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ELECTRICAL/ELECTRONICS

PI Controller

$V_a = 0$

$I_1 + I_2 = 0$ ——— (1)

$I_3 - I_2 = 0$ ——— (2)

Current through the capacitor.

$I_c = C\frac{\delta V_c}{dt}$.

$I_1 = \frac{V_e - V_a}{R_1}$ $(V_a = 0)$

$= \frac{V_e}{R_1}$

$I_2 = \frac{V_o - V_a}{R_2}$ $(V_a = 0)$

$= \frac{V_o}{R_2}$

$I_3 = \frac{C\delta}{\delta t}(V_{out1} - V_o)$

Sub into equ (1) & equ(2).

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \quad \text{—— (1)}$$

$$\frac{Cd}{dt}(V_{out1} - V_b) - \frac{V_b}{R_2} = 0 \quad \text{—— (2)}$$

From eq (1)

$$\frac{V_b}{R_2} = -\frac{V_e}{R_1}$$

$$V_b = -\frac{R_2}{R_1} V_e$$

taking laplace transform of equa (2)

$$SC(V_{out1}(s) - V_b(s)) - \frac{V_b(s)}{R_2} = 0$$

$$SC\,V_{out1}(s) = SC\,V_b(s) + \frac{V_b(s)}{R_2}$$

$$SC\,V_{out1}(s) = V_b(s)\left(SC + \frac{1}{R_2}\right)$$

Recall, $V_b = -\frac{R_2}{R_1} V_e$

$$SC\,V_{out1}(s) = -\frac{R_2}{R_1} V_e(s)\left(SC + \frac{R_1}{R_2}\right)$$

$$V_{out1}(s) = -\frac{R_2}{sCR_1} V_e(s) \left(sC + \frac{1}{R_2}\right)$$

$$V_{out1}(s) = -\frac{R_2}{R_1} C V_e(s) \left(sC + \frac{1}{R_2}\right)$$

$$V_{out1}(s) = -\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1} \frac{1}{sCR_2} V_e(s).$$

form the inverting circuit

$V_{out1} = -V_{out}$

$$\therefore V_{out}(s) = -\left(-\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1} \frac{1}{sCR_2} V_e(s)\right).$$

$$V_{out}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1} \frac{1}{sCR_2} V_e(s).$$

✓ taking Inverse Laplace.

$$V_{out} = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1} \frac{1}{R_2} \int_0^t V_e(t)\,dt + V_{(0)}.$$

(When $\frac{1}{s} = \int_0^t 1\,dt + K$)

$V_{out} = G_p V_e + G_p G_I \int_0^t V_e\,dt + V_{ci}$

When $G_p = \frac{R_2}{R_1}$

$G_I = \frac{1}{R_2 C}$.

PD controller.

$I_1 + I_2 = I_3$ —— (1)

$I_3 + I_4 = 0$ —— (2)

$$I_1 = \frac{V_c - V_a}{R_3}$$

$$I_2 = \frac{C\delta}{dt}(V_c - V_a)$$

$$I_3 = \frac{V_a - V_b}{R_1} \quad (V_b = 0)$$

$$= \frac{V_a}{R_1}$$

$$I_4 = \frac{V_{out1} - V_b}{R_2} \quad (V_b = 0)$$

$$= \frac{V_{out1}}{R_2}$$

$R = \frac{R_1 R_3}{R_1 + R_3}$ —— Effective Resistance.

Sub into eqn's ξ eq(a)

$$\frac{V_i - V_a}{R_3} + \frac{Cd}{dt}(V_i - V_a) = \frac{V_a}{R_f} \quad \text{— (*)}$$

$$V_a/R_1 + V_{out1}/R_2 = 0 \quad \text{— (**)}$$

$$\frac{V_a}{R_1} = -\frac{V_{out1}}{R_2}$$

from equation (**)

$$\frac{V_a}{R_1} = -\frac{V_{out1}}{R_2}$$

$$V_a = -\frac{R_1}{R_2} V_{out1}$$

Rearranging eq*

$$\frac{V_i - V_a}{R_3} + \frac{Cd}{dt}(V_i - V_a) = \frac{V_a}{R_f}$$

taking Laplace transform

$$\frac{V_i(s) - V_a(s)}{R_3} + sC\,(V_i(s) - V_a(s)) =$$

Initial conditions go to zero.

$$\frac{V_e(s)}{R_3} + sCV_e(s) = \frac{V_a(s)}{R_1} + \frac{V_a(s)}{R_3} + sCV_a(s).$$

$$V_e(s)\left(\frac{1}{R_3} + sC\right) = V_a(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right).$$

Recall, $V_a = -\frac{R_1}{R_2} V_{out}$

$$V_e(s)\left(\frac{1}{R_3} + sC\right) = -\frac{R_1}{R_2} V_{out}(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right).$$

taking lcm.

$$V_e(s)\left(\frac{1 + R_3 sC}{R_3}\right) = -\frac{R_1}{R_2} V_{out}(s)\left(\frac{R_3 + R_1 + sCR_1R_3}{R_1R_3}\right)$$

$$V_e(s)(1 + sCR_3) = -\frac{V_{out}(s)}{R_2}(R_3 + R_1 + sCR_1R_3)$$

$$-V_{out}(s) = \frac{V_e(s)(1 + sCR_3)R_2}{(R_1 + R_3 + sCR_1R_3)}.$$

$$-V_{out}(s) = \frac{V_e(s)(R_2 + sCR_2R_3)}{(R_1 + R_3 + sCR_1R_3)}.$$

dividing Nom and denom by R_1+R_3 ..

$$-V_{out}(s) = \frac{V_c(s)(R_2 + sCR_2R_3)/R_1+R_3}{\frac{R_1+R_3}{R_1+R_3} + \frac{sCR_1R_3}{R_1+R_3}}.$$

recall $R = \frac{R_1R_3}{R_1+R_3}$

$$-V_{out}(s) = \frac{V_c(s)(R_2 + sCR_2R_3)/R_1+R_3}{1+sCR}$$

If $sCR << 1$

$$-V_{out}(s) = \frac{V_c(s)(R_2 + sCR_2R_3)}{R_1+R_3}$$

From the Inverting Circuit
$V_{out_1} = V_{out} + V_c$

$$\therefore -(-V_{out}(s) + V_c(s)) = \frac{V_c(R_2 + sCR_2R_3)}{R_1+R_3}.$$

$$V_{out}(s) - V_c(s) = \frac{V_c(s)R_2}{R_1+R_3} + \frac{sCR_2R_3}{R_1+R_3}V_c(s)$$

$$V_{out}(s) = \frac{R_2}{R_1+R_3} V_e(s) + \frac{R_2}{R_1+R_3} R_3 C s V_e(s) + V(0)$$

taking inverse Laplace.

$$V_{out} \Rightarrow \frac{R_2}{R_1+R_3} V_e + \frac{R_2}{R_1+R_3} R_3 C \frac{dV_e}{dt} + dV_0$$

$$V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_0$$

Where; $G_P = \frac{R_2}{R_1+R_3}$

$G_D = R_3 C$