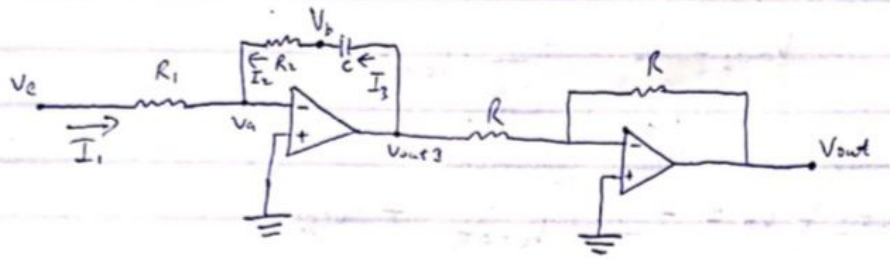
Assignment Solution

Proportional - Integral mode Controlle (PI Controller)



$$V_a = 0$$

using Ohm's law

$$I_1 + I_2 = 0 \quad \text{-----} \quad (1)$$

$$I_3 - I_2 = 0 \quad \text{-----} \quad (2)$$

$$I_1 = \frac{V_e}{R_1}$$

$$I_2 = \frac{V_b}{R_2}$$

$$I_3 = C\frac{d}{dt}[V_{out3} - V_b]$$

from eqn (1)

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0$$

$$V_b = -\frac{R_2}{R_1} \cdot V_e$$

$\therefore$ adding eqn (1) & (2) we have

$$I_1 + I_3 = 0$$

$$\therefore \quad \frac{V_e}{R_1} + C\frac{d[V_{out1} - V_b]}{dt} = 0$$

$$\therefore \quad C\frac{d}{dt}[V_{out1} - V_b] = -\frac{V_e}{R_1}$$

$$d[V_{out1} - V_b] = -\frac{1}{CR_1}.V_e\,dt$$

~~taking the~~ Integrating Both sides

$$\int d[V_{out1} - V_b] = \int -\frac{1}{CR_1}.V_e\,dt$$

$$\therefore \quad V_{out1} - V_b = -\frac{1}{CR_1}\int V_e\,dt$$

Considering initial Conditions

$$V_{out1} - V_b = -\frac{1}{CR_1}\left(\int_0^t V_e.dt + V_{(0)}\right)$$

$$\therefore \quad V_{out1} = V_b - \frac{1}{CR_1}\left(\int_0^t V_e.dt + V_{(0)}\right)$$

$$V_{out1} = -\frac{R_2}{R_1}.V_e - \frac{1}{CR_1}\int_0^t V_e\,dt + V_{(0)}$$

$$\therefore \quad V_{out1} = -\left[\frac{R_2}{R_1}V_e + \frac{R_2}{R_1}.\frac{1}{R_2C}\int_0^t V_e\,dt + V_{(0)}\right]$$

$$\therefore \quad V_{out1} - V_b = -\frac{1}{CR_1}\int V_e\,dt$$

$$V_{out1} = V_b - \frac{1}{CR_1}\int V_e\,dt$$

$$V_{out1} = -\frac{R_2}{R_1}V_e - \frac{1}{CR_1}\int V_e\,dt$$

$$V_{out1} = -\left[\frac{R_2}{R_1}V_e + \frac{1}{CR_1}\int V_e\,dt\right]$$

After inverting

$$V_{out} = -V_{out1}$$

$$\therefore \quad V_{out} = \left[\frac{R_2}{R_1}V_e + \frac{R_2}{R_1}.\frac{1}{R_2C}\int V_e\,dt\right]$$

$\therefore$ take initial Conditions into consideration

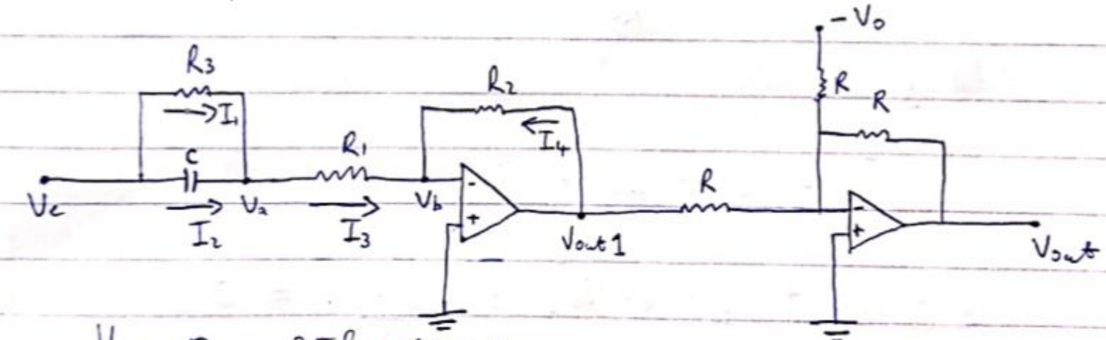
$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot \frac{1}{R_2 C} \int_0^t V_e \, dt + V_{(0)}$$

$$G_P = \frac{R_2}{R_1} \quad \text{- - - - - - Proportional Gain}$$

$$G_I = \frac{1}{R_2 C} \quad \text{- - - - - - Integral Gain}$$

$$\therefore \quad V_{out} = G_P V_e + G_P G_I \int_0^t V_e \, dt + V_{(0)}$$

Proportional - Derivative mode Controller (PD Controller)



