from eq **

$$\left[V_a = -\frac{R_1}{R_2} V_{out1} \right]$$

rearranging eq *

$$\frac{V_e - V_a}{R_3} + Cd\frac{}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$$

taking laplace transform

$$\frac{V_e(s) - V_a(s)}{R_3} + Sc(V_e(s) - V_a(s)) - \frac{V_a(s)}{R_1} = 0$$

Initial conditions going to zero

$$V_e(s)\left(\frac{1}{R_3} + Sc\right) = V_a(s)\left(\frac{1}{R_1} + \frac{1}{R_2} + Sc\right)$$

recall, $V_a = -\frac{R_1}{R_2} V_{out1}$

$$V_c(s)\left(\frac{1}{R_3} + Sc\right) = -\frac{R_1}{R_2} V_{out1}\left(\frac{1}{R_1} + \frac{1}{R_3} + Sc\right)$$

taking Lcm

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \quad \text{— (1)}$$

$$C\frac{d}{dt}(V_{out1} - V_b) - \frac{V_b}{R_2} = 0 \quad \text{— (2)}$$

from eq (1)

$$\frac{V_b}{R_2} = \frac{V_e}{R_1}$$

$$\boxed{V_b = -\frac{R_2 V_e}{R_1}}$$

taking laplace transform of eq(2)

$$sC(V_{out1}(s) - V_b(s)) - \frac{V_b(s)}{R_2} = 0$$

$$sCV_{out1}(s) = sC(V_b(s) + \frac{V_b(s)}{R_2}$$

$$sCV_{out1}(s) = V_b(s)\left(sC + \frac{1}{R_2}\right)$$

To call: $V_b = -\frac{R_2}{R_1} V_e$

$$sCV_{out1}(s) = -\frac{R_2}{R_1} V_e(s)\left(sC + \frac{1}{R_2}\right)$$

From the Inverting circuit

$$V_{out1} - V_{out} + V_0$$

$$\therefore -(-V_{out(s)} + V_{(0)}) = V_e \frac{(R_2 + S_c R_2 R_3)}{R_1 + R_3}$$

$$V_{out(s)} - V_{(0)} = \frac{V_{e(s)} R_2}{R_1 + R_3} + \frac{S C R_2 R_3 \; V_{e(s)}}{R_1 + R_3}$$

$$V_{out(s)} = \frac{R_2}{R_1 + R_3} V_{e(s)} + \frac{R_2}{R_1 + R_3} R_2 C_s V_{e(s)} + V_0$$

taking Inverse Laplace

$$V_{out} = \frac{R_2}{R_1 + R_3} V_e + \frac{R_2}{R_1 + R_3} R C \frac{dV_e}{dt} + V_0$$

$$\boxed{V_{out} = G_p V_e + G_p G_D \frac{dV_e}{dt} + V_0}$$

where $G_p = \frac{R_2}{R_1 + R_3}$

$G_D = R_3 C$

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16/ENG04/060
ELECT/ELECT

QUESTION 1

PI Controller

$V_a = 0$

$I_1 + I_2 = 0 \quad \longrightarrow (1)$

$I_3 - I_2 = 0 \quad \longrightarrow (2)$

Current through the capacitor

$$I_c = \frac{C\,dV_c}{dt}$$

$$I_1 = \frac{V_e - V_a}{R_1} \quad (V_a = 0)$$

$$I_2 = \frac{V_o - V_a}{R_2} \quad (V_a = 0)$$

$$\pm \frac{V_b}{R_2} = V_b / R_2$$

$$I_3 = C\,d/dt\,(V_{out} - V_b)$$

Sub into eq(1) & eq(2)

QUESTION 2

PD controller

$I_1 + I_2 = I_3$ — (1)

$I_3 + I_4 =$ — (2)

$I_1 = \frac{V_e - V_a}{R_3}$

$I_2 = C\,d/dt\,(V_e - V_a)$

$I_3 = \frac{V_a - V_b}{R_1}$ $(V_b = 0)$

$= V_a / R_1$

I_4 $\frac{V_{out1} - V_b}{R_2}$ $(V_b = 0)$

$= \frac{V_{out1}}{R_2}$

$R = \frac{R_1 R_3}{R_1 + R_3}$ —— effective resistance

Sub into eq(i) & eq(2)

$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}(V_e - V_a) = V_a / R_1$ — *

$V_b / R_1 + \frac{V_{out1}}{R_2} = 0$ — **