EZEOBIDI CLEMENTINA ONYINYECHUKWU 17/Eng04/027

Servo mechanism & Control systems 400Level 29-12-2020

Assignment EEE441

Onyinye Obidi.

1) Briefly explain the Root locus Technique

=> root locus analysis: is a graphical method for examining how the roots of a system change with variation of a certain system parameter, commonly a gain within a feedback system.

This is a technique used as a stability criterion in the field of classical control theory developed by Walter R. Evans which can determine stability of the system.

Hence Root locus is the method of graphically displaying the roots of a polynomial equation having the following form on the complex plane when the parameter K varies from 0 to ∞

$$1 + K \cdot G(s) = 0 \quad \text{or} \quad 1 + K \cdot \frac{N(s)}{\Delta(s)} = 0$$

where $N(s)$ and $\Delta(s)$ are known polynomials in factorized form:

$$N(s) = (s - z_1)(s - z_2) \cdots (s - z_{N_z})$$
$$\Delta(s) = (s - p_1)(s - p_2) \cdots (s - p_{N_p})$$

Conventionally, the N_i roots of the polynomial $N(s)$; $z_1; z_2; \cdots, z_k$ are called the finite "open-loop zeros". The N_p roots of the polynomial $\Delta(s)$, $p_1, p_2, \cdots, p_{N_p}$ are called the finite open-loop poles

NOTE: By transforming the closed loop characteristic equation of a feedback controlled system with a single positive design parameter k into the above standard form, one can use the Root locus Technique to determine the range of k that have CL poles in the performance region.

Root locus sketching Rules

$$1 + K \cdot \frac{N(s)}{\Delta(s)} = 0$$

$$1 + K \cdot \frac{(s - z_1)(s - z_2) \cdots (s - z_{N_z})}{(s - p_1)(s - p_2) \cdots (s - p_{N_p})} = 0$$

Rule 1: The numbe of branches of the roots locus is equal to the numbe of Closed-loop poles (or roots of the characteristics Equation). In other words, the number of branches is equal to the numbe of open-loop poles or open-loop zeros, whichever is greater

$$D(s) + KN(s) = 0$$

Rule 2: Root locus starts at open-loop (when $K=0$) and ends at open loop zeros (when $K=\infty$); if the number of open loop poles as is greater than the number of open-loop zeros, some branches starting from finite open-loop poles will teminate at zeros at infinity (ie go to infinity). if the reverse is true, some branches will start at poles at infinity and teminates at the finite open-loop zeros

$$D(s) + KN(s) = 0 \quad , K=0? \quad K=\infty?$$

Rule 3: Root locus is symmetric about the real axis, which reflects the fact that closed-loop poles appear in complex conjugate pairs.

Rule 4: Along the real axis, the root includes all segments that are to the left of an odd number of finite real open-loop poles and zero.

$$\Rightarrow \text{To Check The phases} = \angle K \frac{N(s)}{D(s)} - \angle -1 = \pi\,[rad] = 180^\circ$$

Rule 5: if number of poles N_p exceeds the number of zeros N_z, then as $K \to \infty$, $(N_p - N_z)$ branches will becomes asymptotic to straight lines. These straight lines intersect the real axis with angles θ_K at σ_0

$$\sigma_0 = \frac{\sum p_i - \sum z_i}{N_p - N_z} = \frac{\text{Sum of open-loop poles} - \text{Sum of open loop zeros}}{\#\text{ of open-loop poles} - \#\text{ of open loop zeros}}$$

$$\theta_K = (2K+1)\frac{\pi}{N_p - N_z}\,[rad] = (2K+1)\frac{180^\circ}{N_p - N_z}\,[deg], \quad K = 0, 1, 2 \ldots$$

if N_z exceeds N_p, then as $K \to 0$, $(N_z - N_p)$ branches behave

Rule 6: Breakaway and/or break-in (arrival) points should be the solutions to the following equations.

$$\frac{d}{ds}\left(\frac{N(s)}{\Delta(s)}\right)=0 \quad \text{OR} \quad \frac{d}{ds}\left(\frac{\Delta(s)}{N(s)}\right)=0$$

Rule 7: The departure angle for a pole P_i (The arrival angle for a zero Z_i) can be calculated by slightly modifying the following equation:

$$\angle(s-Z_1)+\angle(s-Z_2)+\dots+\angle(s-Z_{N_2})-\angle(s-P_1)-\angle(s-P_2)-\dots$$
$$\angle(s-P_{N_p})=180^\circ$$

↑
Angle criterion

~~Rule~~: The departure angle from the pole P_i can be calculated by replacing the term with q_i and replacing all the s's with P_i in the other terms.

Rule 8: if the root locus passes through the imaginary axis (the stability boundary), the crossing point $j\omega$ and the corresponding gain K can be found as follows.

- replace s in the left side of the closed-loop characteristic equation with $j\omega$ to obtain the real and imaginary parts of the resulting complex number
- set the real and imaginary parts to zero and solve for ω and K. This will tell you at what values of K and at what point on the $j\omega$ axis the roots will cross

magnitude criterion →
$$K=\frac{|s-P_1||s-P_2|\dots|s-P_{N_p}|}{|s-Z_1||s-Z_2|\dots|s-Z_{N_z}|}$$

(2) Describe the use of Routh Hurwitz to find the stability of closed loop system when:

a) entire row is zero on the Routh table.
b) to determine the poles on the $J\omega$ axis

a) if The entire row of zeros appear on the Routh table.

1) form a new polynomial using the entries in The row above zeros. The polynomial will start with power of s in that row and Continue by skipping every other power of s.

2) differentiate The polynomial with respect to s.

3) finally The row with all zeros in The Routh table is replaced with The coefficients in

example:

$$\frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56} = T(s)$$

soln

s^5	1	6	8
s^4	$\cancel{7}$ 1	$\cancel{42}$ 6	$\cancel{56}$ 8
s^3	0	0	0

from the above, we are faced with the problem of entire zero in the Third row.

Hence: $P(s) = s^4 + 6s^2 + 8$

$$\frac{dP(s)}{ds} = 4s^3 + 12 + 0 \quad \text{— i)}$$

Therefore replacing s^3 with eqn i)

s^5	1	6	8	0
s^4	$\cancel{7}$ 1	$\cancel{42}$ 6	$\cancel{56}$ 8	0
s^3	$\cancel{4}$ 1	$\cancel{12}$ 3	$\cancel{0}$ 0	
s^2	3	8	0	
s^1	0.3	0	0	
s^0	8	0	0	

Therefore: from the above table, there's no sign changes - as a result no right-hand poles

Scanned by CamScanner

5) The entries from the row before the row ~~of~~ zeros to the last row are looking at the even polynomials and there are no sign changes therefore all poles belong to the $j\omega$ axis.