Olabode Tony Pelumi
18/ENG 04/082
EEE 441

(1) In control theory & stability theory, root locus analysis is a graphical method for examining how the roots of a system change with variation of a certain system parameter, commonly a gain with a feedback system. This is a technique used as a stability criteria in the field of classical control theory. The root locus plots the poles of closed loop transfer function as a function of a gain parameter.

A graphical method that uses a special protractor called a Spirule was once used to determine angles & draw the root loci.

2.) Fractional layout for Routh's table

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	b_1	b_2	0
s^1	c_1	0	0
s^0	d_1	0	0

$$b_1 = \frac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} \qquad b_2 = \frac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3}$$

$$b_3 = \frac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$$

In that order, $c_1 = \frac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1}$ $\quad c_2 = \frac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$

$$d_1 = \frac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$$

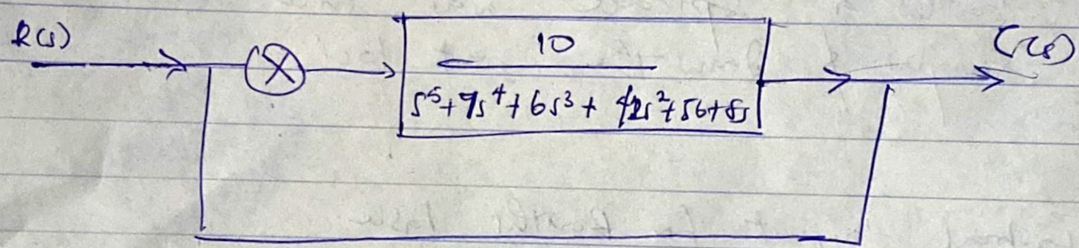
Therefore standard eqn will be

$$b_k = \frac{-\det\begin{vmatrix} a_n & a_{n-2k} \\ a_{n-1} & a_{n-2k} \end{vmatrix}}{a_{n-1}}$$

where $f(s) = a_n s^n + a_{n-1} s^{n-1} + \dots a_1 s + a_0$

Example:-

In a case where a row becomes zero



$$T(s) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 56s + 8s}$$

$$\begin{array}{l|c|c|c}
s^5 & 1 & 6 & 8 \\
s^4 & 7 & 42 & 56 \\
s^3 & \frac{-\begin{vmatrix} 1 & 6 \\ 7 & 42 \end{vmatrix}}{7} = 0 & \frac{-\begin{vmatrix} 1 & 8 \\ 7 & 56 \end{vmatrix}}{7} = 0 & \frac{-\begin{vmatrix} 1 & 0 \\ 7 & 0 \end{vmatrix}}{7} = 0
\end{array} \Bigg\} \text{ Entire row} = 0$$

when a row of zero's appear, we develop an auxiliary polynomial

$$P(s) = 7s^4 + 42s^2 + 56 \quad — \quad (1)$$

$$\frac{d[P(s)]}{ds} = 28s^3 + 84s \quad — \quad (2)$$

We continue with auxiliary equation

$$\begin{array}{llll}
s^3 & 28 & 84 & 0 \\
s^2 & \dfrac{-\begin{vmatrix} 7 & 42 \\ 28 & 84 \end{vmatrix}}{28} & \dfrac{-\begin{vmatrix} 7 & 56 \\ 28 & 0 \end{vmatrix}}{28} & \dfrac{-\begin{vmatrix} 7 & 0 \\ 28 & 0 \end{vmatrix}}{28} \\
 & -21 & = 56 & = 0
\end{array}$$

$$s^1 = \dfrac{-\begin{vmatrix} 28 & 84 \\ 21 & 56 \end{vmatrix}}{21} \quad \dfrac{-\begin{vmatrix} 28 & 0 \\ 21 & 0 \end{vmatrix}}{21} \quad 0$$

$$= 28/3 \qquad 0$$

$$s^0 \quad \dfrac{\begin{vmatrix} 28 & 56 \\ 28/3 & 0 \end{vmatrix}}{28/3} = 56 \qquad 0 \qquad 0$$

There is no change in sign. Therefore the system is marginally stable.

(3) To determine poles on $j\omega$ axis

Ans:-

When the entries from the row before the row of the zero to the last row are looking at the even polynomials & there are no sign changes then all the poles there belong to the $j\omega$ axis.