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16/ENG04/029 ELCG/ELCG ENG
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EEE 561 ASSIGNMENT (PROCESS CONTROL & AUTOMATION

(1) PI controller

$V_a = 0$, $I_1 + I_2^{(1)} = 0$, $I_3 - I_2^{(2)} = 0$

Current through the capacitor

$$I_c = C\frac{dV_c}{dt}$$

$$I_1 = \frac{V_e - V_a}{R_1} \quad (V_a = 0)$$

$$= V_e/R_1$$

$$I_2 = V_b - V_a/R_2 \quad (V_a = 0)$$

$$= V_b/R_2$$

$$I_3 = C\frac{d(V_{out1} - V_b)}{dt}$$

Sub into eqn (1) and (2)

$$V_e/R_1 + V_b/R_2 = 0 \quad \text{---(3)}$$

$$C\frac{d(V_{out2} - V_b)}{dt} - V_b/R_2 = 0 \quad \text{---(4)}$$

from (3)

$$V_b/R_2 = -V_e/R_1$$

$$V_b = -R_2/R_1\, V_e$$

taking laplace transform of (4)

$$sC(V_{out1}(s) - V_b(s)) - (V_b(s)/R_2) = 0$$

$$sC\,V_{out1}(s) = sC\,V_b(s) + V_b(s)/R_2$$

$$sC\,V_{out1}(s) = V_b(s)(sC + 1/R_2)$$

recall, $V_b = -R_2/R_1\, V_e$

$$sC\,V_{out1}(s) = -R_2/R_1\, V_e(s)(sC + 1/R_2)$$

$$V_{out1}(s) = -R_2/sCR_1\, V_e(s)(sC + 1/R_2)$$

$$V_{out1}(s) = -R_2/R_1\, V_e(s) - (R_2/R_1)(1/sCR_2)\, V_e(s)$$

from the inverting circuit

$$V_{out1} = -V_{out}$$

$$\therefore V_{out}(s) = -\left(-R_2/R_1\, V_e(s) - \frac{R_2}{R_1}\frac{1}{sCR_2} V_e(s)\right)$$

$$V_{out}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{sCR_2} V_e(s)$$

taking the inverse laplace

$$V_{out} = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1 R_2 C} \int_0^t V_e(t)\,dt + V_0$$

$$\left(\frac{1}{s} = \int_0^t dt + K\right)$$

$$V_{out} = G_p V_e + G_p G_I \int_0^t V_e\,dt + V_0$$

where $G_p = R_2/R_1$

$G_I = 1/R_2 C$

2) PD controller

$I_1 + I_2 = I_3$ ---(1)

$I_3 + I_4 = 0$ ---(2)

$I_1 = \frac{V_e - V_a}{R_3}$

$I_2 = \frac{C d(V_e - V_a)}{dt}$

$I_3 = \frac{V_a - V_b}{R_1}$ $(V_b = 0)$

$= V_a/R_1$

$I_4 = \frac{V_{out1} - V_b}{R_2}$ $(V_b = 0)$

$= \frac{V_{out1}}{R_2}$

* $R = \frac{R_1 R_3}{R_1 + R_3}$ — effective resistance

Sub into eq(1) and eq(2)

$$\frac{V_e - V_a}{R_3} + C\frac{d(V_e - V_a)}{dt} = \frac{V_a}{R_1} \quad \text{----(3)}$$

$$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0 \quad \text{----(4)}$$

from (4)

$V_a/R_1 = \frac{-V_{out1}}{R_2}$

$V_a = -R_1/R_2 V_{out1}$

rearranging (3)

$$\frac{V_e - V_a}{R_3} + C\frac{d(V_e - V_a)}{dt} - \frac{V_a}{R_1} = 0$$

taking laplace transform

$$\frac{V_e(s) - V_a(s)}{R_3} + sC(V_e(s) - V_a(s)) - \frac{V_a(s)}{R_1} = 0$$

(initial conditions go to zero)

$$\frac{V_e(s)}{R_3} + sCV_e(s) = \frac{V_a(s)}{R_1} + \frac{V_a(s)}{R_3} + sCV_a(s)$$

$$V_e(s)(1/R_3 + sC) = V_a(s)(1/R_1 + 1/R_3 + sC)$$

recall $V_a = -R_1/R_2 V_{out1}$

$$V_e(s)(1/R_3 + sC) = -R_1/R_2 V_{out1}(s)(1/R_1 + 1/R_3 + sC)$$

taking the LCM

$$V_e(s)\left(\frac{1+R_3sC}{R_3}\right) = \frac{-R_1}{R_2} V_{out1}(s)\left(\frac{R_3 + R_1 + sCR_1R_3}{R_1R_3}\right)$$

$$V_e(s)(1+sCR_3) = \frac{-V_{out1}(s)}{R_2}(R_3 + R_1 + sCR_1R_3)$$

making $-V_{out1}(s)$ subject of formula

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + sCR_2R_3)}{(R_1 + R_3 + sCR_1R_3)}$$

dividing the numerator and denominator by $(R_1 + R_3)$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + sCR_2R_3)/(R_1+R_3)}{\frac{R_1+R_3}{R_1+R_3} + \frac{sCR_1R_3}{R_1+R_3}}$$

recall, $R = \frac{R_1R_3}{R_1+R_3}$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + sCR_2R_3)/(R_1+R_3)}{1 + sCR}$$

if $sCR \ll 1$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + sCR_2R_3)}{R_1 + R_3}$$

from the inverting circuit, $V_{out1} = -V_{out} + V_o$

$$\therefore -(-V_{out}(s) + V_o) = \frac{V_e(s)(R_2 + sCR_2R_3)}{R_1 + R_3}$$

$$V_{out}(s) - V_0 = \frac{V_e(s) R_2}{R_1 + R_3} + \frac{sCR_2R_3 V_e(s)}{R_1 + R_3}$$

$$V_{out}(s) = \frac{V_e(s) R_2}{R_1 + R_3} + V_e(s) \frac{sCR_2R_3}{R_1 + R_3} + V_0$$

taking inverse laplace

$$V_{out} = \frac{R_2}{R_1 + R_3} V_e(t) + \frac{R_2}{R_1 + R_3} R_3 C \frac{dV_e}{dt} + V_0$$

comparing with

$$V_{out} = G_p V_e + G_p G_D \frac{dV_e}{dt} + V_0$$

where $G_p = \frac{R_2}{R_1 + R_3}$

$G_D = R_3 C$