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161ENG04I031
ELECT/ELECT ENGR.
EEE 561 ASSIGNMENT

Question: Derive the analysis for the Output voltage in using Operational Amplifier for;
i) Proportional Integral Controller mode.
ii) Proportional Derivative Controller mode.

Soln

i) Propotional Integral Controller Mode;

Eqn 1:- $I_1 + I_2 = 0$

Eqn 2:- $I_3 - I_2 = 0$

$I_c = C\dfrac{dV_c}{dt}$ {The Current through capacitor}.

for I_1;

$$= \frac{V_e - V_a}{R_1}$$

as $V_a = 0$

$$\Rightarrow I_1 = \frac{V_e}{R_1}.$$

for I_2;

$$= \frac{V_b - V_a}{R_2}$$

as $V_a = 0$

$$\Rightarrow I_2 = \frac{V_b}{R_2}.$$

for I_3;

$$= \frac{C\,d(V_{out1} - V_b)}{dt}$$

inputting the above ~~vars~~ into eqn 1 & 2

in eqn 1;

$$\Rightarrow \frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \text{ ——— Eqn } 3$$

in eqn 2;

$$\Rightarrow \frac{C\,d(V_{out1} - V_b)}{dt} - \frac{V_b}{R_2} = 0 \text{ ——— Eqn } 4$$

From eqn 3;

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0$$

→ Making V_b subject of this formula;

$$\Rightarrow \frac{V_e}{R_1} = -\frac{V_b}{R_2}$$ {Multiplying both sides}.

$$\Rightarrow V_e R_2 = -V_b R_1$$

$$\Rightarrow V_b = -\frac{V_e R_2}{R_1} \text{ ——— Eqn } *$$

Taking the laplace transform of eqn 4

$$\mathcal{L}\left[\frac{C\,d(V_{out1} - V_b)}{dt} - V_b/R_2 = 0\right]$$

where, G_P = Propotional gain.
G_I = Integral gain.

as $\frac{R_2}{R_1} = G_P$, $\frac{1}{R_2 C} = G_I$.

$$\therefore V_{out} = G_P G_I \int_0^t V_e \cdot dt + V_{cos} + G_P V_e .$$

i) Propotional Derivative Controller Mode;

Eqn 1: $I_1 + I_2 = I_3$

Eqn 2: $I_3 + I_4 = 0$

For I_1;

$$= \frac{V_e - V_a}{R_3}$$

For I_2;

$$= \frac{C d(V_e - V_a)}{dt}$$

For I_3;

$$= \frac{V_a - V_b}{R_1}$$

as $V_b = 0$

$$\Rightarrow I_3 = \frac{V_a}{R_1} .$$

For I_4;

$$= \frac{V_{out1} - V_b}{R_2}$$

as $V_b = 0$

$$\Rightarrow I_4 = \frac{V_{out1}}{R_2} .$$

The effective Resistance, $R = \frac{R_1 R_3}{R_1 + R_3}$.

~~Substituting~~ ~~inputting~~ ~~eqn~~ inputting the above in eqn 1 & 2 .

For eqn 1;

$$\frac{V_e - V_a}{R_3} + \frac{C d(V_e - V_a)}{dt} = \frac{V_a}{R_1} \text{ ——— Eqn } 3$$

for eqn (2);

$$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0 \quad \text{———— Eqn (4)}$$

~~from eqn~~ (4) from eqn (4)

$$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0$$

→ Making Va subject of formular;

$$\Rightarrow \frac{V_a}{R_1} = \frac{V_{out1}}{R_2}$$

$$\therefore V_a = \frac{-R_1 V_{out1}}{R_2}.$$

rearranging eqn (3)

$$\frac{V_e - V_a}{R_3} + \frac{Cd}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0.$$

taking the Laplace transform of the above eqn (3).

$$\frac{V_{e(s)} - V_{a(s)}}{R_3} + SC(V_{e(s)} - V_{a(s)}) - \frac{V_{a(s)}}{R_1} = 0 \text{ \{initial Conditions to 0\}}$$

