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16/ENG05/033

MECHATRONICS

MCT 511

Derive the analysis for the output voltage in using Operational Amplifier for:

1. Proportional integral Controller mode:
2. Proportional Derivative Controller Mode.

Answer =>

$$C\frac{d}{dt}[V_{out} - V_b] - \frac{V_b}{R_2} = 0 \quad \text{——— (1) PI}$$

$$\left.\begin{aligned}\frac{V_e - V_a}{R_3} + C\frac{d}{dt}[V_e - V_a] - \frac{V_a}{R_1} = 0 \\ \frac{V_{out}}{R_2} + \frac{V_a}{R_1} = 0\end{aligned}\right\} (2)$$

PI

$$C\frac{d}{dt}[V_{out} - V_b] - \frac{V_b}{R_2} = 0$$

$$C\frac{d}{dt}[V_{out} - V_b] = \frac{V_b}{R_2}$$

$$\frac{d}{dt}[V_{out} - V_b] = \frac{V_b}{R_2 C}$$

Recall

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0$$

$$\frac{V_b}{R_2} = \frac{V_e}{R_1} \quad \therefore V_b = \frac{R_2}{R_1} V_e \quad \text{——— *}$$

$$\int d\left[V_{out} - V_b\right] = \int \frac{V_b}{R_2 C}\, dt$$

$$V_{out} - V_b = \int \frac{V_b}{R_2 C}\, dt$$

$$V_{out} = \int \frac{V_b}{R_2 C}\, dt + V_b$$

$$= \frac{1}{R_2 C} \int -\frac{R_2}{R_1} V_e\, dt + \left(\frac{-R_2 V_e}{R_1}\right)$$

$$= -\frac{1}{R_2 C} \frac{R}{R_1} \int_0^t V_e\, dt - \frac{R_2}{R_1} V_e$$

$$= -\left[\frac{R_2}{R_1} V_e + \frac{R_2}{R_1} + \frac{1}{R_2 C} \int_0^t V_e\, dt\right] + V_{(0)}$$

After inverting

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_e}{R_1} \frac{1}{R_2 C} \int_0^t V_e\, dt + V_{(0)}$$

$$V_{out} = G_p V_e + G_p G_i \int_0^t V_e\, dt + V_{(0)}$$

Where,

$G_p = \frac{R_2}{R_1}$ Proportional Gain

$G_i = \frac{1}{R_2 C}$ Integral Gain

P. D

$$\frac{V_e - V_b}{R_3} + C \frac{d}{dt}\left[V_e - V_a\right] - \frac{V_a}{R_1} = 0 \quad — *$$

$$\frac{V_{out}}{R_2} + \frac{V_0}{R_1} = 0 \quad \text{———— **}$$

from **

$$V_a = -\frac{R_1}{R_2} V_{out}$$

Substituting into eq *

$$\frac{V_e}{R_3} - \left[-\frac{R_1}{R_2} V_{out}\right] \frac{1}{R_3} + \frac{C\,dV_e}{dt} - \frac{Cd}{dt}\left[\frac{-R_1}{R_2} V_{out}\right] - \left[\frac{-R_1}{R_2} V_{out}\right] \frac{1}{R}$$

$$= 0$$

$$\frac{V_e}{R_3} + \frac{R_1}{R_1R_3} V_{out} + \frac{C\,dV_e}{dt} + \frac{R_1}{R_2} \frac{C\,d}{dt} V_{out} + \frac{1}{R_2} V_{out} = 0$$

Multiply all through by R_3

$$V_e + \frac{R_1}{R_2} V_{out} + R_3 \frac{C\,dV_e}{dt} + \frac{R_1R_3}{R_2} \frac{C\,dV_{out}}{dt} + \frac{R_1}{R_2} V_{out} = 0$$

$$\frac{R_1 V_{out}}{R_2} + \frac{R_1R_3}{R_2} \frac{C\,dV_{out}}{dt} + \frac{R_3}{R_2} V_{out} = -V_e - R_3 \frac{C\,dV_e}{dt}$$

$$\frac{R\,V_{out} + R_3 V_{out}}{R_2} + \frac{R_1R_3}{R_2} \frac{C\,dV_{out}}{dt} = -V_e - R_3 \frac{C\,dV_e}{dt}$$

$$\frac{R_1 V_{out} + R_3 V_{out}}{R_2} + \frac{R_1R_3}{R_2} \frac{C\,dV_{out}}{dt} = -V_e - R_3 \frac{C\,dV_e}{dt}$$

$$\left[\frac{R_1 + R_3}{R_2}\right] V_{out} + \frac{R_1R_3}{R_2} \frac{C\,dV_{out}}{dt} = -V_e - R_3 \frac{C\,dV_e}{dt}$$

Multiply all through by $\frac{R_2}{R_1+R_3}$

$$V_{out} + \left[\frac{R_1}{R_1+R_3}\right] R_3 C \frac{dV_{out}}{dt} = -\left[\frac{R_2}{R_1+R_3}\right] V_e - \left[\frac{R_2}{R_1+R_3}\right] R_3 C \frac{dV_{e}}{dt}$$

After inverting

$$V_{out} + \left[\frac{R_1}{R_1+R_3}\right] R_3 C \frac{dV_{out}}{dt} = \left(\frac{R_2}{R_1+R_3}\right) V_e + \left(\frac{R_2}{R_1+R_3}\right) R_3 C \frac{dV_e}{dt}$$

$$V_{out} = \left(\frac{R_2}{R_1+R_3}\right) V_e + \left(\frac{R_2}{R_1+R_3}\right) R_3 C \frac{dV_e}{dt} + V(0)$$

$$V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_0$$

Where

$G_P = \frac{R_2}{R_1+R_3}$ — Proportional Gain

$G_D = R_3 C$ — Derivative gain.