Assignment 19/SA01/035

1. Find the eigen values and eigen vectors of $\begin{pmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{pmatrix}$

Solution

for eigen values $|(A - \lambda I)| = 0$

$$\left| \begin{pmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{vmatrix} = 0$$

$$2-\lambda \begin{vmatrix} 3-\lambda & 1 \\ 0 & 8-\lambda \end{vmatrix} - 7 \begin{vmatrix} 1 & 1 \\ 5 & 8-\lambda \end{vmatrix} + 0 \begin{vmatrix} 1 & 3-5 \\ 5 & 6 \end{vmatrix} = 0$$

$$2-\lambda \left((8-\lambda)(3-\lambda) \right) - 7\left((8-\lambda) - 5 \right) + 0 = 0$$

$$2-\lambda \left((8-\lambda)(3-\lambda) \right) - 7(3-\lambda) + 0 = 0$$

$$(3-\lambda)\left[(8-\lambda)(2-\lambda) - 7 \right] = 0$$

$$(3-\lambda)\left[16 - 8\lambda - 2\lambda + \lambda^2 - 7 \right] = 0$$

$$(3-\lambda)\left[\lambda^2 - 10\lambda + 9 \right] = 0$$

$$(3-\lambda)(\lambda - 9)(\lambda - 1) = 0$$

$\therefore \lambda = 3$ or $\lambda = 9$ or $\lambda = 1$

$\therefore$ the eigen values are 3, 9, 1

For eigen vector $(A - \lambda I)x = 0$

$$\begin{pmatrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

for $\lambda = 3$

$$\begin{pmatrix} 2-3 & 7 & 0 \\ 1 & 3-3 & 1 \\ 5 & 0 & 8-3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 7 & 0 \\ 1 & 0 & 1 \\ 5 & 0 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-x_1 + 7x_2 = 0$ — I

$x_1 + x_3 = 0$ — II

$5x_1 + 5x_3 = 0$ — III

from equation III $\quad 5x_1 + 5x_3 = 0$

$5x_1 = -5x_3$

$x_1/x_3 = -5/5$

$x_1/x_3 = -1/1 \quad \therefore x_1 = -1,\ x_3 = 1$

from equation I $\quad -x_1 + 7x_2 = 0$

$-(-1) + 7x_2 = 0$

$1 + 7x_2 = 0$

$x_2 = -1/7$

$$X_1 = \begin{pmatrix} -1 \\ -1/7 \\ 1 \end{pmatrix}$$

for $\lambda = 9$

$$\begin{pmatrix} 2-9 & 7 & 0 \\ 1 & 3-9 & 1 \\ 5 & 0 & 8-9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -7 & 7 & 0 \\ 1 & -6 & 1 \\ 5 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-7x_1 + 7x_2 = 0$ — I

$x_1 - 6x_2 + x_3 = 0$ — II

$5x_1 - x_3 = 0$ — III

from equation I $\quad -7x_1 + 7x_2 = 0$

$-7x_1 = 7x_2$

$x_1/x_2 = 1/1$

$\therefore x_1 = 1, \quad x_2 = 1$

from equation III $\quad 5x_1 - x_3 = 0$

$5(1) = x_3$

$5 = x_3$

$$X_2 = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$$

for $\lambda = 1$

$$\begin{pmatrix} 2-1 & 7 & 0 \\ 1 & 3-1 & 1 \\ 5 & 0 & 8-1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 7 & 0 \\ 1 & 2 & 1 \\ 5 & 0 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 + 7x_2 = 0$ — I

$x_1 + 2x_2 + x_3 = 0$ — II

$5x_1 + 7x_3 = 0$ — III

from equation I $\quad x_1 + 7x_2 = 0$

$$x_1 = -7x_2$$

$$x_1 / x_2 = -7/1$$

$\therefore \quad x_1 = -7, \quad x_2 = 1$

from equation III $\quad 5x_1 + 7x_3 = 0$

$$7(-7) + 7x_3 = 0$$

$$-49 + 7x_3 = 0$$

$$7x_3 = 49$$

$$x_3 = 7$$

$$X_3 = \begin{pmatrix} -7 \\ 1 \\ 7 \end{pmatrix}$$

2. State two properties of a diagonal matrix

(I) If A is the matrix, the diagonal of matrix A is the same as the transpose of matrix A

i.e $A = A^T$

(II) Under multiplication, diagonal matrices are commutative

i.e $PQ = QP$