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PI Controller

$V_d = 0$

$I_1 + I_2 = 0$ —— ①

$I_3 - I_2 = 0$ —— ②

current through the capacitor

$I_c = C\frac{dv_c}{dt}$

$I_1 = \frac{V_R - V_d}{R_1}$ $(V_d = 0)$

$= \frac{V_R}{R_1}$

$I_2 = \frac{V_b - V_d}{R_2}$ $(V_d = 0)$

$= \frac{V_b}{R_2}$

$I_3 = Cd\,(V_{out} - V_b)$

Sub into eqn① & eqn②

$\frac{V_R}{R_1} + \frac{V_b}{R_2} = 0$ —— ①

$$\frac{C\,d}{dt}\left(V_{out_1} - V_b\right) - \frac{V_b}{R} = 0 \quad \text{--- (2)}$$

from eqr (1)

$$\frac{V_b}{R_2} = -\frac{V_a}{R_1}$$

$$\boxed{V_b = -\frac{R_2}{R_1} V_C}$$

taking la place transform of eqr (2)

$$SC\left(V_{out_1}(s) - V_b(s)\right) - \frac{V_b(s)}{R_2} = 0$$

$$SC\,V_{out_1}(s) = SC\,V_b(s) + \frac{V_b(s)}{R_2}$$

$$S_C\,V_{out_1}(s) = V_b(s)\left(SC + \frac{1}{R_2}\right)$$

recall, $V_b = -\frac{R_2}{R_1} V_C$

$$S_C\,V_{out_1}(s) = \frac{R_2}{R_1} V_B(s)\left(SC + \frac{B_1}{R_2}\right)$$

$$V_{out_1}(s) = -\frac{R_2}{SCR_1} V_R(s)\left(SC + \frac{1}{R_1}\right)$$

$$V_{out}(s) = -\frac{R_2}{R_1} V_C(s) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_R(s)$$

from the Inverting circuit

$$V_{out_1} = -V_{out}$$

$$\therefore V_{out(s)} = -\left(-\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)\right)$$

$$V_{out_1}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)$$

~~from the~~ on taking inverse laplace

$$V_{out} = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{R_2}\int_0^t V_e(t)\,dt + V(0)$$

$$\left(\text{where } \frac{1}{s} = \int_s^t dt + \kappa\right)$$

$$\boxed{V_{out} = G_p V_e + G_p G_s \int_0^t V_e\,dt + V(0)}$$

Where $G_p = \frac{R_2}{R_1}$

$G_I = \frac{1}{R_2 C}$

PD controller

$I_1 + I_2 = I_3$ — ①

$I_3 + I_4 = 0$ — ②

$I_1 = \frac{v_c - v_a}{R_3}$

$I_2 = c\frac{d}{dt}(v_c - v_a)$

$I_3 = \frac{v_a - v_b}{R_1}$ $(v_b = 0)$

$= \frac{v_a}{R_1}$

$I_4 = \frac{v_{out1} - v_b}{R_2}$ $(v_b = 0)$

$= \frac{v_{out1}}{R_2}$

$R = \frac{R_1 R_3}{R_1 + R_3}$ ⟵ effective resistance

Sub into eq(1) & eq(2)

$\frac{v_c - v_a}{R_3} + c\frac{d}{dt}(v_c - v_a) = \frac{v_a}{R_1}$ — * ①

$\frac{v_a}{R_1} + \frac{v_{out}}{R_2} = 0$ — * ②

from eq②

$$\frac{v_a}{R_1} = -\frac{v_{out}}{R_2}$$

$$v_a = -\frac{R_1}{R_2} v_{out}$$

~~[illegible]~~ rearranging eq

$$\frac{v_c - v_a}{R_3} + \frac{cd}{dt}(v_c - v_a) - \frac{v_a}{R_1} = 0$$

~~taking~~ laplace transformation

$$\frac{v_c - v_a}{R_3} + sc\,(v_c(s) - v_a(s)) - \frac{v_a(s)}{R_1} = 0$$

(initial inductions go to zero)

$$v_c(s) + sc\,(v_c(s)) = \frac{v_a(s)}{R_1} + \frac{v_a(s)}{R_3} + R_c\, v_a(s)$$

$$v_c(s)\left(\frac{1}{R_3} + sc\right) + v_a(c)\left(\frac{1}{R_1} + \frac{1}{R_3} + sc\right)$$

recall, $v_a = \frac{-R_1}{R_2} v_{out}$ I

$$V_o(s)\left(\frac{1}{R_3}+sC\right) = -\frac{R_1}{R_2}V_{out1}(s)\left(\frac{1}{R_1}+\frac{1}{R_3}+sC\right)$$

taking lcm

$$V_C(s)\left(1+\frac{R_3 sC}{\cancel{R_3}}\right) = -\frac{\cancel{R_1}}{R_2}V_{out1}(s)\left(\frac{R_3+R_1+sCR_1R_3}{\cancel{R_1 R_3}}\right)$$

$$V_C(s)\,(1+sC\,R_3) = -\frac{V_{out1}(s)}{R_2}\,(R_3+R_1+sCR_1R_3)$$

$$-V_{out1}(s) = \frac{V_o(s)\,(1+sCR_3)\,R_2}{(R_1+R_3+sCR_1R_3)}$$

$$-V_{out1}(s) = \frac{V_R(s)\,(R_2+sCR_2R_3)}{(R_1+R_2+sCR_1R_3)}$$

dividing num & denom by R_1+R_3

$$-V_{out1}(s) = \frac{V_C(s)\,(R_2+sCR_2R_3)/R_1+R_3}{\frac{R_1+R_3}{R_1+R_3}+\frac{sCR_1R_3}{R_1+R_3}}$$

recall, $R = \frac{R_1 R_3}{R_1+R_3}$

$$-V_{out1}(s) = \frac{V_2(s)\,(R_2+sCR_2R_3)/R_1+R_3}{1+sCR}$$

If $sCR << 1$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + sCR_2R_3)}{R_1 + R_3}$$

from the Inverting ~~element~~ circuit

$$V_{out1} = -V_{out} + V_0$$

$$\therefore -(-V_{out}(s) + V(0)) = \frac{V_e(R_2 + sCR_2R_3)}{R_1 + R_3}$$

$$V_{out}(s) - V(0) = \frac{V_eR_2}{R_1 + R_3} + \frac{sCR_2R_3V_e(s)}{R_1 + R_3}$$

$$V_{out}(s) = \frac{R_2}{R_1 + R_3}V_e(s) + \frac{R_2}{R_1 + R_3}R_3 C s V_e(s) + V(0)$$

taking inverse Laplace

$$V_{out} = \frac{R_2}{R_1 + R_3}V_e + \frac{R_2}{R_1 + R_3}R_3 C\frac{dV_e}{dt} + V_0$$

$$\boxed{V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_0}$$

where; $G_P = \frac{R_2}{R_1 + R_3}$ $\quad G_D = R_3 C$