Ejike David Chinedu
18/ENG04/078
Elect/Elect

EEE441

Question

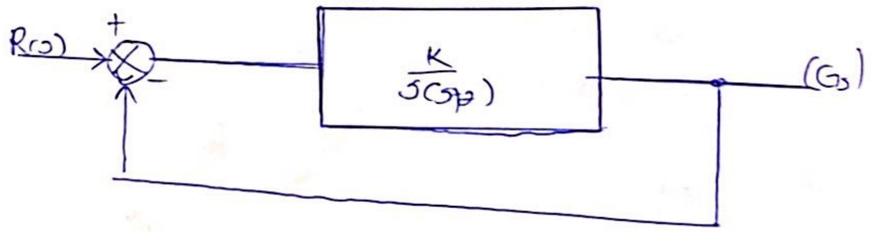
① Briefly explain root locus technique

② Describe the use Routh Hurwitz to find the stability of a closed loop system when: Entire row is zero on the routh table
To determine the poles on the jω axis

Solution

Root locus technique :- Root locus ⇒ closed loop poles path by varying the gain K from zero to infinity
- locus ⇒ path
- Root means root of characteristic equation (poles of the closed loop)

• Root locus is a graphical method in which roots of the characteristic equation are plotted in the s-plane for the different value of parameter, the locus of the roots of the characteristic equation when the gain is varied from zero to infinity. Consider a unity feedback as shown below:



$\frac{C(s)}{R(s)}$ · where $G = \frac{K}{s(s+2)}$; $H(s) = 1$

$1 + \frac{K}{s(s+2)} \cdot 1 = 0$

∴ the root of the equation

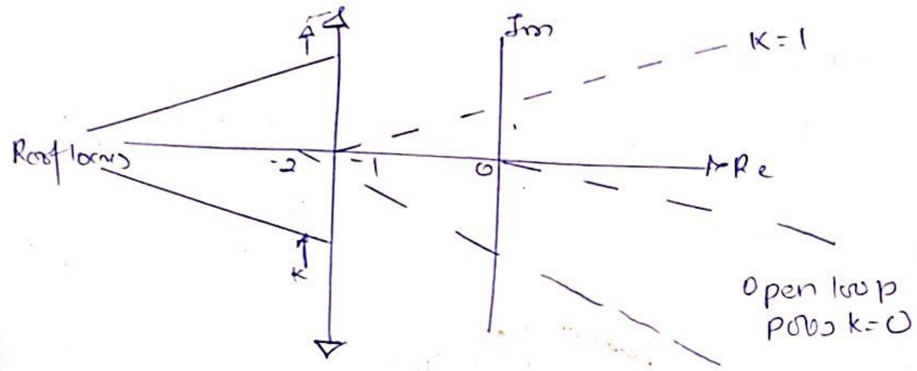
$s_1 = -1 + \sqrt{1-K}$, $s_2 = -1 - \sqrt{1-K}$

As 'K' is varied, the two roots given the locii in s-plane for the various values of K, the location of the roots are

① When $0 < K < 1$, the root are real and distinct
② When $K = 0$, the two roots are $s_1 = 0$ and $s_2 = 0$ (open loop poles)
③ When $K = 1$ both roots are real and equal
④ When $K > 1$ the root are complex conjugate with real part = 1

* Plotting root of a characteristic equation

- $K = 0$; $s = 0$ & $s = -2$
- $K = 1$; Both root $s = -1$
- $K > 1$; the roots break away from the real axis and become complex conjugate having negative real part equal to -1, $s_{1,2} = -1 \pm j\sqrt{K-1}$



(2) Describe the use of Routh Hurwitz to find the stability of a closed loop system

Routh Hurwitz method is a quick mathematical way to determine what gains are allowed whilst achieving stability. It doesn't show the path of the poles but it tells the stability of a system and its relation to a gain factor. This method can tell how many closed loop system poles are in the left half plane in the right half plane and on the $j\omega$ axis

* Two methods required in Routh Hurwitz
(1) Operate a data table called Routh table
(2) Interprete the routh table to tell how many closed loop systems poles are in the left half plane the right half plane and on the $j\omega$ axis

