NAME: Tomilayo Aluko

MATRIC NO: 17/ENG05/042

COURSE CODE: MCT 511

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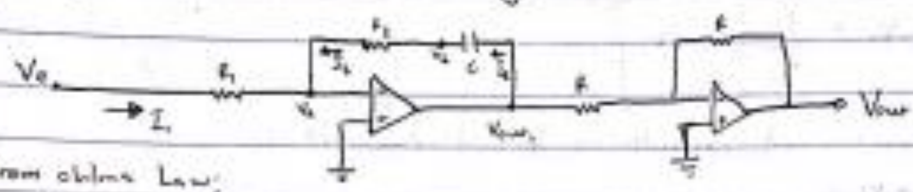
Assignment

Proving for Proportional - Integral mode [PI]



From ohms Law,

$V_b = 0$

Current through the capacitor

$I_1 + I_2 = 0$ —— ①

$I_c = C \frac{\partial V_c}{\partial t}$

$I_3 - I_2 = 0$ —— ②

* Combining this with ohm's law

$I_1 = \frac{V_e}{R_1}$ $\quad I_2 = \frac{V_b}{R_2}$

$I_3 = C\frac{d}{dt}\left[V_{out_1} - V_b\right]$

* Plugging back into eqn 1

$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0$

$V_b = -\frac{R_2}{R_1} \cdot V_e$

* Plugging back into eqn 2

$C\frac{d}{dt}\left[V_{out_1} - V_b\right] - \frac{V_b}{R_2} = 0$

* We then add eqn ① & ②

Which will give $I_1 + I_3 = 0$

$\therefore \frac{V_e}{R_1} + C\frac{d}{dt}\left[V_{out_1} - V_b\right] = 0$

$C\frac{d}{dt}\left[V_{out_1} - V_b\right] = -\frac{V_e}{R_1}$

$d\left[V_{out_1} - V_b\right] = -\frac{1}{CR_1} \cdot V_e\,dt$

$V_{out} - V_b = -\frac{1}{CR_1}\int V_e\,dt$

$$V_{out,1} - V_b = -\frac{1}{CR_1}\int V_e\,dt$$

$$V_{out,1} = V_b - \frac{1}{CR_1}\int V_e\,dt$$

$$V_{out,1} = -\frac{R_2}{R_1}V_e - \frac{1}{CR_1}\int V_e\,dt$$

$$V_{out,1} = -\left[\frac{R_2}{R_1}V_e + \frac{1}{CR_1}\int V_e\,dt\right]$$

* After Inverting circuit and considering Initial Conditions

$$V_{out} = -V_{out,1}$$

$$V_{out} = \frac{R_2}{R_1}V_e + \frac{R_2}{R_1}\cdot\frac{1}{R_2C}\int V_e\,dt$$

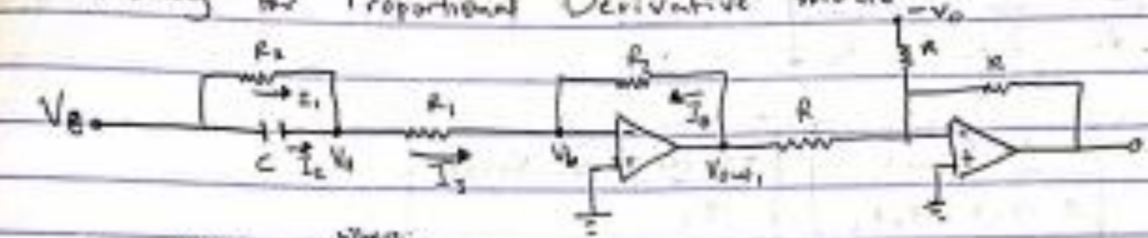
$$V_{out} = \frac{R_2}{R_1}V_e + \frac{R_2}{R_1}\frac{1}{R_2C}\int_0^t V_e\,dt + V_{(0)}$$

