14/SC101/035

Assignment

1. Find the eigen values and corresponding eigen vectors of $\begin{pmatrix} -3 & 0 & 6 \\ 4 & 5 & 3 \\ 1 & 2 & 1 \end{pmatrix}$

Solution

for eigen values $|(A - \lambda I)| = 0$

$$\left| \begin{pmatrix} -3 & 0 & 6 \\ 4 & 5 & 3 \\ 1 & 2 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} -3-\lambda & 0 & 6 \\ 4 & 5-\lambda & 3 \\ 1 & 2 & 1-\lambda \end{vmatrix} = 0$$

$$-3-\lambda \begin{vmatrix} 5-\lambda & 3 \\ 2 & 1-\lambda \end{vmatrix} - 0 \begin{vmatrix} 4 & 3 \\ 1 & 1-\lambda \end{vmatrix} + 6 \begin{vmatrix} 4 & 5-\lambda \\ 1 & 2 \end{vmatrix} = 0$$

$$-3-\lambda((5-\lambda)(1-\lambda)-6) - 0 + 6(8-(5-\lambda)) = 0$$

$$[(-3-\lambda)(5-\lambda)(1-\lambda)] + 18 + 6\lambda + 48 - 6(5-\lambda) = 0$$

$$[(-3-\lambda)(5-\lambda)(1-\lambda)] + 66 + 6\lambda - 30 + 6\lambda = 0$$

$$[(-3-\lambda)(5-\lambda)(1-\lambda)] + 36 + 12y = 0$$

$$[(-3-\lambda)(5-\lambda)(1-\lambda)] - 12(-3-\lambda) = 0$$

$$(-3-\lambda)\ [(5-\lambda)(1-\lambda) - 12] = 0$$

$$(-3-\lambda)\ [\lambda^2 - 6\lambda - 7] = 0$$

$$(-3-\lambda)\ [(\lambda-7)(\lambda+1)] = 0$$

$\therefore \lambda = -3$ or $\lambda = 7$ or $\lambda = -1$

$\therefore$ the eigenvalues are $-3, 7, 1$

for eigen vectors $(A - \lambda I)x = 0$

for $\lambda = -3$

$$\begin{pmatrix} -3+3 & 0 & 6 \\ 4 & 5+3 & 3 \\ 1 & 2 & 1+4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 6 \\ 4 & 8 & 3 \\ 1 & 2 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_3 = 0$ ——— I

$4x_1 + 8x_2 + 3x_3 = 0$ ——— II

$x_1 + 2x_2 + 5x_3 = 0$ ——— III

from equation III $\quad x_1 + 2x_2 + 5x_3 = 0$

$x_1 + 2x_2 + 5(0) = 0$

$x_1 + 2x_2 = 0$

$x_1/x_2 = -2/1$

$\therefore x_1 = -2, \quad x_2 = 1$

$$X_1 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

for $\lambda = 7$

$$\begin{pmatrix} -3-7 & 0 & 6 \\ 4 & 5-7 & 3 \\ 1 & 2 & 1-7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -10 & 0 & 6 \\ 4 & -2 & 3 \\ 1 & 2 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-10x_1 + 6x_3 = 0$ —— I

$4x_1 - 2x_2 + 3x_3 = 0$ —— II

$x_1 + 2x_2 - 6x_3 = 0$ —— III

from equation I $\quad -10x_1 + 6x_3 = 0$

$-10x_1 = -6x_3$

$x_1 / x_3 = 6/10 = 3/5$

$\therefore x_1 = 3 \quad x_3 = 5$

from equation III $\quad x_1 + 2x_2 - 6x_3 = 0$

$3 + 2x_2 - 6(5) = 0$

$\frac{2x_2}{2} = \frac{27}{2}$

$x_2 = 13.5$

$$X_2 = \begin{pmatrix} 3 \\ 13.5 \\ 5 \end{pmatrix}$$

for $\lambda = -1$

$$\begin{pmatrix} -3+1 & 0 & 6 \\ 4 & 5+1 & 3 \\ 1 & 2 & 1+1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 & 6 \\ 4 & 6 & 3 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$$

