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Elect

PI controller

$V_a = 0$

$I_1 + I_2 = 0$ —— (1)

$I_3 - I_2 = 0$ —— (2)

Current through the capacitor

$$I_c = \frac{c\,dV_c}{dt}$$

$$I_1 = \frac{V_e - V_a}{R_1} \quad (V_a = 0)$$

$$= \frac{V_e}{R_1}$$

then $I_2 = \frac{V_b - V_a}{R_2}$ $(V_a = 0)$

$$= \frac{V_b}{R_2}$$

$$I_3 = \frac{cd}{dt}(V_{out} - V_b)$$

Sub into eqn(1) & eqn 2

$$-V_{out1}(s) = \frac{V_e(s)\left(R_2 + SCR_3R_2\right)/R_1+R_3}{1+SCR}$$

If $SCR << 1$

$$-V_{out1}(s) = \frac{V_e(s)\left(R_2 + SCR_2R_3\right)}{R_1+R_3}$$

from the inverting circuit

$$V_{out1} = -V_{out} + V_e$$

$$\therefore -\left(-V_{out}(s) + V(0)\right) = \frac{V_e\left(R_2 + SCR_2R_3\right)}{R_1+R_3}$$

$$V_{out}(s) - V(0) = \frac{V_e(s)\,R_2}{R_1+R_3} + \frac{SCR_2R_3}{R_1+R_3}V_e(s)$$

$$V_{out}(s) = \frac{R_2}{R_1+R_3}V_e(s) + \frac{R_2}{R_1+R_3}R_3C\left(sV_e(s)\right) + V_{(0)}$$

taking inverse laplace

$$V_{out} = \frac{R_2}{R_1+R_3}V_e + \frac{R_2}{R_1+R_3}R_3C\frac{dV_e}{dt} + V_0$$

$$V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_0$$

where, $G_P = \frac{R_2}{R_1+R_3}$

$G_D = R_3C$

recall, $V_s = \frac{-R_1}{R_2} V_{out1}$

$$V_e(s)\left(\frac{1}{R_3}+sC\right) = \frac{-R_1}{R^2} V_{out1}(s)\left(\frac{1}{R_1}+\frac{1}{R_3}+sC\right)$$

taking the LCM

$$V_e(s)\left(\frac{1+R_3 sC}{\cancel{R_3}}\right) = \frac{-\cancel{R_1}}{R_2} V_{out1}(s)\left(\frac{R_3+R_1+sCR_1R_3}{\cancel{R_1}\cancel{R_3}}\right)$$

$$V_e(s)(1+sCR_3) = \frac{-V_{out1}(s)}{R_2}(R_3+R_1+sCR_1R_3)$$

$$-V_{out1}(s) = \frac{V_e(s)(1+sCR_3)R_2}{(R_1+R_3+sCR_1R_3)}$$

$$-V_{out1}(s) = \frac{V_e(s)(R_2+sCR_2R_3)}{(R_1+R_3+sCR_1R_3)}$$

dividing ~~num~~ both numerator and denominator by R_1+R_3

$$-V_{out1}(s) = \frac{V_e(s)(R_2+sCR_2R_3)/R_1+R_3}{\frac{R_1+R_3}{R_1+R_3}+\frac{sCR_1R_3}{R_1+R_3}}$$

recall, $R = \frac{R_1R_3}{R_1+R_3}$

$$\frac{V_e - V_a}{R_3} + \frac{C\,d(V_e - V_a)}{dt} = \frac{V_a}{R_1} \quad --- \; i$$

$$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0 \quad --- \; ii$$

from eqn ii

$$\frac{V_a}{R_1} = \frac{-V_{out1}}{R_2}$$

$$V_a = \frac{-R_1}{R_2} V_{out1}$$

~~Rearrang~~ eqn (i)

$$\frac{V_e - V_a}{R_3} + \frac{C\,d}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$$

taking ~~the~~ Laplace transform

$$\frac{V_e(s) - V_a(s)}{R_3} + sC\,(V_e(s) - V_a(s)) - \frac{V_a(s)}{R_1} = 0$$

(initial conditions to zero)

$$\frac{V_e(s)}{R_3} + sC\,V_e(s) = \frac{V_a(s)}{R_1} + \frac{V_a(s)}{R_3} + sC\,V_a(s)$$

$$V_e(s)\left(\frac{1}{R_3} + sC\right) = V_a(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right)$$

PD Controller

$I_1 + I_2 = I_3$ —①

$I_3 + I_4 = 0$ ②

$I_1 = \frac{V_e - V_a}{R_3}$

$I_2 = \frac{c\,dv}{dt} = \frac{c\,d(V_e - V_a)}{dt}$

$I_3 = \frac{V_a - V_b}{R_1}$ but ($V_b = 0$)

$= \frac{V_a}{R_1}$

$I_4 = \frac{V_{out1} - V_b}{R_2}$; $V_b = 0$

$= \frac{V_{out1}}{R_2}$

$R = \frac{R_1 R_3}{R_1 + R_3}$ ← effective resistance

Sub into eqn (i) & eqn (2)

recall; $V_b = \frac{-R_2}{R_1} V_e$

$$S_C V_{out1}(s) = \frac{-R_2}{R_1} V_e(s) \left(SC + \frac{1}{R_2} \right)$$

$$V_{out1}(s) = \frac{-R_2}{SCR_1} V_e(s) \left(SC + \frac{1}{R_2} \right)$$

$$V_{out1}(s) = \frac{-R_2}{R_1} V_e(s) - \frac{R_2}{R_1} \frac{1}{SCR_2} V_e(s)$$

from the inverting current:

$V_{out1} = -V_{out}$

$$\therefore V_{out}(s) = -\left(\frac{-R_2}{R_1} V_e(s) - \frac{R_2}{R_1} \frac{1}{SCR_2} V_e(s) \right)$$

$$V_{out}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1} \frac{1}{SCR_2} V_e(s)$$

taking inverse laplace:

$$V_{out} = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1} \frac{1}{R_2} \int_0^t V_e(t)\,dt + V(0)$$

$$\left(\text{where } \frac{1}{s} = \int_0^t dt + C \right)$$

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e\,dt + V_{(0)}$$

where $G_P = \frac{R_2}{R_1}$

$G_I = \frac{1}{R_2 C}$

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \quad -- (i)$$

$$C\frac{d}{dt}(V_{out1} - V_b) - \frac{V_b}{R_2} = 0 \quad — (2)$$

from eqn (1)

$$\frac{V_b}{R_2} = \frac{-V_e}{R_1}$$

$$\boxed{V_b = \frac{-R_2 V_e}{R_1}}$$

transform eqn 2

$$SC(V_{out1}(s) - V_b(s)) - \frac{V_b(s)}{R_2} = 0$$

$$SC\,V_{out1}(s) = SC\,V_b(s) + \frac{V_b(s)}{R_2}$$

$$SC\,V_{out1}(s) = SC\,V_b(s) + \frac{V_b(s)}{R_2}$$

$$SC\,V_{out(1)}(s) = SC\,V_b(s) + \frac{V_b}{R_2}(s)$$

$$SC\,V_{out1}(s) = V_b(s)\left(SC + \frac{1}{R_2}\right)$$