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16/ENG04/008
ELECT/ELECT
EEE 561
Process Control & Automation
ASSIGNMENT

1.) Proportional Integral Controller Mode

$V_a = 0$

$I_1 + I_2 = 0$ —— 1

$I_3 - I_2 = 0$ —— 2

Current through the capacitor

$$I_c = C\frac{dV_c}{dt}$$

$$I_1 = \frac{V_e - V_a}{R_1}$$

Recall $V_a = 0$

$$I_1 = \frac{V_e}{R_1}$$

$$I_2 = \frac{V_b - V_a}{R_2}$$

$$= \frac{V_b}{R_2}$$

$$I_3 = \frac{Cd}{dt}(V_{out} - V_b)$$

Sub into eq 1 & eq. 2

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \text{ —— } 3$$

$$\frac{Cd}{dt}(V_{out} - V_b) - \frac{V_b}{R_2} = 0 \text{ —— } 4$$

from eq 3

$$\frac{V_b}{R_2} = \frac{-V_e}{R_1} \qquad \therefore \quad V_b = \frac{-R_2}{R_1} V_e$$

taking laplace transform of eq. 4

$SC(V_{out} - V_b) - \frac{V_b}{R_2} = 0$; $SC\,V_{out} - SC\,V_b - \frac{V_b}{R_2} = 0$

$$SC\,V_{out}(s) = V_b(s)\left(SC + \frac{1}{R_2}\right)$$

recall $V_b = \frac{-R_2}{R_1} V_e$

$$\therefore \; SC\,V_{out}(s) = -\frac{R_2}{R_1} V_e(s)\left(SC + \frac{1}{R_2}\right)$$

$$V_{out1}(s) = -\frac{R_2}{SCR_1} V_e(s)\left(SC + \frac{1}{R_2}\right)$$

$$V_{out1}(s) = -\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)$$

from the inverting circuit.

$V_{out1} = -V_{out}$

$$\therefore V_{out}(s) = -\left(-\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)\right)$$

$$V_{out}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)$$

taking inverse laplace.

$$V_{out} = \frac{R_2}{R_1} V_e(t) + \frac{R_2}{R_1}\frac{1}{R_2}\int_0^t V_e(t)\,dt + V(0)$$

where $\frac{1}{S} = \int_0^t dt + k$

$$V_{out} = G_p V_e + G_p G_i \int_0^t V_e\,dt + V(0)$$

where $G_p = \frac{R_2}{R_1}$ $\quad G_i = \frac{1}{R_2 C}$

Proportional Derivative Controller Mode.

$I_1 + I_2 = I_3$ $\quad V_b = 0$

$I_3 + I_4 = 0$

$I_1 = \frac{V_e - V_a}{R_3}$

$I_2 = \frac{Cd}{dt}(V_e - V_a)$

$I_3 = \frac{V_a - V_b}{R_1}$ $(V_b = 0)$

$= \frac{V_a}{R_1}$

$I_4 = \frac{V_{out1} - V_b}{R_2}$ $(V_b = 0)$

$= \frac{V_{out1}}{R_2}$

$R = \frac{R_1 R_3}{R_1 + R_3}$ — effective resistance

Sub into eq. 1 & 2

$\frac{V_e - V_a}{R_3} + \frac{Cd}{dt}(V_e - V_a) = \frac{V_a}{R_1}$ ——— 3

$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0$ ——— 4.

from eq-4

$\frac{V_a}{R_1} = \frac{-V_{out1}}{R_2}$

$\therefore V_a = -\frac{R_1}{R_2} V_{out1}$

Rearrange eq-3

$\frac{V_e - V_a}{R_3} + \frac{Cd}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$

Taking laplace transform

$\frac{V_e - V_a}{R_3} + SC\,(V_e(s) - V_a(s)) - \frac{V_a(s)}{R_1} = 0$

$$\frac{V_e(s)}{R_3} + SC\,V_e(s) = \frac{V_1(s)}{R_1} + \frac{V_1(s)}{R_3} + SC\,V_1(s)$$

$$V_e(s)\left(\frac{1}{R_3} + SC\right) = V_1(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + SC\right)$$ Recall $V_1 = -\frac{R_1}{R_2} V_{out}$

$$V_e\left(\frac{1}{R_3} + SC\right) = -\frac{R_1}{R_2} V_{out1}\left(\frac{1}{R_1} + \frac{1}{R_3} + SC\right)$$

$$V_e(s)\left(\frac{1 + R_3SC}{R_3}\right) = \frac{R_1}{R_2} V_{out1}(s)\left(\frac{R_3 + R_1 + SCR_1R_3}{R_1R_3}\right)$$

$$V_e(s)(1 + R_3SC) = -\frac{V_{out1}(s)}{R_2}(R_3 + R_1 + SCR_1R_3)$$

$$-V_{out1} = \frac{V_e(s)(1 + SCR_3)R_2}{R_1 + R_3 + SCR_1R_3}$$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + SCR_2R_3)}{(R_1 + R_3 + SCR_1R_3)}$$

dividing numerator & denominator by $R_1 + R_3$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + SCR_2R_3)/R_1+R_3}{\frac{R_1+R_3}{R_1+R_3} + \frac{SCR_1R_3}{R_1+R_3}}$$

Recall effective resistance $\rightarrow \frac{R_1R_3}{R_1+R_3}$

$$-V_{out}(s) = \frac{V_e(s)(R_2 + SCR_2R_3)/R_1+R_3}{1 + SCR}$$

If $SCR \ll 1$ $\therefore -V_{out1}(s) = \frac{V_e(s)(R_2 + SCR_2R_3)}{R_1 + R_3}$

from inverting circuit: $V_{out1} = -V_{out} + V_0$; $\therefore -(-V_{out}(s) + V_0) = \frac{V_e(R_2 + SCR_2R_3)}{R_1 + R_3}$

$$V_{out}(s) - V(0) = \frac{V_e(s)R_2}{R_1+R_3} + \frac{SCR_2R_3}{R_1+R_3} V_e(s)$$

$$V_{out}(s) = R_2/(R_1+R_3)\,V_e(s) + R_2/R_1+R_3\,V_e(s) + V(0)$$

taking inverse laplace: $V_{out} = \frac{R_2}{R_1+R_3} V_e + \frac{R_2}{R_1+R_3} R_3C\frac{dV_e}{dt} + V_0$

$V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_0$; where $G_P = \frac{R_2}{R_1+R_3}$, $G_D = R_3C$