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17/ENG04/078
Elect/Elect

- PI controller

$V_a = 0$

$I_1 + I_2 = 0$ —— (1)

$I_3 - I_2 = 0$ —— (2)

Current through the Capacitor

$$I_c = \frac{C\,dV_c}{dt}$$

$$I_1 = \frac{V_c - V_a}{R_1} \quad (V_a = 0)$$

$$= \frac{V_c}{R_1}$$

$$I_2 = \frac{V_b - V_a}{R_2} \quad (V_a = 0)$$

$$= \frac{V_b}{R_2}$$

$$I_3 = \frac{C\,d(V_{out1} - V_b)}{dt}$$

Sub into eqn (1) and eqn (2)

$$\frac{V_c}{R_1} + \frac{V_b}{R_2} = 0 \text{ —— (1)}$$

$$C\frac{d(V_{out1} - V_b)}{dt} - \frac{V_b}{R_2} = 0 \text{ —— (2)}$$

From eqn (1)

$$\frac{V_b}{R_2} = \frac{-V_c}{R_1}$$

$$V_b = -\frac{R_2}{R_1} \cdot V_c$$

taking laplace transform of eqn (2)

$$SC(V_{out1}(s) - V_b(s)) - \frac{V_b(s)}{R_2} = 0$$

$$SC\,V_{out1}(s) = SC\,V_b(s) + \frac{V_b(s)}{R_2}$$

$$SC\,V_{out1}(s) = V_b(s)\left(SC + \frac{1}{R_2}\right)$$

recall, $V_b = \frac{-R_2}{R_1} V_c$

$$SC\,V_{out1}(s) = \frac{-R_2}{R_1} V_c(s)\left(SC + \frac{R_1}{R_2}\right)$$

$$V_{out1}(s) = \frac{-R_2}{ScR_1} V_e(s) \left(Sc + \frac{1}{R_2} \right)$$

$$V_{out1}(s) = \frac{-R_2}{R_1} V_e(s) - \frac{R_2}{R_1} \frac{1}{ScR_2} V_e(S)$$

From the inverting current

$$V_{out1} = -V_{out}$$

$$\therefore V_{out}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1} \frac{1}{ScR_2} V_e(s)$$

$$V_{out}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1} \cdot \frac{1}{ScR_2} V_e(s)$$

Taking inverse laplace

$$V_{out1} = \cancel{V_{out}} \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1} \frac{1}{R_2} \int_0^t V_e(t)\,dt + V(0)$$

$$\left(\text{where } \frac{1}{s} = \int_0^t dt + K\right)$$

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e\,dt + V(0)$$

Where $G_P = \frac{R_2}{R_1}$

$$G_I = \frac{1}{R_2 C_1}$$

- PD controller

$$I_1 + I_2 = I_3 \quad \text{--- (1)}$$

$$I_3 + I_4 = 0 \quad \text{--- (2)}$$

$$I_1 = \frac{V_e - V_a}{R_3}$$

$$I_2 = \frac{c\,d(V_e - V_a)}{dt}$$

$$I_3 = \frac{V_a - V_b}{R_1} \quad (V_b = 0)$$

$$= \frac{V_a}{R_1}$$

$$I_4 = \frac{V_{out1} - V_b}{R_2} \quad (V_b = 0)$$

$$= \frac{V_{out1}}{R_2}$$

$$R = \frac{R_1 R_3}{R_1 + R_3} \leftarrow \text{effective resistance}$$

Sub into eq.n (1) and eq.n (2)

$$\frac{V_e - V_a}{R_3} + \frac{Cd}{dt}(V_e - V_a) = \frac{V_a}{R_1} \quad \text{---} *$$

$$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0 \quad \text{---} **$$

From eqn **

$$\frac{V_a}{R_1} = \frac{-V_{out1}}{R_2}$$

$$V_a = \frac{-R_1}{R_2} V_{out1} //$$

rearrange eqn *

$$\frac{V_c - V_a}{R_3} + C\frac{d}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$$

Taking laplace transform

$$\frac{V_e(s) - V_a(s)}{R_3} + SC(V_e(s) - V_a(s)) - \frac{V_a(s)}{R_1} = 0$$

(initial conductions go to zero)

$$\frac{V_e(s)}{R_3} + SCV_e(s) = \frac{V_a(s)}{R_1} + \frac{V_a(s)}{R_3} + SCV_a(s)$$

$$V_e(s)\left(\frac{1}{R_3} + SC\right) = V_a(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + SC\right)$$

recall, $V_a = -\frac{R_1}{R_2} V_{out1}$

$$V_e(s)\left(\frac{1}{R_3} + SC\right) = \frac{-R_1}{R_2} V_{out1}(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + SC\right)$$

Taking

$$V_e(s)\left(\frac{1 + R_3 SC}{\cancel{R_3}}\right) = \frac{-\cancel{R_1}}{R_2} V_{out1}(s)\left(\frac{R_3 + R_1 + SCR_1R_3}{\cancel{R_1R_3}}\right)$$

$$V_e(s)(1 + SCR_3) = \frac{-V_{out1}(s)}{R_2}(R_3 + R_1 + SCR_1R_3)$$

$$-V_{out1}(s) = \frac{V_e(s)(1 + SCR_3)R_2}{(R_1 + R_3 + SCR_1R_3)}$$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + SCR_2R_3)}{(R_1 + R_3 + SCR_1R_3)}$$

Dividing num and denum by $R_1 + R_3$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + SCR_2R_3)/R_1 + R_3}{\frac{R_1+R_3}{R_1+R_3} + \frac{SCR_1R_3}{R_1+R_3}}$$

recall, $R = \frac{R_1R_3}{R_1+R_3}$

$$-V_{out1}(s) = \frac{V_e(s)\left(R_2 + SC\,R_2R_3/R_1+R_3\right)}{1 + SCR}$$

If $SCR << 1$

$$\cancel{-V_{out1}(s) = \frac{V_e(s)\left(R_2 + SC R_2R_3\right)/R_1+R_3}{1 + SCR}}$$

$$-V_{out1}(s) = \frac{V_e(s)\left(R_2 + SC(R_2R_3)\right)}{R_1 + R_3}$$

From the inverting circuit

$V_{out1} = -V_{out} + V_e$

$$-\left(-V_{out}(s) + V_{(0)}\right) = \frac{V_e\left(R_2 + SCR_2R_3\right)}{R_1+R_3}$$

$$\cancel{V_{out}}\; V_{out}(s) - V_{(s)} = \frac{V_e(s)R_2}{R_1+R_3} + \frac{SC\,R_2R_3\,V_e(s)}{R_1+R_3}$$

$$V_{out}(s) = \frac{R_2}{R_1+R_3}V_e(s) + \frac{R_2}{R_1+R_3}R_3(S\,V_e(s) + V_{(s)})$$

Taking inverse laplace

$$V_{out} = \frac{R_2}{R_1+R_3}V_e + \frac{R_2}{R_1+R_3}R_3 C\frac{dV_e}{dt} + V_0$$

$$V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_0 \,//$$

where; $G_P = \frac{R_2}{R_1+R_3}$

$G_D = R_3 C$