Topic:

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ELECT / ELECT

PI Controller

$V_2 = 0$

$I_1 + I_2 = 0$ — (1)

$I_3 - I_2 = 0$ — (2)

Current through the Capacitor

$I_c = C \, dv/dt$

$I_1 = \frac{V_e - V_2}{R_1}$ $(V_2 = 0)$

$= V_e/R_1$

$I_2 = \frac{V_o - V_2}{R_2}$

$= V_o/R_2$

$I_3 = C\, d/dt\, (V_{out1} - V_o)$

Sub into eqn (1) & (2)

$V_e/R_1 + V_o/R_2 = 0$ ——— (1)

$C\, d/dt\, (V_{out1} - V_o) - V_o/R_2 = 0$ — (2)

from eqn (1)

$V_o/R_2 = -V_e/R_1$

$V_o = -R_2/R_1 \, V_e$

Taking Laplace transform of eqn (2)

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$SC(V_{out1}(s) - V_b(s)) - V_b(s)/R_2 = 0$

$SC(V_{out1}(s)) = SC\,V_b(s) + V_b(s)/R_2$

$SC(V_{out1}(s)) = V_b(s)(SC + 1/R_2)$

recall; $V_b = -R_2/R_1 (V_e)$

$SC\,V_{out1}(s) = -R_2/R_1\, V_e(s)(SC + R_1/R_2)$

$V_{out1}(s) = -R_2/SCR_1\, V_e(s)(SC + 1/R_2)$

$V_{out1}(s) = -R_2/R_1\, V_e(s) - R_2/R_1\, 1/SCR_2\, V_e(s)$

from the inverting circuit

$V_{out1} = -V_{out}$

$\therefore V_{out}(s) = -(-R_2/R_1\, V_e(s) - R_2/R_1\, 1/SCR_2\, V_e(s))$

$V_{out}(s) = R_2/R_1\, V_e(s) + R_2/R_1\, 1/SCR_2\, V_e(s)$

Taking inverse Laplace

$V_{out} = R_2/R_1\, V_e(t) + R_2/R_1\, 1/R_2 \int_0^t V_e(t)\,\delta t + V(0)$

where $1/s = \int_0^t \delta t + K$

$V_{out} = G_p V_e + G_p G_I \int_0^t V_e\,\delta t + V(0)$

where $G_p = R_2/R_1$

$G_I = 1/R_2 C$

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PD Controller

$I_1 + I_2 = I_3$ —— ①

$I_3 + I_4 = 0$ —— ②

$I_1 = V_e - V_a / R_3$

$I_2 = C\delta/\delta t\ (V_e - V_a)$

$I_3 = V_a - V_b / R_1 \quad (V_b = 0)$

$= V_a / R_1$

$I_4 = V_{out1} - V_b / R_2 \quad (V_b = 0)$

$= V_{out1} / R_2$

$R = R_1 R_3 / R_1 + R_3 \implies$ Effective Resistance

Sub into eqn ① & ②

$V_e - V_a / R_3 + C\delta/\delta t\ (V_e - V_a) = V_a / R_1$ —— ⓐ

$V_a / R_1 + V_{out1} / R_2 = 0$ —— ⓑ

from eqn ⓑ

$V_a / R_1 = - V_{out1} / R_2$

$V_a = -R_1 / R_2\ V_{out1}$

Rearranging eqn ⓐ

$V_e - V_a / R_3 + C\delta/\delta t\ (V_e - V_a) - V_a / R_1 = 0$

Taking Laplace transform

$V_{e(s)} - V_{a(s)} / R_3 + sC\ (V_{e(s)} - V_{a(s)}) - V_{a(s)} / R_1 = 0$

(initial conditions go to zero)

$V_{e(s)} / R_3 + sC\ V_{e(s)} = V_{a(s)} / R_1 + V_a / R_3 + sC\ V_{a(s)}$

$V_{e(s)} \left(1/R_3 + sC \right) = V_{a(s)} \left(1/R_1 + 1/R_3 + sC \right)$

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Recall ; $V_0 = {}^{+R_1}/_{R_2}\, V_{out1}$

$V_e(s) \left({}^{1}/_{R_3} + sC \right) = {}^{-R_1}/_{R_2}\, V_{out1}(s) \left({}^{1}/_{R_1} + {}^{1}/_{R_3} + sC \right)$

$V_e(s) \left({}^{1 + R_3 sC}/_{R_3} \right) = {}^{-R_1}/_{R_2}\, V_{out1}(s) \left({}^{R_3 + R_1 + sCR_1R_3}/_{R_1R_3} \right)$

$V_e(s) \left(1 + sCR_3 \right) = {}^{-V_{out1}(s)}/_{R_2} \left(R_3 + R_1 + sCR_1R_3 \right)$

$-V_{out1}(s) = {}^{V_e(s)(1 + sCR_3)R_2}/_{R_1 + R_3 + sCR_1R_3}$

$-V_{out1}(s) = {}^{V_e(s)(R_2 + sCR_2R_3)}/_{R_1 + R_3 + sCR_1R_3}$

Dividing RHS by $R_1 + R_3$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + sCR_2R_3)/R_1 + R_3}{{}^{R_1+R_3}/_{R_1+R_3} + {}^{sCR_1R_3}/_{R_1+R_3}}$$

recall, $R = {}^{R_1R_3}/_{R_1+R_3}$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + sCR_2R_3)/R_1 + R_3}{1 + sCR}$$

if $sCR << 1$

$-V_{out1}(s) = {}^{V_e(s)(R_2 + sCR_2R_3)}/_{R_1 + R_3}$

from the inverting circuit

$V_{out1} = -V_{out} + V_0$

$\therefore -(-V_{out}(s) + V_{(0)}) = {}^{V_e(R_2 + sCR_2R_3)}/_{R_1+R_3}$

$V_{out}(s) - V_{(0)} = {}^{V_e(s)R_2}/_{R_1+R_3} + {}^{sCR_2R_3}/_{R_1+R_3} V_e(s)$

$V_{out}(s) = {}^{R_2}/_{R_1+R_3} (V_e(s)) + {}^{R_2}/_{R_1+R_3}\, R_3 Cs\, (V_e(s)) + V_{(0)}$

Taking inverse Laplace

$V_{out} = {}^{R_2}/_{R_1+R_3}\, V_e + {}^{R_2}/_{R_1+R_3}\, R_3 C\, {}^{dV_e}/_{dt} + V_0$

$V_{out} = G_P V_e + G_P G_D\, {}^{dV_e}/_{dt} + V_0$

where ; $G_P = {}^{R_2}/_{R_1+R_3}$ and $G_D = R_3 C$