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MECHATRONICS ENGR.

1) Root Locus Technique

In the Routh hurwitz stability criterion, we can know whether the closed loop poles are in the left half of the 's' plane or the right half of the 's' plane or on an imaginary axis. But we cant find out the nature of the control system. This brings about the root locus technique which is used to overcome this limitation.

Root locus is a graphical presentation of the closed loop poles as a system parameter is varied. It is used to describe qualitatively the performance of a system as various parameters are changed e.g varying gain upon percent overshoot, settling time and peak time can be vividly displayed.

The root locus is the locus of the roots of the characteristic equation by varying system gain K from zero to infinity.

We know that, the characteristic equation of the closed loop control system is:

$$1 + G(s)H(s) = 0$$

we can represent $G(s)H(s)$ as $G(s)H(s) = K\frac{N(s)}{D(s)}$

where K – multiplying factor

N(s) – Numerator term having (factored) nth order polynomial of 's'

D(s) – Denominator term having (factored) mth order polynomial of 's'

Substitute $G(s)H(s)$ value in the characteristic equation

$$1 + K\frac{N(s)}{D(s)} = 0$$

$$\Rightarrow D(s) + KN(s) = 0$$

Case 1; K = 0

If $K = 0$ then, $D(s) = 0$

That means, the closed loop poles are equal to open loop poles when K is zero

Case 2; K = ∞

Re-write the above characteristic equation as:

$$K\left(\frac{1}{K} + \frac{N(s)}{D(s)}\right) = 0 \Rightarrow \frac{1}{K} + \frac{N(s)}{D(s)} = 0$$

Substitute, $K = \infty$ in the above equation.

$$\frac{1}{\infty} + \frac{N(s)}{D(s)} = 0 \Rightarrow \frac{N(s)}{D(s)} = 0 \Rightarrow N(s)$$

If $K = \infty$, Then $N(s) = 0$ · It means the closed loop poles are equal to the open loop zeros when K is infinity.

From the above two cases, we can conclude that the root locus branches start at open loop poles and end at open loop zeros.

Angle condition and Magnitude condition:

The points on the root locus branches satisfy the angle condition. so, the angle condition is used to know whether the point exist on root locus branch or not. we can find the value of K for the points on the root locus branches by using magnitude condition for the points, and this satisfies the angle condition.

characteristic equation of closed loop control system is:

$$1 + G(s) H(s) = 0$$

$$\Rightarrow G(s) H(s) = -1 + j0$$

The phase angle of $G(s) H(s)$ is $\Rightarrow \angle G(s) H(s) = \tan^{-1}\left(\frac{0}{-1}\right)$

$$= (2n+1)\pi$$

The angle condition is the point at which the angle of the open loop transfer function is an odd multiple of $180°$

magnitude of $G(s) H(s)$ is $|G(s) H(s)| = \sqrt{(-1)^2 + 0^2} = 1$

The magnitude condition is that the point (which satisfied the angle condition) at which the magnitude of the open loop transfer function is one.

2 a) When the entire row is zero on the Routh table the following steps are taken.

i) Write the auxiliary equation, A(s) of the row which is just above the row of zeros

ii) Differentiate the auxiliary eqn A(s) with respect to s. fill the row of zeros with these coefficients.

For example: lets find the stability of a control system having characteristic equation

$$s^5 + 3s^4 + s^3 + 3s^2 + s + 3 = 0$$

Forming the routh table;

s^5	1	1	1
s^4	~~3~~ 1	~~3~~ 1	~~3~~ 1
s^3	$\frac{(1\times1)-(1\times1)}{1} = 0$	$\frac{(1\times1)-(1\times1)}{1} = 0$	
s^2			
s^1			
s^0			

→ dividing through by 3

The row s^4 elements have the common factor 3, so all these elements are divided by 3.

Since all the elements of row s^3 are zero, we write the auxiliary equation $A(s)$ of the row s^4

$$A(s) = s^4 + s^2 + 1$$

Differentiating the above equation with respect to s:

$$\frac{dA(s)}{ds} = 4s^3 + 2s$$

then placing these coefficients in row s^3.

s^5	1	1	1
s^4	1	1	1
s^3	$\cancel{4}\ 2$	$\cancel{2}\ 1$	
s^2	$\frac{(2\times1)-(1\times1)}{2} = 0.5$	$\frac{(2\times1)-(0\times1)}{2} = 1$	
s^1	$\frac{(0.5\times1)-(1\times2)}{0.5} = -3$		
s^0	1		

→ dividing through by 2

Now verifying the condition for Routh-Hurwitz stability:
There are two (2) sign changes in the first column of the routh table therefore the system is unstable.

(2) b) determine the poles on the $j\omega$ axis:

An entire row of zeros will appear on a routh table when a purely even polynomial or purely odd polynomial is a factor of the original polynomial.
e.g $s^4 + 5s^2 + 7$ is an even polynomial; it has only even powers of s.
Even polynomials only have roots that are symmetrical about the origin.

It is this even polynomial that causes the row zeros to appear. Thus the row of zeros tells us of the existence of an even polynomial whose roots are symmetric about the origin. Some of these roots could be on the $j\omega$ - axis. On the other hand, since $j\omega$ roots are symmetric about the origin. If we do not have a row of zeros, we cannot possibly have $j\omega$ roots.

Another characteristic of the Routh table for the case in question is that the row previous to the row of zeros contains the even polynomial that is a factor of the original polynomial. Finally, everything from the row containing the even polynomial down to the end of the Routh table is a test of only the even polynomial.

Using the example in 2a, we draw out a table summarizing the pole locations

Location	Even (Fourth order)	Other (First order)	Total (Fifth order)
Right half-plane	2	0	2
Left half-plane	2	0	2
$j\omega$	0	1	1

The remaining roots of the total polynomial is evaluated at the S^5 row where we notice no sign change signifying the root is in the $j\omega$ axis (no root in the right plane means no root in left plane because of symmetry).

The even polynomial is then tested from the row above the row of zeros i.e. S^4 to S^0 which shows two sign changes i.e two in the right half plane and two in the left.

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