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Elect/Elect

PI controller

$V_a = 0$

$I_1 + I_2 = 0 \quad \cdots \cdots (1)$

$I_3 - I_2 = 0 \quad \cdots \cdots (2)$

current through capacitor

$$I_c = \frac{C dV_c}{dt}.$$

$$I_1 = \frac{V_e - V_a}{R_1} \quad (V_a = 0)$$

$$= \frac{V_e}{R_1}$$

$$I_2 = \frac{V_0 - V_a}{R_2} \quad (V_a = 0)$$

$$= \frac{V_b}{R_2}$$

$$I_3 = \frac{Cd}{dt}(V_{out_1} - V_b)$$

sub into eqn(1) and eqn(2)

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \quad \cdots \cdots (1)$$

$$\frac{Cd}{dt}(V_{out_1} - V_b) - \frac{V_b}{R_2} = 0 \quad \cdots \cdots (2)$$

from eqn (1)

$$\frac{V_b}{R_2} = \frac{-V_e}{R_1}$$

$$V_b = \frac{-R_2 V_e}{R_1}$$

Taking the laplace transform of eqn(2)

$$sC\left(V_{out_1}(s) - V_b(s)\right) - \frac{V_b(s)}{R_2} = 0$$

$$sC\ V_{out1}(s) = sC\ V_b(s) + \frac{V_b(s)}{R_2}$$

$$sC\ V_{out1}(s) = V_b(s)\left(sC + \frac{1}{R_2}\right)$$

recall; $V_b = -\frac{R_2}{R_1} V_e$

$$sC\ V_{out_1}(s) = -\frac{R_2}{R_1} V_e(s)\left(sC + \frac{1}{R_2}\right)$$

$$V_{out_1}(s) = \frac{-R_2}{sCR_1} V_e(s)\left(sC + \frac{1}{R_2}\right)$$

$$V_{out_1}(s) = -\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{sCR_2} V_e(s)$$

from the inverting circuit

$$V_{out1} = N_{out}$$

$$\therefore V_{out}(s) = -\left(-\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{sCR_2} V_e(s)\right)$$

Taking the inverse Laplace

$$V_{out} = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{R_2}\int_0^t V_e(t)\,dt + V(0)$$

$$\left(\text{where } V_0 = \int_0^t dt + K\right.$$

$$\boxed{V_{out} = G_p V_e + G_p G_I \int_0^t V_e\,dt + V(0)}$$

where $G_p = \frac{R_2}{R_1}$

$G_I = \frac{1}{R_2 C_1}$

PD controller

$$I_1 + I_2 = I_3 \quad \ldots\ldots (1)$$

$$I_3 + I_4 = 0 \quad \ldots\ldots (2)$$

$$I_1 = \frac{V_e - V_a}{R_3}$$

$$I_2 = \frac{Cd}{dt}(V_e - V_a)$$

$$I_3 = \frac{V_a - V_b}{R_1} \quad (V_b = 0)$$

$$= V_a / R_1$$

$$I_4 = \frac{V_{out\,1} - V_b}{R_2} \quad (V_b = 0)$$

$$= \frac{V_{out\,1}}{R_2}$$

$$R = \frac{R_1 R_3}{R_1 + R_3}$$ — effective Resistance

sub into eqn (i) & eqn (2)

$$\frac{V_e - V_a}{R_3} + \frac{Cd}{dt}(V_e - V_a) = \frac{V_a}{R_1} \quad \ldots\ldots \#$$

$$\frac{V_a}{R_1} + \frac{V_{out\,1}}{R_2} = 0 \quad \ldots\ldots \#\#$$

from eqn ##

$$\frac{V_a}{R_1} = -\frac{V_{out\,1}}{R_2}$$

$$\boxed{V_a = -\frac{R_1}{R_2} V_{out\,1}}$$

Rearranging eqn #

$$\frac{V_c - V_a}{R_3} + \frac{C\,d}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$$

Taking the laplace transform

$$\frac{V_e(s) - V_a(s)}{R_3} + sC\,(V_e(s) - V_a(s)) - \frac{V_a(s)}{R_1} = 0$$

(initial conditions go to zero)

$$V_e(s) + sC\,V_e(s) = \frac{V_a(s)}{R_1} + \frac{V_a(s)}{R_3} + sC\,V_a(s)$$

$$V_e(s)\left(1/R_3 + sC\right) = V_a(s)\left(1/R_1 + 1/R_3 + sC\right)$$

recall, $V_a = -R_1/R_2\ V_{out\,1}$

$$V_e(s)\left(1/R_3 + sC\right) = -R_1/R_2\ V_{out\,1}(s)\left(1/R_1 + 1/R_3 + sC\right)$$

Taking lcm

$$V_e(s)\left(\frac{1 + R_3 sC}{R_3}\right) = -R_1/R_3\ V_{out\,1}(s)\left(\frac{R_3 + R_1 + sCR_1R_3}{R_1R_3}\right)$$

$$V_e(s)\,(1 + sC\,R_3) = -\frac{V_{out\,1}(s)}{R_2}(R_3 + R_1 + sCR_1R_3)$$

$$-V_{out_1}(s) = \frac{V_e(s)\,(1 + sC\,R_3)\,R_2}{(R_1 + R_3 + sC\,R_1R_3)}$$

$$-V_{out_1}(s) = \frac{V_e(s)\,(R_2 + sCR_2R_3)}{(R_1 + R_3 + sC\,R_1R_3)}$$

dividing number & denominator by $R_1 + R_3$

$$-V_{out,1}(s) = \frac{V_e(s)\,(R_2 + sCR_2R_3)/R_1+R_3}{\frac{R_1+R_3}{R_1+R_3} + \frac{sCR_1R_3}{R_1+R_3}}$$

recall, $R = \frac{R_1 R_3}{R_1 + R_3}$

$$-V_{out,1}(s) = \frac{V_e(s)(R_3 + sC R_2 R_3)}{R_1 + R_3}$$

from the inverting circuit

$$V_{out\,1} = -V_{out} + V_0$$

$$\therefore \; +(+V_{out}(s) + V_{(0)}) = \frac{V_e\,(R_2 + sCR_2R_3)}{R_1 + R_3}$$

$$V_{out}(s) - V_{(0)} = \frac{V_e(s)\,R_2}{R_1+R_3} + \frac{sCR_2R_3}{R_1+R_3}\,V_e(s)$$

$$V_{out}(s) = \frac{R_2}{R_1+R_3}\,V_e(s) + \frac{R_2}{R_1+R_3}\,R_3 C \frac{dv_e}{dt} + V_0$$

$$\boxed{V_{out} = G_p V_e + G_p G_D \frac{dv_e}{dt} + v_0}$$

where; $G_p = R_2/R_1 + R_3$

$G_D = R_3 C$