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19/SCI01/034

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MAT203

Find the eigen values and the corresponding eigen vectors of the following

1) $\begin{bmatrix} -3 & 0 & 6 \\ 4 & 5 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

for eigenvalues, $|A - \lambda I| = 0$

$$\left| \begin{bmatrix} -3 & 0 & 6 \\ 4 & 5 & 3 \\ 1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -3-\lambda & 0 & 6 \\ 4 & 5-\lambda & 3 \\ 1 & 2 & 1-\lambda \end{vmatrix} = 0$$

$$-3 - \lambda [(5-\lambda)(1-\lambda) - 6] + 6[8 - (5-\lambda)] = 0$$

$$-3 - \lambda [5 - 5\lambda - \lambda + \lambda^2 - 6] + 6[8 - 5 + \lambda] = 0$$

$$-3 - \lambda [\lambda^2 - 6\lambda - 1] + 18 + 6\lambda = 0$$

$$-3\lambda^2 + 18\lambda + 3 - \lambda^3 + 6\lambda^2 + \lambda + 18 + 6\lambda = 0$$

$$-\lambda^3 + 3\lambda^2 + 25\lambda + 21 = 0$$

$\therefore$ The eigen values are

$7, -3, -1$

For eigenvectors

$(A - \lambda I)x = 0$

$$\left[\begin{bmatrix} -3 & 0 & 6 \\ 4 & 5 & 3 \\ 1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3-\lambda & 0 & 6 \\ 4 & 5-\lambda & 3 \\ 1 & 2 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

2 $\begin{bmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{bmatrix}$

For eigenvalues $|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{vmatrix} = 0$$

$2-\lambda[(3-\lambda)(8-\lambda)] - 7[8-\lambda-5] = 0$

$2-\lambda[(3-\lambda)(8-\lambda)] - 7[3-\lambda] = 0$

$2-\lambda[24 - 3\lambda - 8\lambda + \lambda^2] - 21 + 7\lambda = 0$

$48 - 6\lambda - 16\lambda + 2\lambda^2 - 24\lambda + 3\lambda^2 + 8\lambda^2 - \lambda^3 - 21 + 7\lambda = 0$

$-\lambda^3 + 13\lambda^2 - 39\lambda + 27 = 0$

$\therefore$ The eigen values are
9, 1, 3

When $\lambda = 9$

$$\begin{bmatrix} -7 & 7 & 0 \\ 1 & -6 & 1 \\ 5 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-7x_1 + 7x_2 = 0$ — (i)

$x_1 - 6x_2 + x_3 = 0$ — (ii)

$5x_1 - x_3 = 0$ — (iii)

from (i)

$-7x_1 = -7x_2$

$x_1 = x_2$

$x_1 : x_2 = 1 : 1$

$x_1 = 1, \ x_2 = 1$

eqn (iii) becomes

$5(1) - x_3 = 0$

$5 - x_3 = 0$

$-x_3 = -5$

$x_3 = 5$

$$\lambda_1 = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

When $\lambda = 1$

$$\begin{bmatrix} 1 & 7 & 0 \\ 1 & 2 & 1 \\ 5 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + 7x_2 = 0$ — (i)

$x_1 + 2x_2 + x_3 = 0$ — (ii)

$5x_1 + 7x_3 = 0$ — (iii)

from (i)

$x_1 = -7x_2$

$x_1 : x_2 = -7 : 1$

$\therefore x_1 = -7, \ x_2 = 1$

~~from~~

eqn (11) becomes

$5(-7) + 7x_3 = 0$

$35 + 7x_3 = 0$

$7x_3 = -35$

$x_3 = -5$

$$X_2 = \begin{bmatrix} -7 \\ 1 \\ -5 \end{bmatrix}$$

When $\lambda = 3$

$$\begin{bmatrix} -1 & 7 & 0 \\ 1 & 0 & 1 \\ 5 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1 + 7x_2 = 0$ — (1)

$x_1 + x_3 = 0$ — (11)

$5x_1 + 5x_3 = 0$ — (111)

from eqn (11)

$5x_1 + 5x_3 = 0$

$5x_1 = -5x_3$

$x_1 = -x_3$

$x_1 : x_3 = -1 : 1$

$x_1 = -1, \quad x_3 = 1$

eqn (1) becomes

$-(-1) + 7x_2 = 0$

$1 + 7x_2 = 0$

$7x_2 = -1$

$x_2 = -1/7$

$$X_3 = \begin{bmatrix} -1 \\ -1/7 \\ 1 \end{bmatrix}$$