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17/ENG04/087
Elect / Elect
EEE 561



Derive the analysis for the output Voltage in using Operational Amplifier for :-

1.) Proportional Integral controller Mode
2.) Proportional Derivative Controller Mode.

1.) Proportional Integral controller Mode.

$$V_a = 0$$

$$I_1 + I_2 = 0 \quad \ldots\ldots\ldots\ldots 1$$

$$I_3 - I_2 = 0 \quad \ldots\ldots\ldots\ldots 2$$

Current through the capacitor

$$I_c = C\frac{dV_c}{dt}$$

$$I_1 = \frac{V_e - V_a}{R_1} \quad (V_a = 0)$$

$$= V_e / R_1$$

$$I_2 = \frac{V_b - V_a}{R_2} \quad \text{where } V_a = 0$$

$$\therefore = V_b / R_2$$

$$I_3 = C\,d/dt\,(V_{out_1} - V_b)$$

Sub into equation 1 and equation 2

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \quad \text{.......... 3}$$

$$C \frac{d}{dt}(V_{out1} - V_b) - \frac{V_b}{R_2} = 0 \quad \text{.......... 4}$$

from equation 3

$$\frac{V_b}{R_2} = -\frac{V_e}{R_1}$$

$$V_b = -\frac{R_2}{R_1} V_e$$

Taking laplace transform of equation 4

$$sC\,(V_{out1}(s) - V_b(s)) - \frac{V_b(s)}{R_2} = 0$$

$$sC\,V_{out1}(s) = sC\,V_b(s) + \frac{V_b(s)}{R_2}$$

$$sC\,V_{out1}(s) = V_b(s)\left[sC + \frac{1}{R_2}\right]$$

Recall; $V_b = -\frac{R_2}{R_1} V_e$

$$sC\,V_{out1}(s) = -\frac{R_2}{R_1} V_e(s)\left[sC + \frac{R_1}{R_2}\right]$$

$$V_{out1}(s) = -\frac{R_2}{sCR_1} V_e(s)\left[sC + \frac{1}{R_2}\right]$$

$$V_{out_1(s)} = -\frac{R_2}{R_1} V_{e(s)} - \frac{R_2}{R_1} \frac{1}{SCR_2} V_{e(s)}$$

from the Inverting Circuit

$$V_{out_1} = -V_{out}$$

$$\therefore V_{out(s)} = -\left[-\frac{R_2}{R_1} V_{e(s)} - \frac{R_2}{R_1} \frac{1}{SCR_2} V_{e(s)}\right]$$

$$V_{out(s)} = \frac{R_2}{R_1} V_{e(s)} + \frac{R_2}{R_1} \frac{1}{SCR_2} V_{e(s)}$$

taking the inverse Laplace

$$V_{out} = \frac{R_2}{R_1} V_{e(s)} + \frac{R_2}{R_1} \frac{1}{R_2} \int_0^t V_{e(t)}\,dt + V_{(0)}$$

$$\left\{ \text{where } \frac{1}{s} = \int_0^t dt + k_1 \right\}$$

$$\therefore V_{out} = G_P V_e + G_P G_I \int_0^t V_e\,dt + V_{(0)}$$

where $G_P = \frac{R_2}{R_1}$

$$G_I = \frac{1}{R_2 C}$$

2.) Proportional Derivative Controller Mode

$$V_b = 0$$

$$I_1 + I_2 = I_3 \quad \text{.............} \quad 1$$

$$I_3 + I_4 = 0 \quad \text{.............} \quad 2$$

$$I_1 = \frac{V_e - V_a}{R_3}$$

$$I_2 = C\,d/dt\,(V_e - V_a)$$

$$I_3 = \frac{V_a - V_b}{R_1} \quad \text{where } V_b = 0$$

$$= V_a / R_1$$

$$I_4 = \frac{V_{out_1} - V_b}{R_2} \quad \text{Where } V_b = 0$$

$$= V_{out_1} / R_2$$

$$R = \frac{R_1 R_3}{R_1 + R_3} \quad \text{....... is known as Effective Resistance.}$$

substituting into equation 1 and equation 2

$$\frac{V_e - V_a}{R_3} + C\,d/dt\,(V_e - V_a) = V_a / R_1 \quad \text{.................} \quad 3$$

$$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0 \quad \text{------------------------} \quad 4$$

from equation 4

$$\frac{V_a}{R_1} = \frac{-V_{out1}}{R_2}$$

$$V_a = -\frac{R_1}{R_2} V_{out1}$$

Rearranging equation 3

$$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$$

Taking laplace transform of the above.

