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COURSE: EEE 561 (Process Control & Automation)
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DEPT; ELECTRICAL & ELECTRONICS

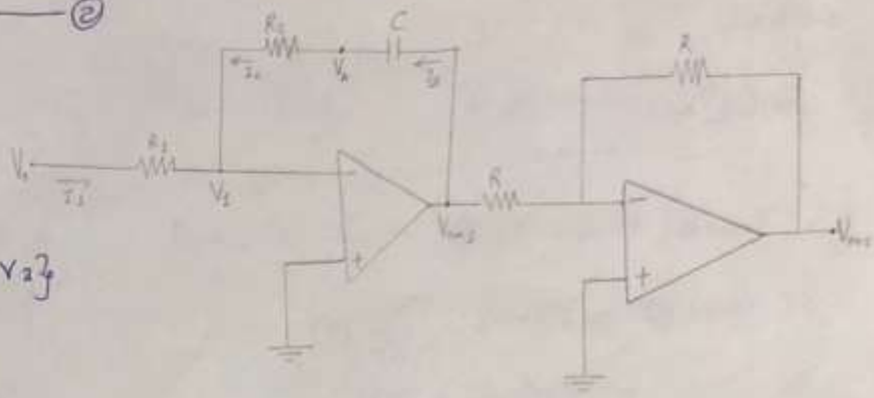
Derive the analysis for the output voltage using Operational Amplifier for:
① Proportional Integral controller mode (PI Mode)

ⓐ from; $I_1 + I_2 = 0$ ——— ①

$I_3 - I_2 = 0$ ——— ②

But; $V_a = 0$

$I_c = C\frac{dV_c}{dt}$



$I_1 = \frac{V_e - V_a}{R_1}$ {subt. V_a}

$= \frac{V_e}{R_1}$ ——— ⓐ

$I_2 = \frac{V_b - V_a}{R_2} \Rightarrow \frac{V_e - 0}{R_2}$

$= \frac{V_b}{R_2}$ ——— ⓑ

$I_3 = \frac{C\,d(V_{out1} - V_b)}{dt}$ ——— ⓒ

subt. equ. a, b, c into equ. 1 & 2;

We have;

$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0$ ——————— (*)

$\frac{C\,d(V_{out1} - V_b)}{dt} - \frac{V_b}{R_2} = 0$ ——— (**)

from equ. *

Making V_b subject of formula;

$\frac{V_b}{R_2} = -\frac{V_e}{R_1}$

$$V_b = -\frac{V_e}{R_1} \times R_2$$

$$V_b = -\frac{R_2}{R_1} V_e$$

Taking the Laplace transform of equ. (** **),
We have;

$$SC[V_{out1}(s) - V_b(s)] - \frac{V_b}{R_2}(s) = 0$$

$$[SCV_{out1}(s) - SCV_b(s)] - \frac{V_b}{R_2}(s) = 0$$

$$SCV_{out1}(s) = SCV_b(s) + \frac{V_b}{R_2}(s)$$

$$= V_b(s)(SC + \tfrac{1}{R_2})$$

Subt. V_b;

$$SCV_{out1}(s) = -\frac{R_2}{R_1} V_e(s)\left[SC + \frac{1}{R_2}\right]$$

$$SCV_{out1}(s) = -\frac{\cancel{R_2}}{R_1} SCV_e(s) - \frac{R_2}{R_1}\frac{1}{R_2} V_e(s)$$ (Dividing both sides by SC)

$$V_{out1}(s) = -\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{R_2}\cdot\frac{1}{SC} V_e(s)$$

But $V_{out}(1) = -V_{out}$ (from Inverting circuit)

$$-V_{out}(s) = -\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{CR_2}\cdot\frac{1}{S} V_e(s)$$

$$V_{out}(s) = -\left[-\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{R_2C}\frac{1}{S} V_e(s)\right]$$

$$= \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{R_2C}\frac{1}{S} V_e(s)$$

Taking the Laplace Inverse,

We have;

$$V_{out} = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1} \frac{1}{R_2 C} \int_0^t V_e(t) \cdot dt + V(0)$$

But; $\frac{1}{S} = \int_0^t dt + K)$

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e \, dt + V(0)$$

where $G_P = \frac{R_2}{R_1}$

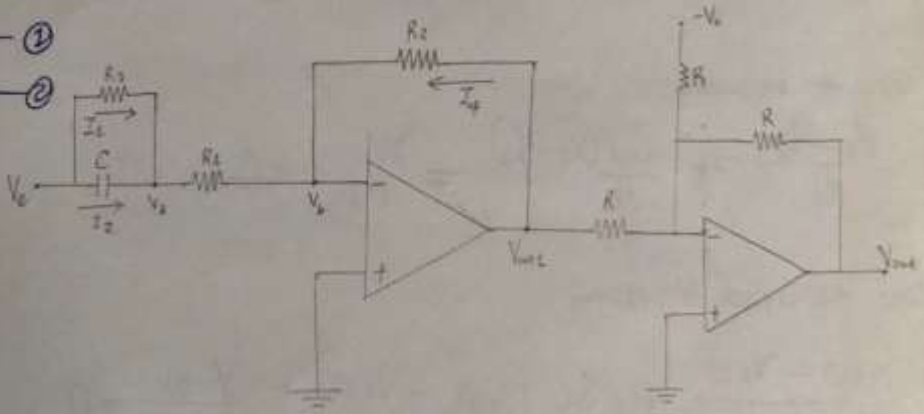
& $G_I = \frac{1}{R_2 C}$

(2) Proportional Differential controller mode (PD mode).

