

KORAWOLE Joseph O.

17ENG04038

Elect/Elect

EEE 441

1) In control theory & stability theory, root locus analysis is a graphical method for examining how the roots of a system change with variation of a certain system parameter, commonly a gain with a feedback system. This is a technique used as a stability criterion in the field of classical control theory. The root locus plots the poles of the closed loop transfer function in the complex s -plane as a function of a gain parameter.

A graphical method that was a special protractor called a "sprake" was once used to determine angles and draw the root loci.

2) A
Initial layout for Routh's table

| | | | |
|-------|-------|-------|-------|
| s^4 | a_4 | a_2 | a_0 |
| s^3 | a_3 | a_1 | 0 |
| s^2 | b_1 | b_2 | 0 |
| s^1 | c_1 | 0 | 0 |
| s^0 | d_1 | 0 | 0 |

$$b_1 = - \frac{\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} \quad b_2 = - \frac{\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3}$$

$$b_3 = - \frac{\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$$

In that order, $c_1 = - \frac{\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1}$ $c_2 = - \frac{\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$

$$d_1 = - \frac{\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1'$$

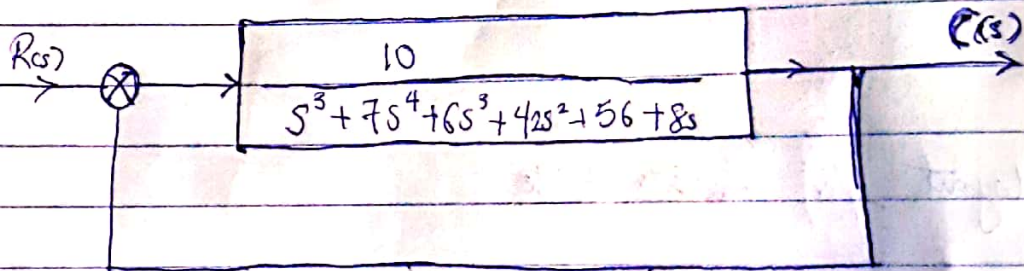
Therefore standard equation will be

$$d_n = - \det \begin{vmatrix} d_n & d_{n-2k} \\ d_{n-1} & d_{n-2k-1} \end{vmatrix}$$

where $d(s) = d_n s^n + d_{n-1} s^{n-1} + \dots + d_1 s + d_0$

Example :

In a case where a whole row becomes zero



$$T(s) = \frac{10}{s^3 + 7s^4 + 6s^3 + 42s^2 + 56 + 8s}$$

$$\begin{array}{l} s^5 : 1 \quad | \quad 6 \quad | \quad 8 \\ s^4 : 7 \quad | \quad 42 \quad | \quad 56 \\ s^3 : - \frac{\begin{vmatrix} 6 & 8 \\ 7 & 42 \end{vmatrix}}{7} = 0 \quad | \quad - \frac{\begin{vmatrix} 8 & 56 \\ 7 & 56 \end{vmatrix}}{7} = 0 \quad | \quad - \frac{\begin{vmatrix} 0 & 0 \\ 7 & 0 \end{vmatrix}}{7} = 0 \end{array} \left. \vphantom{\begin{array}{l} s^5 \\ s^4 \\ s^3 \end{array}} \right\} \text{Entire row is Zero}$$

When a row of zero's appears, we develop an auxiliary polynomial from s^4, s_0 we have :

$$P(s) = 7s^4 + 42s^2 + 56 \quad \text{--- (1)}$$

$$\frac{dP(s)}{ds} = 28s^3 + 84s \quad \text{--- (2)}$$

∴ We continue with auxiliary equation

$$\begin{array}{l} s^3 \quad 28 \qquad \qquad 84 \qquad \qquad 0 \\ s^2 \quad - \begin{array}{|cc|} \hline 7 & 42 \\ \hline 28 & 84 \\ \hline \end{array} \quad - \begin{array}{|cc|} \hline 7 & 56 \\ \hline 28 & 0 \\ \hline \end{array} \quad - \begin{array}{|cc|} \hline 7 & 0 \\ \hline 28 & 0 \\ \hline \end{array} \\ \qquad \qquad 28 \qquad \qquad 28 \qquad \qquad 28 \\ \qquad \qquad = 21 \qquad \qquad = 56 \qquad \qquad = 0 \end{array}$$

$$\begin{array}{l} s^1 \quad - \begin{array}{|cc|} \hline 28 & 84 \\ \hline 21 & 56 \\ \hline \end{array} \quad - \begin{array}{|cc|} \hline 28 & 0 \\ \hline 21 & 0 \\ \hline \end{array} \quad 0 \\ \qquad \qquad 21 \qquad \qquad 21 \qquad \qquad 0 \\ \qquad \qquad = 28/3 \qquad \qquad 0 \end{array}$$

$$\begin{array}{l} s^0 \quad \begin{array}{|cc|} \hline 24 & 56 \\ \hline 28/3 & 0 \\ \hline \end{array} \quad 0 \quad 0 \\ \qquad \qquad 28/3 \qquad \qquad = 56 \end{array}$$

There is no sign change therefore, the system is marginally stable.

B To determine the poles on $j\omega$ axis.

ANSWER: When the entries from the row before the row of the zero to the last row are looking at the even polynomials and there are no sign changes then all the poles there belong to the $j\omega$ axis.