Umoh Oobong Ezekiel
16/Eng06/072
Elect

PI CONTROLLER

$V_a = 0$

$I_1 + I_2 = 0$ --- (1)

$I_3 - I_2 = 0$ --- (2)

Current through the capacitor

$$I_c = \frac{c\,dV_c}{dt}$$

$$I_1 = \frac{V_e - V_a}{R_1} \quad (V_a = 0)$$

$$= \frac{V_o}{R_1}$$

$$I_2 = \frac{V_s - V_a}{R_2} \quad (V_a = 0)$$

$$= \frac{V_b}{R_2}$$

$$I_8 = \frac{Cd\,(V_{out,} - V_b)}{dt}$$

sub into eqn ① & eqn ②

$$\frac{V_c}{R_1} + \frac{V_b}{R_2} = 0 \quad -- ①$$

$$\frac{Cd}{dt}(V_{out,} - V_b) - \frac{V_b}{R_2} = 0 \quad -- ②$$

from eqn 1

$$\frac{V_b}{R_2} = -\frac{V_c}{R_1}$$

$$\boxed{V_b = -\frac{R_2 V_c}{R_1}}$$

Taking laplace transform of eqn 2

$$SC\,(V_{out,} - V_b) - \frac{V_{b(s)}}{R_2} = 0$$

$$SC\,(V_{out1}(s)) = SC\,V_{b(s)} + \frac{V_{b(s)}}{R_2}$$

$$SC\,V_{out1}(s) = V_{b(s)}\left(SC + \frac{1}{R_2}\right)$$

recall; $V_b = -\frac{R_2}{R_1} V_c$

$$S_C V_{out1}(s) = \frac{-R_2}{R_1} V_e(s) \left(SC + \frac{1}{R_2} \right)$$

$$V_{out1}(s) = \frac{-R_2 V_e(s)}{SCR_1} \left(SC + \frac{1}{R_2} \right)$$

$$V_{out1}(s) = \frac{-R_2}{R_1} V_e(s) - \frac{R_2}{R_1} \frac{1}{SCR_2} V_e(s)$$

from the inverting circuit

$$V_{out1} = -V_{out}$$

$$\therefore V_{out}(s) = -\left(-\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1} \frac{1}{SCR_2} V_e(s) \right)$$

$$V_{out}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1} \frac{1}{SCR_2} V_e(s)$$

taking inverse laplace

$$V_{out} = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1} \frac{1}{R_2} \int_0^t \cancel{V_e(t)\, dt + V_0}$$

$$\left(\text{where } 1/s = \int_0^t dt + K \right)$$

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e\, dt + V(s)$$

where $G_P = \frac{R_2}{R_1}$ $\quad G_I = \frac{1}{R_2 C}$

PD CONTROLLER

$$I_1 + I_2 = I_3 \quad --- \quad (1)$$

$$I_3 + I_4 = 0 \quad --- \quad (2)$$

$$I_1 = \frac{V_e - V_a}{R_3}$$

$$I_2 = \frac{Cd}{dt}(V_e - V_a)$$

$$I_3 = \frac{V_a - V_b}{R_1} \quad (V_b = 0)$$

$$= \frac{V_a}{R_1}$$

$$I_4 = \frac{V_{out1} - V_b}{R_2} \quad (V_b = 0)$$

$$= \frac{V_{out1}}{R_2}$$

$$R = \frac{R_1 R_3}{R_1 + R_3}$$ ← effective resistance

Sub into eqn 1 & eqn 2

$$\frac{V_e - V_a}{R_3} + \frac{Cd}{dt}(V_e - V_a) = \frac{V_a}{R_1} \quad -- \quad *$$

$$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0 \quad - - \; *$$

from eqn * *

$$\frac{V_a}{R_1} = -\frac{V_{out1}}{R_2}$$

$$\boxed{V_a = \frac{-R_1}{R_2} V_{out1}}$$

rearranging eqn *

$$\frac{V_e - V_a}{R_3} + \frac{Cd}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$$

taking Laplace transform

$$\frac{V_e(s) - V_a(s)}{R_3} + sC(V_e(s) - V_a(s)) - \frac{V_a(s)}{R_1} = 0$$

(initial conditions go to zero)

$$\frac{V_e(s)}{R_3} + sC\,V_e(s) = \frac{V_a(s)}{R_1} + \frac{V_a(s)}{R_3} + sC\,V_a(s)$$

$$V_e(s)\left(\frac{1}{R_3} + sC\right) = V_a(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right)$$

recall, $V_a = \frac{-R_1}{R_2} V_{out1}$

$$V_e(s)\left(\frac{1}{R_3} + sC\right) = -\frac{R_1}{R_2} V_{out1}(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right)$$

taking lcm

$$V_e(s)\left(\frac{1 + R_3 sC}{R_3}\right) = \frac{-R_1}{R_2} V_{out1}(s)\left(\frac{R_3 + R_1 + sCR_1R_3}{R_1R_3}\right)$$

$$V_e(s)(1 + sCR_3) = \frac{-V_{out1}(s)}{R_2}(R_3 + R_1 + sCR_1R_3)$$

$$-V_{out1}(s) = \frac{V_e(s)(1 + sCR_3)R_2}{(R_1 + R_3 + sCR_1R_3)}$$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + sCR_2R_3)}{(R_1 + R_3 + sCR_1R_3)}$$

dividing num & denom by $R_1 + R_3$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + sCR_2R_3)/R_1 + R_3}{\frac{R_1 + R_3}{R_1 + R_3} + \frac{sCR_1R_3}{R_1 + R_3}}$$

recall $R = \frac{R_1R_3}{R_1 + R_3}$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + sCR_2R_3)/R_1 + R_3}{1 + sCR}$$

If $SCR << 1$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + SCR_2R_3)}{R_1 + R_3}$$

from the inverting circuit

$$V_{out1} = -V_{out} + V_0$$

$$\therefore -(-V_{out}(s) + V(0)) = \frac{V_e(R_2 + SCR_2R_3)}{R_1 + R_3}$$

$$V_{out}(s) - V(0) = \frac{V_e(s)R_2}{R_1 + R_3} + \frac{SCR_2R_3 V_e(s)}{R_1 + R_3}$$

$$V_{out}(s) = \frac{R_2}{R_1 + R_3} V_e(s) + \frac{R_2}{R_1 + R_3} R_3 CSV_e(s) + V(0)$$

taking inverse laplace

$$V_{out} = \frac{R_2}{R_1 + R_3} V_e + \frac{R_2}{R_1 + R_3} R_3 C \frac{dV_e}{dt} + V_0$$

$$\boxed{V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_0}$$

where $G_P = \frac{R_2}{R_1 + R_3}$ $\quad G_D = R_3C$