Proportional Gain $(G_P) = \frac{R_2}{R_1}$

Integral Gain $(G_I) = \frac{1}{R_2C}$

$$V_{out} = G_P V_e + G_P G_I\int_0^t V_e\,dt + V_{(0)}$$

Proving for Proportional Derivative mode



where;

$R = \frac{R_1 R_3}{R_1 + R_3}$ R = Effective resistance

the condition becomes as usual → $2\pi f_{max} R_c = 0.1$

by kirchoff's current law

$$I_1 + I_2 = I_3$$

$$I_1 + I_2 - I_3 = 0 \quad \text{———— (1)}$$

$$I_4 + I_3 = 0 \quad \text{———— (2)}$$

$I_1 = \frac{V_e - V_a}{R_3}$ $I_2 = C\frac{d}{dt}[V_e - V_a]$ $I_3 = \frac{V_a}{R_1}$ $I_4 = \frac{V_{out1}}{R_2}$

* Putting back into eqn 1

$$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}[V_e - V_a] - \frac{V_a}{R_1} = 0$$

* Putting back into eqn 2

$$\frac{V_{out1}}{R_2} + \frac{V_a}{R_1} = 0$$

$$V_a = -\frac{R_1}{R_2} V_{out1}$$

adding eqn (1) & (2)

$$I_1 + I_2 + I_4 = 0$$

Hence;

$$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}[V_e - V_a] + \frac{V_{out1}}{R_2} = 0$$

* taking Laplace transform

$$\frac{V_e(s) - V_a(s)}{R_3} + SC[V_e(s) - V_a(s)] + \frac{V_{out1}(s)}{R_2} = 0$$

$$\frac{V_{out1}(s)}{R_2} - \frac{1}{R_3}\left(-\frac{R_1 V_{out1}(s)}{R_2}\right) - SC\left(-\frac{R_1}{R_2} V_{out1}(s)\right) = \frac{V_E(s)}{R_3} - SC\, V_E(s)$$

$$V_{out,1}(s)\left[\frac{1}{R_2}+\frac{R_1}{R_3R_2}+\frac{SCR_1}{R_2}\right]=-\frac{V_e(s)}{R_3}-SCV_e(s)$$

$$V_{out,1}(s)=\left[\frac{R_2R_3}{R_1+R_3+SCR_1R_3}\right]\left[\frac{-V_e(s)}{R_3}-SCV_e(s)\right]$$

$$V_{out,1}(s)=\left[\frac{\frac{R_2R_3}{R_1+R_3}}{1+SCR}\right]\left[-\frac{V_e(s)}{R_3}-SCV_e(s)\right]$$

*assume $SCR << 1$

$$V_{out1}(s)=\left[\frac{R_2R_3}{R_1+R_3}\right]\left[-\frac{V_e(s)}{R_3}-SCV_e(s)\right]$$

$$V_{out1}(s)=-\left[\frac{R_2}{R_1+R_3}\right]V_e(s)-\left[\frac{R_2R_3}{R_1+R_3}\right]SCV_e(s)$$

* Inverse Laplace will give

$$V_{out1}=-\left[\frac{R_2}{R_1+R_3}\right]V_e-\left[\frac{R_2R_3}{R_1+R_3}\right]C\frac{dV_e}{dt}$$

* from inverter circuit we have

$$V_{out_2}=-V_{out_1}+V_o$$

$$V_{out_1}=-\left[\frac{R_2}{R_1+R_3}\right]V_e-\left[\frac{R_2R_3}{R_1+R_3}\right]C\frac{dV_e}{dt}$$

hence

$$V_{out}=\left[\frac{R_2}{R_1+R_3}\right]V_e+\left[\frac{R_2}{R_1+R_3}\right]R_3C\frac{dV_e}{dt}+V_o$$

*

$$G_P=\frac{R_2}{R_1+R_3}$$

$$G_D=R_3C$$

then;

$$V_{out}=G_PV_e+G_PG_D\frac{dV_e}{dt}+V_o.$$