

Abimran Adepoju

IT/ENG05/002

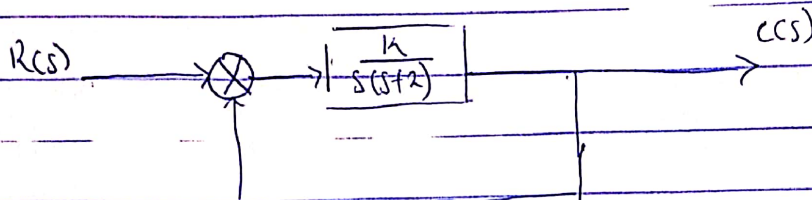
Mechatronics Engineering

(1)

• Explain the Root Locus Technique?

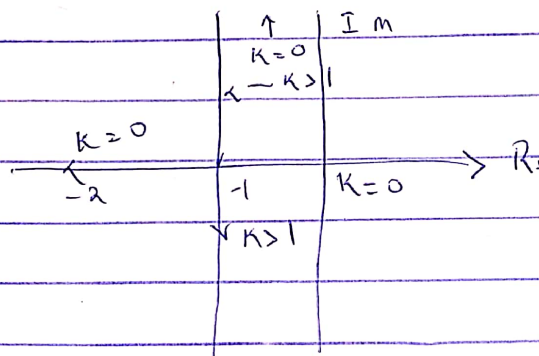
Root Locus is known as a graphical method in which roots of the characteristic equation are plotted in s-plane for the different values of parameters

The Locus of the roots of the characteristic equation when gain is varied from zero to infinity.



$$G(s) = \frac{k}{s(s+2)}, \quad H(s) = 1$$

$$1 + \frac{k}{s(s+2)} = 0 \text{ or } s^2 + 2s + k = 0$$



As k is varied, the two roots give the Loci in s-plane. For various values of k , the location of the roots are:

1) When $0 < k < 1$, The roots are real and distinct

2) When $k = 0$ The two roots are $s_1 = 0$ & $s_2 = -2$ These are also the open loop poles

3) When $k > 1$ Both roots are complex conjugate with real part = -1

When " k " is varying the roots locus is shown in the figure above

4) When $k = 1$, both roots are real and square

3) a) When $k=0$, two branches of root locus starts from $s=0$ and $s=-2$

b) When $k=1$, both roots meet at $s=-1$.

c) When $k>1$, the roots breakaway from the real axis and become complex conjugate having negative real part equal to -1

(2) (A)

$T(s) =$

$$\frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56}$$

s^5	1	6	8
s^4	7 $\rightarrow 1$	42 $\rightarrow 6$	56 $\rightarrow 8$
s^3	0 \rightarrow	0 \rightarrow	0 \rightarrow
s^2			
s^1			
s^0			

\therefore Third row consists of zeros.

By using figure 8 method

s^5	1	6	8
s^4	2	10	12
s^3	1	2	0
s^2	6	12	0
s^1	0	0	0
s^0			

s^n	a_n	a_{n-2}	a_{n-4}
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}
s^{n-2}	b_1	b_2	b_3
	c_1	c_2	c_3

$$\therefore b_1 = \frac{a_{n-1} \cdot a_{n-2} - a_n \cdot a_{n-3}}{a_{n-1}}, \quad b_2 = \frac{a_{n-1} \cdot a_{n-4} - a_n \cdot a_{n-5}}{a_{n-1}}$$

$$c_1 = \frac{b_1 \cdot a_{n-3} - a_{n-1} \cdot b_2}{b_1}, \quad c_2 = \frac{b_1 \cdot a_{n-5} - a_{n-1} \cdot b_3}{b_1}, \quad c_3 = \text{etc}$$

The table is continued horizontally & vertically until only zeros are obtained. Any row can be multiplied by a positive constant before the next row is computed without disturbing the properties of the table.

a) When entire row is zero in the r th table.

An entire row of zeros will appear in the Routh table when a purely even or purely odd polynomial is a factor of the original polynomial E.g.

$$\text{Let } T(s) = 10s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56$$

From the Routh table

s^5	1	6	5
s^4	7	42	56
s^3	0	0	0

If an ~~entire~~ even ~~po~~ entire row consists of zero setup. We return the row of zero and form auxiliary polynomial using entries now as coefficients. The polynomial will stand with power of s in the r th column and centre by stepping away power of s . Thus we have

$$P(s) = 7s^4 + 42s^2 + 56 \text{ which can be}$$

$$P(s) = s^4 + 6s^2 + 8$$

Differentiate (1) with respect to s .

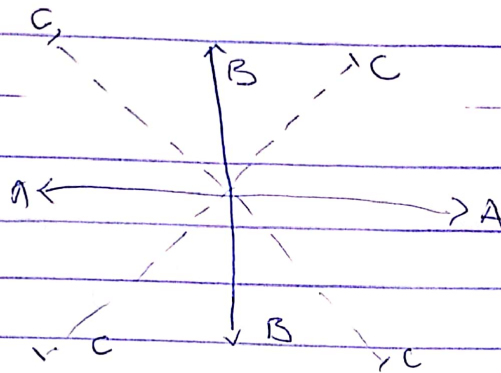
$$\frac{dP(s)}{ds} = 4s^3 + 12s + 0$$

s^3	4	12	0
s^2	3	8	0
s^1	$\frac{1}{3}$	0	0
s^0	8	0	0

There is no sign change \therefore The system is marginally stable

The eqn (1) is an even polynomial which only has roots that are symmetrical about the origin. The symmetry can occur under conditions of root position

- i) Root are symmetrical & imaginary
- ii) Roots are quadrilateral
- iii) Symmetrical and real



AA = Real & symmetrical about the origin.

\overline{BB} = Imaginary and symmetrical about the origin

\overline{CC} = Quadrilateral and symmetrical about the origin

1 (2B)

To determine the poles on the jw axis

Solu:

When the entries from the row before the row of zeros to the last row of the routh table are looking at even polynomials and there are no sign changes then all the poles that belong to the jw axis like the example used to illustrate the previous question.