NB. If the characteristic equation of the table is : $a_1 s^n + a_2 s^{n-1} + a_3 s^{n-2} + \dots = 0$

$$
\begin{array}{c|cccc|}
s^n & a_1 & a_3 & a_5 & a_7 \\
s^{n-1} & a_2 & a_4 & a_6 & a_8 \\
s^{n-2} & b_1 & b_2 & b_3 & b_4 \\
s^{n-3} & c_1 & c_2 & c_3 &
\end{array}
$$

where $b_1 = -\dfrac{\begin{vmatrix} a_1 & a_3 \\ a_2 & a_4 \end{vmatrix}}{a_2}$, $b_2 = \dfrac{\begin{vmatrix} a_1 & a_5 \\ a_2 & a_6 \end{vmatrix}}{a_2}$, $b_3 = \dfrac{\begin{vmatrix} a_1 & a_7 \\ a_2 & a_8 \end{vmatrix}}{a_2}$

$C_1 = \dfrac{\begin{vmatrix} a_2 & a_4 \\ b_1 & b_2 \end{vmatrix}}{b_1}$, $C_2 = \dfrac{\begin{vmatrix} a_2 & a_6 \\ b_1 & b_3 \end{vmatrix}}{b_1}$, $C_3 = \dfrac{\begin{vmatrix} a_2 & a_8 \\ b_1 & b_4 \end{vmatrix}}{b_1}$

Special case: Entire Row in zero on the Routh table

In this case, the entire row consists of zero because there is an even polynomial that is a factor of the original - it indicates the equation has atleast one pair of roots which lie vertically opposite each other and equidistant from origin. The array can be completed by forming the auxillary polynomial

The polynomial whose coefficient are elements of the row just above the zeros in Routh array is called an auxillary polynomial

An example will be used to demonstrate how to construct and interpret the routh table

Consider the following characteristic equation

$$\overset{a_0}{s^6} + \overset{a_2}{2s^5} + \overset{a_3}{8s^4} + \overset{a_4}{12s^3} + \overset{a_5}{20s^2} + \overset{a_6}{16s} + \overset{a_7}{16} = 0$$

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	0	0	0	0
s^2				
s^1				

From the above table it is clear that the fourth row is zero. Hence the auxillary equation, $A(s)$ can be form from the coefficients of the row just above the fourth row as shown

Auxillary equation: $A(s) = 2s^4 + 12s^2 + 16 = 0$

$$\therefore \frac{dA(s)}{ds} = 8s^3 + 24s$$

Once the order of characteristic eqn is six in order of eqn for the two roots (6-4)=2 lie to the left half of the s-plane and four roots of auxillary eqn are dominant

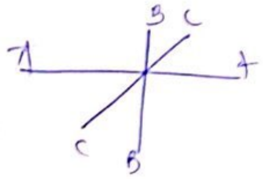
s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	8	24	0	0
s^2	6	16	0	
s^1	2.67	0		
	16			

Result

1. No sign change in first column
2. The system is stable (marginal stability)
3. Roots lie on the imaginary axis
4. The two roots lie on the left side of the s-plane
5. The four roots are dominant

* To determine the poles on the $j\omega$ axis

The poles on the $j\omega$ axis means that there is a row of zeros in a closed loop system with only imaginary axis poles of multiplicity 1 and poles in the left half plane. It is marginally stable. The roots of such system is imaginary and symmetrical about the origin



From the example above: Determine the poles of the $j\omega$ axis

Since there is a row of zeros, it means the roots are lying on the imaginary axis. The poles can be determined by equating the auxiliary axis to zero; $A(s)=0$

$$\therefore A(s): 2s^4 + 12s^2 + 16 = 0$$

$$\text{let } s^2 = x$$

$$\therefore 2x^2 + 12x + 16 = 0$$

$$2x^2 + 12x + 16 = 0$$

$$x^2 + 6x + 8 = 0$$

$$(x+2)(x+4) = 0$$

$$x+2 = 0 \qquad x+4 = 0$$

$$x = -2 \qquad x = -4$$

Recall: $x = s^2$

$$\therefore s^2 = -2 \qquad s^2 = -4$$

$$s = \pm\sqrt{-2} \qquad s^2 = -\sqrt{-4}$$

$$s = \pm j\sqrt{2} \qquad s = \pm j2$$

$\therefore$ the poles on the imaginary axis are given as $\pm\sqrt{2}$, ± 2

Since the roots are non-repeated on the imaginary axis, hence the system is marginally stable

No of roots with positive real part = 0

No of roots with negative real part = 2

No of roots with zero real part = 4