$V_b = 0 \qquad 2\pi f_{max} RC = 0.1$

Using KCL (Kirchoff's Current law)

$$I_1 + I_2 = I_3$$

$$I_1 + I_2 - I_3 = 0 \quad \text{- - - - - - (1)}$$

$$I_4 + I_3 = 0 \quad \text{- - - - - - (2)}$$

effective Resistance, $\bar{R} = \dfrac{R_1 R_3}{R_1 + R_3}$

$$I_1 = \frac{V_e - V_a}{R_3}$$

$$I_2 = C\frac{d}{dt}[V_e - V_a]$$

$$I_3 = \frac{V_a}{R_1}$$

$$I_4 = \frac{V_{out1}}{R_2}$$

from equation (1)

$$\therefore \frac{V_e - V_a}{R_3} + C\frac{d}{dt}[V_e - V_a] - \frac{V_a}{R_1} = 0$$

from equation (2)

$$\frac{V_{out1}}{R_2} + \frac{V_a}{R_1} = 0$$

$$\therefore V_a = -\frac{R_1}{R_2} V_{out1}$$

adding eqn (1) & (2)

$$I_1 + I_2 + I_4 = 0$$

$$\therefore \frac{V_e - V_a}{R_3} + C\frac{d}{dt}[V_e - V_a] + \frac{V_{out1}}{R_2} = 0$$

taking the Laplace transform

$$\frac{V_e(s) - V_a(s)}{R_3} + SC[V_e(s) - V_a(s)] + \frac{V_{out1}(s)}{R_2} = 0$$

$$\therefore \frac{V_{out1}(s)}{R_2} - \frac{V_a(s)}{R_3} - SC(V_a(s)) = -\frac{V_e(s)}{R_3} \cancel{+} - SC V_e(s)$$

$$\frac{V_{out1}(s)}{R_2} - \frac{1}{R_3}\left(-\frac{R_1}{R_2} V_{out1}(s)\right) - SC\left(-\frac{R_1}{R_2} V_{out1}(s)\right) = -\frac{V_e(s)}{R_3} - SC V_e(s)$$

$$\frac{V_{out1}(s)}{R_2} + \frac{R_1 V_{out1}(s)}{R_3 R_2} + \frac{SCR_1 V_{out1}(s)}{R_2} = -\frac{V_e(s)}{R_3} - SCV_e(s)$$

$$V_{out1}(s)\left[\frac{1}{R_2} + \frac{R_1}{R_3 R_2} + \frac{SCR_1}{R_2}\right] = -\frac{V_e(s)}{R_3} - SCV_e(s)$$

$$V_{out1}(s)\left[\frac{R_3 + R_1 + SCR_1R_3}{R_2 R_3}\right] = -\frac{V_e(s)}{R_3} - SCV_e(s)$$

$$V_{out1}(s) = \left[\frac{R_2 R_3}{R_1 + R_3 + SCR_1R_3}\right]\left[-\frac{V_e(s)}{R_3} - SCV_e(s)\right]$$

$$\therefore V_{out1}(s) = \left[\frac{\frac{R_2R_3}{R_1+R_3}}{\frac{R_1+R_3}{R_1+R_3} + \frac{SCR_1R_3}{R_1+R_3}}\right]\left[-\frac{V_e(s)}{R_3} - SCV_e(s)\right]$$

$$V_{out1}(s) = \left[\frac{\frac{R_2 R_3}{R_1+R_3}}{1 + SCR}\right]\left[-\frac{V_e(s)}{R_3} - SCV_e(s)\right]$$

~~$V_{out1}(s) =$~~ if $SCR << 1$

$$V_{out1}(s) = \left[\frac{R_2 R_3}{R_1+R_3}\right]\left[-\frac{V_e(s)}{R_3} - SCV_e(s)\right]$$

$$V_{out1}(s) = -\left[\frac{R_2}{R_1+R_3}\right]V_e(s) - \left[\frac{R_2R_3}{R_1+R_3}\right]SCV_e(s)$$

taking the inverse laplace Transform

$$V_{out1} = -\left[\frac{R_2}{R_1+R_3}\right]V_e - \left[\frac{R_2R_3}{R_1+R_3}\right]C\frac{dV_e}{dt}$$

from the inverter circuit

$$V_{out1} = -\left(V_{out} + (-V_0)\right)$$

$$\therefore \quad V_{out1} = -V_{out} + V_o$$

$\cancel{V_{out}}$

$$V_{out1} = -\left[\frac{R_2}{R_1 + R_3}\right] V_e - \left[\frac{R_2 R_3}{R_1 + R_3}\right] C \frac{dV_e}{dt}$$

$$\therefore \quad -V_{out} + V_o = -\left[\frac{R_2}{R_1 + R_3}\right] V_e - \left[\frac{R_2 R_3}{R_1 + R_3}\right] C \frac{dV_e}{dt}$$

$$-V_{out} = -\left[\frac{R_2}{R_1 + R_3}\right] V_e - \left[\frac{R_2 R_3}{R_1 + R_3}\right] C \frac{dV_e}{dt} - V_o$$

$$\therefore \quad V_{out} = \left[\frac{R_2}{R_1 + R_3}\right] V_e + \left[\frac{R_2}{R_1 + R_3}\right] R_3 C \frac{dV_e}{dt} + V_o$$

where

$$G_P = \frac{R_2}{R_1 + R_3} \quad \text{- - - - Proportional Gain}$$

$$G_D = R_3 C \quad \text{- - - - - Derivative Gain}$$

$$\therefore \quad V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_o$$