cdot$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

where $a_n, a_{n-1}, \cdots, a_0$ are the coefficient of the characteristic equation and

$$\cancel{b_1 = \frac{a_{n-1}\cdot a_{n-2} - a_n\cdot a_{n-3}}{a_{n-1}}}, \quad \cancel{b_2 = \frac{a_{n-1}\cdot a_{n-4} - a_n a_{n-5}}{a_{n-1}}}$$

$$\cancel{c_1 =}$$

$$b_1 = \frac{a_{n-1}\cdot a_{n-2} - a_n\cdot a_{n-3}}{a_{n-1}}, \quad b_2 = \frac{a_{n-1}\cdot a_{n-4} - a_n\cdot a_{n-5}}{a_{n-1}}, \quad b_3 = \text{etc}$$

$$c_1 = \frac{b_1\cdot a_{n-3} - a_{n-1}\cdot b_2}{b_1}, \quad c_2 = \frac{b_1\cdot a_{n-5} - a_{n-1}\cdot b_3}{b_1}, \quad c_3 = \text{etc.}$$

The table is continued horizontally and vertically until only zeros are obtained. Any row can be multiplied by a positive constant before the next row is computed without disturbing the properties of the table

a) When entire row is zero on the Routh Table

An entire row of zeros will appear in the Routh table when a purely even or purely odd polynomial is a factor of the original polynomial. Lets use an example to illustrate.

Let $T(s) = \dfrac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56}$

We form the Routh Table

s^5	1	6	8
s^4	7	42	56
s^3	0	0	0

$b_1 = \dfrac{42-42}{7}$, $b_2 = \dfrac{56-56}{7}$, $b_3 = \dfrac{0-0}{7}$

If an entire row consists of zeros, we stop. We return to the rows immediately above the rows of zeros and form an auxilliary polynomial, using the entries in the row as coefficients. The polynomial will start with the power of s in the label column and continue by skipping every other power of s. Thus we have:

$P(s) = 7s^4 + 42s^2 + 56$ which can be reduced to

$P(s) = s^4 + 6s^2 + 8$ . . . (i)

differentiate (i) with respect to s

$$\frac{dP(s)}{ds} = 4s^3 + 12s + 0 \quad \ldots \text{(ii)}$$

Finally we replace the rows of zeros with the coefficients of equ (ii) and we then continue the Routh table.

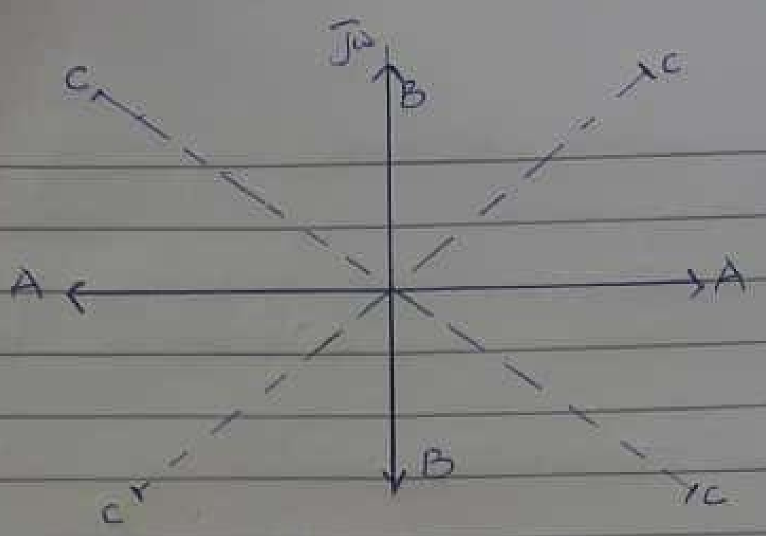
s^3	4	12	0
s^2	3	8	0
s^1	1/3	0	0
s^0	8	0	0

There is no sign change, therefore the system is marginally stable.

The equ (i) is an even polynomial which only have roots that are symmetrical about the origin. This symmetry can occur under three conditions of root position:

i) roots are symmetrical and real
ii) roots are symmetrical and imaginary
iii) roots are quadrantal

Each case or combination of these cases will generate an even polynomial that causes the appearance of the row of zeros.



$\overline{AA}$ = Real and symmetrical about the origin.
$\overline{BB}$ = Imaginary and symmetrical about the origin.
$\overline{CC}$ = Quadrantal and symmetrical about the origin.

b) To determine the poles on the jω axis.

Answer:
When the entries from the row before the row of the zeros to the last row in the Routh table are looking at the even polynomials are there are no sign changes, then all the poles there belong to the jω axis like the example used to illustrate the past question.