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17/ENG05/037
MECHATRONICS ENGINEERING
EEE441

ASSIGNMENT

1\. Root Locus technique is the plot of locus of roots of a characteristic equation with respect to the gain (k) and it gives us an idea about absolute stability and relative stability of a control system

2a Layout for Routh's Table

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	b_1	b_2	0
s^1	c_1	0	0
s^0	d_1	0	0

$$b_1 = \frac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} \qquad b_2 = \frac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3}$$

$$b_3 = \frac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$$

In the same sense;

$$c_1 = \frac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} \qquad c_2 = -\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix} = 0$$

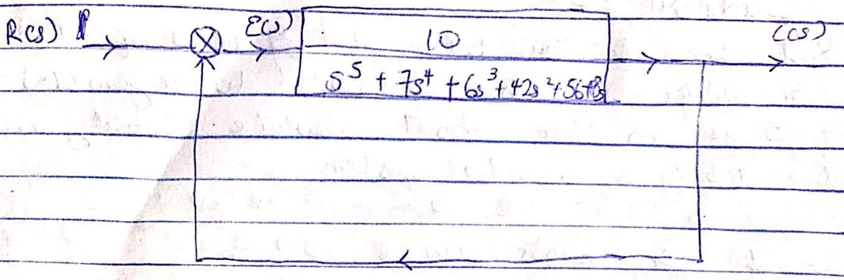
$$d_1 = -\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix} = d_1$$

$\therefore$ the equation will be

$$b_k = \frac{-\det\begin{vmatrix} a_n & a_{n-2k} \\ a_{n-1} & a_{n-2k-1} \end{vmatrix}}{a_{n-1}}$$

where $d(s) = a_n s^n + a_{n-1} s^{n-1} + \dots a_1 s + a_0$

for example, consider a case where a whole row been zero.



$$T(s) = \frac{10}{s^5+7s^4+6s^3+42s^2+56s+8s}$$

$$\begin{array}{l|lll}
s^5 & 1 & 6 & 8 \\
s^4 & 7 & 42 & 56 \\
s^3 & -\dfrac{\begin{vmatrix}1 & 6\\7 & 42\end{vmatrix}}{7} = 0 & -\dfrac{\begin{vmatrix}1 & 8\\7 & 56\end{vmatrix}}{7} = 0 & -\dfrac{\begin{vmatrix}1 & 0\\7 & 0\end{vmatrix}}{7} = 0
\end{array}$$

When a row of zeros appears we develop an auxillary polynomial from s^4. So we have:

$$P(s) = 7s^4 + 42s^2 + 56 \quad --- (i)$$

Differentiate (i)

$$\frac{dP(s)}{ds} = 28s^3 + 84s \quad -- (ii)$$

∴ We continue with the auxillary equation

$$\begin{array}{llll}
s^3 & 28 & 84 & 0 \\
s^2 & -\dfrac{\begin{vmatrix}7 & 42\\28 & 84\end{vmatrix}}{28} = 21 & -\dfrac{\begin{vmatrix}7 & 56\\28 & 0\end{vmatrix}}{28} = 56 & -\dfrac{\begin{vmatrix}7 & 0\\28 & 0\end{vmatrix}}{28} = 0 \\
s^1 & -\dfrac{\begin{vmatrix}28 & 84\\21 & 56\end{vmatrix}}{21} = \dfrac{28}{3} & -\dfrac{\begin{vmatrix}28 & 0\\21 & 0\end{vmatrix}}{21} = 0 & 0
\end{array}$$

s^0 $-\begin{vmatrix} 21 & 56 \\ \frac{28}{3} & 0 \end{vmatrix}$ $\quad 0 \quad 0$

$\dfrac{\quad}{28/3}$

$= 56$

There is no sign change, therefore the system is marginally stable

2b. To determine the poles on the $j\omega$ axis;
When the entires from the row before the row of the zeros to the last row are looking at even polynomials and there are no sign changes then all the poles there belong to the $j\omega$ axis