$I_1 + I_2 = I_3$ —— (1)

$I_3 + I_4 = 0$ —— (2)

$V_b = 0$



$I_1 = \frac{V_e - V_2}{R_3}$

$I_2 = \frac{C \, d(V_e - V_2)}{dt}$

$I_3 = \frac{V_2 - V_b}{R_1}$

$I_3 = \frac{V_2}{R_1}$

$I_4 = \frac{V_{out1} - V_b}{R_2}$

$I_4 \Rightarrow \frac{V_{out1}}{R_2}$

Effective Resistance, $R = \frac{R_1 R_3}{R_1 + R_3}$

Subt I_1, I_2, I_3 & I_4 into equ. ① & ③,

We have;

$$\frac{V_e - V_2}{R_3} + \frac{C\,d(V_e - V_2)}{dt} = \frac{V_2}{R_1} \quad \text{———} (*)$$

$$\frac{V_2}{R_1} + \frac{V_{out1}}{R_2} = 0 \quad \text{———} (**)$$

From equ. 2;

Making V_2 subj. of the formulae.

$$\frac{V_2}{R_1} = \frac{-V_{out1}}{R_2}$$

$$V_2 = -\frac{R_1}{R_2} V_{out1}$$

Equ. $*$ becomes;

$$\frac{V_e - V_2}{R_3} + \frac{C\,d(V_e - V_2)}{dt} \cancel{=} \frac{V_2}{R_1} = 0$$

Taking the Laplace transform;

$$\frac{V_e(s) - V_2(s)}{R_3} + SC\,(V_e(s) - V_2(s)) - \frac{V_2(s)}{R_1} = 0$$

Taking initial conditions to be zero we have;

$$\frac{V_e(s)}{R_3} + SC\,V_e(s) = \frac{V_2(s)}{R_1} + \frac{V_2(s)}{R_3} + SC\,V_2(s)$$

$$V_e(s)\left(\frac{1}{R_3} + SC\right) = V_2(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + SC\right)$$

$\boxed{\text{Subt. } V_2\text{, we have}}$

$$V_{e(s)}\left(\frac{1}{R_3} + SC\right) = -\frac{R_1}{R_2} V_{out1}(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + SC\right)$$

$$V_e(s)\left(\frac{1+R_3SC}{R_3}\right) = -\frac{R_f}{R_2}V_{out1}(s)\left(\frac{R_3+R_1+R_1R_3SC}{R_1R_3}\right)$$

$$V_e(s)\left(\frac{1+R_3SC}{R_3}\right) = -\frac{R_f}{R_2}V_{out1}(s)\times\frac{R_1}{R_1}\left(\frac{R_1+R_3+R_1R_3SC}{R_3}\right)$$

$$V_e(s)\left(\frac{1+R_3SC}{\cancel{R_3}}\right) = -\frac{V_{out1}}{R_2}(s)\left(\frac{R_1+R_3+R_1R_3SC}{\cancel{R_3}}\right)$$

$$V_e(s)\left(1+R_3SC\right) = -\frac{V_{out1}}{R_2}(s)\left(R_1+R_3+R_1R_3SC\right)$$

$$\frac{-V_{out1}(S)}{R_2} = \frac{V_e(s)\left(1+R_3SC\right)}{\left(R_1+R_3+R_1R_3SC\right)}$$

$$-V_{out1}(S) = R_2\times\left[\frac{V_e(s)\left(1+R_3SC\right)}{\left(R_1+R_3+R_1R_3SC\right.}\right] \Rightarrow \frac{V_e(s)\left(R_2+R_3R_2SC\right)}{R_1+R_3+R_1R_3SC.}$$

Dividing the numerator x denominator by R_1+R_3;

$$-V_{out1}(S) = \frac{\frac{V_e(s)\left[R_2+R_2R_3SC\right)}{R_1+R_3}}{\cancel{\frac{R_1+R_3}{R_1+R_3}}+\frac{R_1R_3SC}{R_1+R_3}}$$

But $R = \frac{R_1R_3}{R_1+R_3}$

$$-V_{out1}(S) = \frac{\frac{V_e(s)\cdot\left(R_2+R_2R_3SC\right)}{R_1+R_3}}{1+\cancel{R}SCR}$$

If $SCR \ll 1$

$$-V_{out1}(s) = \frac{V_e(s)\left(R_2+R_2R_3SC\right)}{R_1+R_3}$$

from the Inverting circuit;

$$V_{out1} = -V_{out}+V_o$$

$$-\left(-V_{out}(s)+V_o\right) = \frac{V_e(s)R_2}{R_1+R_3}+\frac{R_2R_3SC}{R_1+R_3}$$

$$V_{out}(s) - V(0) = \frac{R_2}{R_1+R_3} V_e(s) + \frac{R_2}{R_1+R_3} R_3 SC\, V_e(s)$$

$$V_{out}(s) = \frac{R_2}{R_1+R_3} V_e(s) + \frac{R_2}{R_1+R_3} R_3 SC\, V_e(s) + V(0)$$

Taking the Inverse Laplace Transform

$$V_{out}(s) = \frac{R_2}{R_1+R_3} V_e + \frac{R_2}{R_1+R_3} \cdot R_3 C \frac{dV_e}{dt} + V_0$$

$$V_{out}(s) = G_P V_e + G_P \cdot G_D \frac{dV_e}{dt} + V_0$$

Where; $G_P = \frac{R_2}{R_1+R_3}$

and, $G_D = R_3 C$.