$-2x_1 + 6x_3 = 0$ —— I

$4x_1 + 6x_2 + 3x_2 = 0$ —— II

$x_1 + 2x_2 + 2x_3 = 0$ —— III

from equation I $\quad -2x_1 + 6x_3 = 0$

$2x_1 = 6x_3$

$x_1 / x_3 = 6/2 = 3/1 \qquad \therefore x_1 = 3 \quad x_3 = 1$

from equation III $\quad x_1 + 2x_2 + 2x_3 = 0$

$3 + 2x_2 + 2(1) = 0$

$2x_2 + 5 = 0$

$2x_2 = -5$

$x_2 = -5/2 = -2.5 \qquad X_3 = \begin{pmatrix} 3 \\ -2.5 \\ 3 \end{pmatrix}$

2. find the eigen values and eigen vectors of $\begin{pmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{pmatrix}$

Solution

for eigen values $|(A - \lambda I)| = 0$

$$\left| \begin{pmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{vmatrix} = 0$$

$$2-\lambda \begin{vmatrix} 3-\lambda & 1 \\ 0 & 8-\lambda \end{vmatrix} - 7 \begin{vmatrix} 1 & 1 \\ 5 & 8-\lambda \end{vmatrix} + 0 \begin{vmatrix} 1 & 3-5 \\ 5 & 0 \end{vmatrix} = 0$$

$$2-\lambda\ [(8-\lambda)(3-\lambda)] - 7((8-\lambda)-5) + 0 = 0$$

$$2-\lambda\ [(8-\lambda)(3-\lambda)] - 7(3-\lambda) = 0$$

$$(3-\lambda)[(8-\lambda)(2-\lambda) - 7] = 0$$

$$(3-\lambda)[\lambda^2 - 10\lambda + 9] = 0$$

$$(3-\lambda)(\lambda-9)(\lambda-1) = 0$$

$\therefore \lambda = 3$ or $\lambda = 9$ or $\lambda = 1$

$\therefore$ the eigenvalues are 3, 9, 1

for eigen vectors $(A - \lambda I)x = 0$

for $\lambda = 3$

$$\begin{pmatrix} 2-3 & 7 & 0 \\ 1 & 3-3 & 1 \\ 5 & 0 & 8-3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 7 & 0 \\ 1 & 0 & 1 \\ 5 & 0 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-x_1 + 7x_2 = 0$ —— I

$x_1 + x_3 = 0$ —— II

$5x_1 + 5x_3 = 0$ —— III

from equation III $\quad 5x_1 + 5x_3 = 0$

$5x_1 = -5x_3$

$x_1/x_3 = -5/5 = -1/1 \quad \therefore x_1 = -1,\ x_3 = 1$

from equation I $\quad -x_1 + 7x_2 = 0$

$-(-1) + 7x_2 = 0$

$7x_2 = -1$

$x_2 = -1/7 \qquad X_1 = \begin{pmatrix} -1 \\ -1/7 \\ 1 \end{pmatrix}$

for $\lambda = 9$

$$\begin{pmatrix} 2-9 & 7 & 0 \\ 1 & 3-9 & 1 \\ 5 & 0 & 8-9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -7 & 7 & 0 \\ 1 & -6 & 1 \\ 5 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-7x_1 + 7x_2 = 0$ —— I

$x_1 - 6x_2 + x_3 = 0$ —— II

$5x_1 - x_3 = 0$ — III

from equation I $-7x_1 + 7x_2 = 0$

$7x_1 = 7x_2$

$x_1/x_2 = 1/1$ $\therefore x_1 = 1 \quad x_2 = 1$

from equation III $5x_1 - x_3 = 0$

$5(1) = x_3$

$x_3 = 5$ $X_2 = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$

for $\lambda = 1$

$$\begin{pmatrix} 2-1 & 7 & 0 \\ 1 & 3-1 & 1 \\ 5 & 0 & 8-1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 7 & 0 \\ 1 & 2 & 1 \\ 5 & 0 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 + 7x_2 = 0$ — I

$x_1 + 2x_2 + x_3 = 0$ — II

$5x_1 + 7x_3 = 0$ — III

from equation I $x_1 + 7x_2 = 0$

$x_1 = -7x_2$

$x_1/x_2 = -7/1$ $\therefore x_1 = -7, \ x_2 = 1$

from equation III $5x_1 + 7x_3 = 0$

$7(-7) + 7x_3 = 0$

$\frac{7x_3}{7} = \frac{49}{7}$

$x_3 = 7$ $X_3 = \begin{pmatrix} -7 \\ 1 \\ 7 \end{pmatrix}$