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Mechatronics Engineering

$$C\frac{d}{dt}\left[V_{out} - V_{b}\right] - \frac{V_b}{R_2} = 0 \quad \text{—— (1) PI}$$

$$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}\left[V_2 - V_1\right] - \frac{V_a}{R_1} = 0$$

$$\frac{V_{out}}{R_2} + \frac{V_0}{R_1} = 0$$

(2)

PI

$$C\frac{d}{dt}\left[V_{out} - V_b\right] - \frac{V_b}{R_2} = 0$$

$$C\frac{d}{dt}\left[V_{out} - V_b\right] = \frac{V_b}{R_2}$$

$$\frac{d}{dt}\left[V_{out} - V_b\right] = \frac{V_b}{R_2 C}$$

Recall

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0$$

$$\frac{V_b}{R_2} = \frac{V_e}{R_1}$$

$$V_b = \frac{R_2 V_e}{R_1} \quad \text{—— **}$$

$$\int d\left[V_{out} - V_b\right] = \int \frac{V_b}{R_2 C}\, dt$$

$$V_{out} - V_b = \int \frac{V_b}{R_2 C}\, dt$$

$$V_{out} = \int_0^t \frac{V_b}{R_2 C}\, dt + V_b$$

$$= \frac{1}{R_2 C} \int_0^t -\frac{R_e}{R_1} V_e\, dt + \left[-\frac{R_2 V_e}{R_1}\right]$$

$$= \frac{1}{R_2 C} \frac{R}{R_1} \int_0^t V_e\, dt - \frac{R_2}{R_1} V_e$$

$$= -\left[\frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \frac{1}{R_2 C} \int_0^t \left[V_e\, dt\right] + V_{(0)}\right.$$

After Inverting

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_e}{R_1} \frac{1}{R_2 C} \int_0^t V_e\, dt + V_{(0)}$$

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e\, dt + V_{(0)}$$

where

$G_P = \frac{R_2}{R_1}$ ⟶ Proportional Gain

$G_I = \frac{1}{R_2 C}$ ⟶ Integral gain

PD

$$\frac{V_e - V_a}{R_3} + C \frac{d}{dt}\left[V_e - V_a\right] - \frac{V_a}{R_1} = 0 \quad \text{——— } *$$

$$\frac{V_{out}}{R_2} + \frac{V_a}{R_1} = 0 \quad \text{——— } **$$

From **

$$V_a = -\frac{R_1}{R_2} V_{out},$$

Substituting into eqn *

$$\frac{V_e}{R_3} - \left[-\frac{R_1}{R_2} V_{out}\right]\frac{1}{R_3} + C\frac{dv_e}{dt} - C\frac{d}{dt}\left[-\frac{R_1}{R_2} V_{out}\right] - \left[-\frac{R_1 V_{out}}{R_2}\right]\frac{1}{R_1} = 0$$

$$\frac{V_e}{R_3} + \frac{R_1}{R_2 R_3} V_{out} + C\frac{dV_e}{dt} + \frac{R_1}{R_2} C \frac{d}{dt} V_{out} + \frac{1}{R_2} V_{out} = 0$$

Multiply all through by R_3

$$V_e + \frac{R_1}{R_2} V_{out} + R_3 C\frac{dV_e}{dt} + \frac{R_1 R_3}{R_2} C\frac{dV_{out}}{dt} + \frac{R_1}{R_2} V_{out} = 0$$

$$\frac{R_1}{R_2} V_{out} + \frac{R_1 R_3}{R_2} C\frac{dV_{out}}{dt} + \frac{R_3}{R_2} V_{out} = -V_e - R_3 C\frac{dV_e}{dt}$$

$$\frac{R_1 V_{out} + R_3 V_{out}}{R_2} + \frac{R_1 R_3}{R_2} C\frac{dV_{out}}{dt} + \frac{R_3}{R_2} V_{out} = -V_e - \frac{R_3 C dV_e}{dt}$$

$$\frac{R_1 V_{out} + R_3 V_{out}}{R_2} + \frac{R_1 R_3}{R_2} C\frac{dV_{out}}{dt} = -V_e - R_3 C\frac{dV_e}{dt}$$

$$\left[\frac{R_1 + R_3}{R_2}\right] V_{out} + \frac{R_1 R_3}{R_2} C\frac{dV_{out}}{dt} = -V_e - R_3 C\frac{dV_e}{dt}$$

Multiply all through by $\frac{R_2}{R_1 + R_3}$

$$V_{out} + \left[\frac{R_1}{R_1 + R_3}\right] R_3 C\frac{dV_{out}}{dt} = -\left[\frac{R_2}{R_1 + R_3}\right] V_e - \left[\frac{R_2}{R_1 + R_3}\right] R_3 C\frac{dV_e}{dt}$$

After inverting

$$V_{out} + \left[\frac{R_1}{R_2 + R_3}\right] R_3 C \frac{dV_{out}}{dt} = \left(\frac{R_2}{R_1 + R_3}\right) V_e + \left(\frac{R_2}{R_1 + R_3}\right) R_3 C \frac{dV_e}{dt}$$

$$V_{out} = \left(\frac{R_2}{R_1 + R_3}\right) V_e + \left(\frac{R_2}{R_1 + R_3}\right) R_3 C \frac{dV_e}{dt} + V_{o})$$

$$V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_o$$

Where

$G_P = \frac{R_2}{R_1 + R_3}$ ⟶ Proportional Gain

$G_D = R_3 C$ ⟶ Derivative gain.