$$\frac{V_{e(s)} - V_{a(s)}}{R_3} + sC(V_{e(s)} - V_{a(s)}) - \frac{V_{a(s)}}{R_1} = 0$$

(Assume Initial conditions go to Zero)

$$\frac{V_{e(s)}}{R_3} + SCV_{e(s)} = \frac{V_{a(s)}}{R_1} + \frac{V_{a(s)}}{R_3} + sCV_{a(s)}$$

$$V_{e(s)}\left[\frac{1}{R_3} + SC\right] = V_{a(s)}\left[\frac{1}{R_1} + \frac{1}{R_3} + sC\right]$$

Recall that, $V_a = -\frac{R_1}{R_2} V_{out1}$

$$V_{e(s)} \left[ \frac{1}{R_3} + sC \right] = -\frac{R_1}{R_2} V_{out1(s)} \left[ \frac{1}{R_1} + \frac{1}{R_3} + sC \right]$$

Taking LCM of the above

$$V_{e(s)} \left[ \frac{1 + R_3 sC}{R_3} \right] = -\frac{R_1}{R_2} V_{out1(s)} \left[ \frac{R_3 + R_1 + sCR_1R_3}{R_1R_3} \right]$$

$$V_{e(s)} (1 + sCR_3) = -\frac{V_{out1(s)}}{R_2} (R_3 + R_1 + sCR_1R_3)$$

$$-V_{out1(s)} = \frac{V_{e(s)} (1 + sCR_3) R_2}{(R_1 + R_3 + sCR_1R_3)}$$

$$-V_{out1(s)} = \frac{V_{e(s)} (R_2 + sCR_2R_3)}{R_1 + R_3 + sCR_1R_3)}$$

Dividing Both the numerator and denominator by $R_1 + R_3$

$$-V_{out1(s)} = \frac{V_{e(s)} (R_2 + sCR_2R_3) \div R_1 + R_3}{\frac{R_1 + R_3}{R_1 + R_3} + \frac{sCR_1R_3}{R_1 + R_3}}$$

Recall, $R = \frac{R_1 R_3}{R_1 + R_3}$

$$-V_{out1(s)} = \frac{V_{e(s)} (R_2 + sCR_2R_3) / R_1 + R_3}{1 + sCR}$$

if $sCR << 1$

$$-V_{out1}(s) = \frac{V_e(s)\,(R_2 + s(CR_2R_3))}{R_1 + R_3}$$

from inverting circuit

$$V_{out_1} = -V_{out} + V_o$$

$$\therefore\ -(-V_{out}(s) + V(0)) = \frac{V_e\,(R_2 + s\,CR_2R_3)}{R_1 + R_3}$$

$$V_{out}(s) - V(0) = \frac{V_e(s)\,R_2}{R_1 + R_3} + \frac{sCR_2R_3}{R_1 + R_3}\,V_e(s)$$

$$V_{out}(s) = \frac{R_2}{R_1 + R_3}\,V_e(s) + \frac{R_2}{R_1 + R_3}\,R_3\,sC\,V_e(s) + V(0)$$

Taking Inverse laplace

$$V_{out} = \frac{R_2}{R_1 + R_3}\,V_e + \frac{R_2}{R_1 + R_3}\,R_3\,C\,dV_e/dt + V_o$$

$$V_{out} = G_P V_e + G_P G_D\, dV_e/dt + V_o$$

where $G_P = R_2/R_1 + R_3$

$G_D = R_3 C$