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17/ENG05/009
MECHATRONICS ENGINEERING

1. Root Locus Technique

Root locus is the graphical plot of the roots of the characteristic equation by varying system gain, k, from zero to infinity. It shows how a system will change with change in k. It gives an idea of the stability of a control system.

$$1 + G(s)H(s) = 0 \quad \ldots\ldots (1)$$

$$G(s)H(s) = k\frac{N(s)}{D(s)} \quad \ldots\ldots (2)$$

$$1 + k\frac{N(s)}{D(s)} = 0 \quad \ldots\ldots (3)$$

where

k - gain/multiplying factor

N(s) - nth order polynomial of s

D(s) - mth order polynomial of s

Case 1 when $k = 0$

from eqn (3)

$$D(s) + kN(s) = 0$$

$$D(s) = 0$$

∴ when $k = 0$ then closed loop poles = open loop poles

Case 2 when $k = \infty$

from (3)

$$1/k + N(s)/D(s) = 0$$

$$0 + N(s)/D(s) = 0$$

$$N(s) = 0$$

∴ when $k = \infty$ then closed loop poles = closed loop zeros

Root locus branches start at open loop poles and end at open loop zeros

Phase angle $\angle G(s)H(s) = (2n+1)\pi$

Magnitude $|G(s)H(s)| = \sqrt{(-1)^2 + 0^0} = 1$

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Rules for constructing Root Locus

1. Locate the open loop poles and ~~nol~~ zeros on the s plane
2. Determine number of root locus branches (N) you should have

   $N = P$ if $P \geq Z$ where P = no of poles

   $N = Z$ if $P < Z$ Z = no of zeros
3. Draw the real axis locus branches. Root locus starts at open-loop poles and end at open-loop zeros. If $P \neq Z$ then some open poles branches will terminate at ∞ or some zeros branches will start at ∞
4. Find the centroid (α) and angles of asymptotes (θ)

$$\alpha = \frac{\sum \text{open loop poles (real parts)} - \sum \text{open loop zeros (real parts)}}{P - Z}$$

$$\theta = \frac{(2q+1)180}{P-Z}$$

$q = 0, 1, 2, 3, 4 \ldots (P-Z)-1$

5. Find the intersection points of root locus branches with imaginary axis
   - use routh array method: if all elements of any row are zero, then root locus branch intersects $j\omega$ axis
   - identify the row in such a way that if we make the first element as zero, then the elements of the entire row are zero. Find k for this combination
   - Substitute k in the auxiliary equation, you'll get the intersection points
6. Find break ~~in~~ away and break ~~away~~ in points. The are solutions to the following equations respectively $\frac{d}{ds}\left(\frac{N(s)}{D(s)}\right) = 0$ $\frac{d}{ds}\left(\frac{D(s)}{N(s)}\right) = 0$.
7. Find angle of departure (ϕ_d) and the angle of arrival (ϕ_a)

   $\phi_d = 180° - \phi$

   $\phi_a = 180° - \phi$ where $\phi = \sum \phi_p - \sum \phi_z$

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2a. In the case where all the elements of any row of the Routh array are zero, then follow these two steps:

(i) Write the auxilrary equation, $A(s)$, of the row above the row of zeros

(ii) Differentrate the auxiliary equatron, $A(s)$, wrt s. Fill the row of zeros with the coefficrents gotten after differentratron. Then go ahead and complete the table

eg

$$s^5 + 2s^4 + 6s^3 + 10s^2 + 8s + 12$$

s^5	1	1	1
s^4	1	1	1
s^3	0	0	0

$A(s) = s^4$

s^5	1	6	8
s^4	2	10	12
s^3	1	2	6
s^2	6	12	
s^1	0	0	

$A(s) \Rightarrow 6s^2 + 12 = 0$

$s^2 + 2 = 0$

$\frac{dA(s)}{ds} = 2s + 0$

$\therefore$ row s^1 becomes

s^1	2

s^5	1	6	8
s^4	2	10	12
s^3	1	2	6
s^2	6	12	0
s^1	2	0	0
s^0	12		

No sign changes so there are no poles on LHP ~~but~~

2b. If a routh table contains an entire row of zeros there is a possibility that there ~~are~~ poles on the $j\omega$ plane. To confirm this use the above method in 2a to complete the table.

If there are NO SIGN CHANGES then the no of poles ~~of~~ on the $j\omega$ axis is equal to the number of roots of the auxiliary equation

In the example above there are no sign changes, so there are roots on the $j\omega$ axis. The auxiliary eqn was $A(s) \Rightarrow s^2 + 2 = 0$

$s = \pm j\sqrt{2}$ being the roots on the $j\omega$ axis

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