Collecting like terms;

$$\Rightarrow \frac{V_{e(s)} + SCV_{e(s)}}{R_3} - \frac{V_{a(s)}}{R_3} - \frac{V_{a(s)}}{R_1} - SCV_{a(s)} = 0.$$

$$\Rightarrow \frac{V_{e(s)} + SCV_{e(s)}}{R_3} = \frac{V_{a(s)}}{R_3} + \frac{V_{a(s)}}{R_1} + SCV_{a(s)}.$$

$$\left(\frac{1}{R_3} + SC\right)V_{e(s)} = \left(\frac{1}{R_3} + \frac{1}{R_1} + SC\right)V_{a(s)}$$

recall that, $V_a = -\frac{R_1}{R_2} V_{out1}$.

$$\Rightarrow \left(\frac{1}{R_3} + SC\right) V_{e(s)} = \left(\frac{1}{R_3} + \frac{1}{R_1} + SC\right) \cdot -\frac{R_1}{R_2} V_{out1(s)}.$$

taking the lcm of both sides respectively,

$$\left(\frac{1 + SCR_3}{R_3}\right) V_{e(s)} = \left(\frac{R_1 + R_3 + SCR_3R_1}{R_3R_1}\right) \cdot -\frac{R_1}{R_2} V_{out1(s)}.$$

$$(1 + SCR_3) V_{e(s)} = (R_1 + R_3 + SCR_3R_1) \cdot -\frac{V_{out1(s)}}{R_2}.$$

→ Making Vout subject

$$-V_{out1(s)} = \frac{(R_2 + SCR_2R_3) V_{e(s)}}{(R_1 + R_3 + SCR_3R_1)}$$

→ Dividing the numerator and denominator of the RHS with $(R_1 + R_3)$

$$\Rightarrow -V_{out1(s)} = \frac{(R_2 + SCR_2R_3) V_{e(s)} / (R_1 + R_3)}{\left(\frac{R_1 + R_3}{R_1 + R_3} + \frac{SCR_3R_1}{R_1 + R_3}\right)}.$$

recall that;

effective resistance, $R = \frac{R_1R_3}{R_1 + R_3}$

$$\Rightarrow -V_{out1(s)} = \frac{(R_2 + SCR_2R_3) V_{e(s)} / (R_1 + R_3)}{(1 + SCR)}$$

If SCR far less than 1 (i.e $SCR \ll 1$).

$$\Rightarrow -V_{out1(s)} = \frac{V_{e(s)} (R_2 + SCR_2R_3)}{R_1 + R_3},$$

from the inverting Circuit

$V_{out1} = -V_{out} + V_{(0)}$

$$\Rightarrow -(-V_{out}(s) + V_{(0)}) = \frac{(R_2 + SCR_2R_3)V_e(s)}{R_1 + R_3}$$

$$V_{out}(s) - V(0) = \frac{R_2 V_e(s)}{R_1 + R_3} + \frac{SCR_2R_3 V_e(s)}{R_1 + R_3}$$

$$\therefore V_{out}(s) = \frac{R_2 V_e(s)}{R_1 + R_3} + \frac{SCR_2R_3 V_e(s)}{R_1 + R_3} + V(0) \text{ ——— eqn } (*)$$

taking the inverse Laplace transform of eqn (*)

$$\mathcal{L}^{-1}\left[\frac{R_2 V_e(s)}{R_1 + R_3} + \frac{SCR_2R_3 V_e(s)}{R_1 + R_3} + V(0)\right].$$

$$V_{out} = \frac{R_2 V_e}{R_1 + R_3} + \frac{R_2 \cdot R_3 C}{R_1 + R_3}\frac{dV_e}{dt} + V(0).$$

as $\frac{R_2}{R_1 + R_3} = G_P$; $R_3C = G_D$.

G_P = Proportional gain ; G_D = Derivative gain.