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Computer science

MAT 205

Question 1

Find the eigen values and eigen vectors of

$$\begin{bmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{bmatrix}$$

Eigen value

$$\begin{bmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{bmatrix}$$

$$\begin{array}{c|c|c|c|c|c|c|c|c|c|} 2-\lambda & 3-\lambda & 1 & | & -7 & 1 & \oplus & | & +0 & | & 1 & 3-\lambda & | \\ & 0 & 8-\lambda & | & 5 & 8-\lambda & & | & & | & 5 & 0 & | \end{array}$$

$$2-\lambda [(3-\lambda)(8-\lambda)] - 7[(8-\lambda) - 5]$$

$$2-\lambda [24 - 3\lambda - 8\lambda + \lambda^2] - 7(8-\lambda) + 35$$

$$-\lambda^3 + 2\lambda^2 + 3\lambda^2 + 8\lambda^2 - 6\lambda - 16\lambda + 7\lambda - 24\lambda + 48 - 56 + 35$$

$$-\lambda^3 + (3\lambda^2 - 39\lambda + 27) = 0$$

35

Factorise by cubic polynomial

$$x(x-1)(x^2 - 12x + 27) = 0$$

$$(x-1)(x^2 - 9x - 3x + 27) = 0$$

$$(x-1)(x(x-9) - 3(x-9)) = 0$$

$$(x-1)(x-3)(x-9) = 0$$

$$\lambda_1 = 1 \quad \lambda_2 = 3 \quad \lambda_3 = 9$$

Eigen vectors

$$(A - \lambda I)x = 0$$

$$\begin{pmatrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{pmatrix} \text{ when } \lambda = 1$$

$$\begin{pmatrix} 1 & 7 & 0 \\ 1 & 2 & 1 \\ 5 & 0 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x_1 + 7x_2 = 0 \quad - \text{ i}$$

$$x_1 + 2x_2 + x_3 = 0 \quad - \text{ ii}$$

$$5x_1 + 7x_3 = 0 \quad - \text{ iii}$$

from eqn (i)

$$x_1 = -7x_2$$

$$x_1/x_2 = -7/1$$

$$x_1 = -7, \quad x_2 = 1$$

from eq (ii)

$$-7 + 2(1) + x_3 = 0$$

$$x_3 = 5$$

Hence $x_1 = \begin{pmatrix} -7 \\ 1 \\ 5 \end{pmatrix}$

When $\lambda = 3$

$$\begin{array}{ccc|c} 2-\lambda & 7 & 0 & 0 \\ 1 & 3-\lambda & 1 & 0 \\ 5 & 0 & 8-\lambda & 0 \end{array}$$

$$\begin{pmatrix} -1 & 7 & 0 \\ 1 & 0 & 1 \\ 5 & 0 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-x_1 + 7x_2 = 0$$

$$x_1 + x_3 = 0$$

$$5x_1 + 5x_3 = 0$$

from eq (i)

$$\frac{x_1}{x_2} = \frac{2}{-1} = -2$$

$$x_1 = 7, \quad x_2 = 1$$

from eq (iii)

$$5(7) + 5x_3 = 0$$

$$\frac{5x_3}{5} = -\frac{35}{5}$$

$$x_3 = \underline{\underline{-7}}$$

$$\text{hence } x_2 = \begin{pmatrix} 7 \\ 1 \\ -7 \end{pmatrix}$$

When $\lambda = 9$

$$2 - \lambda \quad 7 \quad 0$$

$$1 \quad 3 - \lambda \quad 1$$

$$5 \quad 0 \quad 8 - \lambda$$

$$\begin{pmatrix} -7 & 7 & 0 \\ 1 & -6 & 1 \\ 5 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-7x_1 + 7x_2 = 0 \quad \text{--- i}$$

$$x_1 - 6x_2 + x_3 = 0 \quad \text{--- ii}$$

$$5x_1 - x_3 = 0 \quad \text{--- iii}$$

From eqn (iii)

$$\frac{x_1}{x_3} = \frac{1}{5}$$

$$x_1 = 1 \quad ; \quad x_3 = 5$$

From eqn i

$$-7x_1 + 7x_2 = 0$$

$$\frac{7x_2}{7} = \frac{7}{7}$$

$$x_2 = 1$$

$$\text{Hence } x_3 = \begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix}$$

Question 2

Properties of diagonal matrix

1) If addition / multiplication is being applied on diagonal matrices then the matrices are the same

2) When you transpose a diagonal matrix it is the same as the original