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Matric NO: 19/sci01/083

Assignment

Find the eiguavalues and the corresponding

$$A = \begin{pmatrix} -3 & 0 & 6 \\ 4 & 5 & 3 \\ 1 & 2 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 7 & 0 \\ 1 & 3 & 0 \\ 5 & 0 & 8 \end{pmatrix}$$

Solution

To find the eigvalues $\therefore |A - \lambda I| = 0$.

$$\left| \begin{pmatrix} -3 & 0 & 6 \\ 4 & 5 & 3 \\ 1 & 2 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\left| \begin{pmatrix} -3-\lambda & 0 & 6 \\ 4 & 5-\lambda & 3 \\ 1 & 2 & 1-\lambda \end{pmatrix} \right| = 0$$

$$-3-\lambda((5-\lambda)(1-\lambda) - 6) + 0 + 6(8-(5-\lambda))$$

$$-3-\lambda(\lambda^2 - 6\lambda - 1) + 6(3-\lambda)$$

$$3 + 19\lambda + 3\lambda^2 - \lambda^3 + (18 - 6\lambda) = 0$$

$$-\lambda^3 + 3\lambda^2 + 25\lambda + 21 = 0$$

$$\lambda^3 - 3\lambda^2 - 25\lambda - 21 = 0$$

$$\lambda = -3 \text{ or } -1 \text{ or } 7$$

For eigevectors $(A - \lambda I)x = 0$

$$\left(\begin{pmatrix} -3 & 0 & 6 \\ 4 & 5 & 3 \\ 1 & 2 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} -3-\lambda & 0 & 6 \\ 4 & 5-\lambda & 3 \\ 1 & 2 & 1-\lambda \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

When $\lambda = -3$

$$\begin{pmatrix} -3-(-3) & 0 & 6 \\ 4 & 5-(-3) & 3 \\ 1 & 2 & 1-(-3) \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 6 \\ 4 & 8 & 3 \\ 1 & 2 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$6x_3 = 0 \quad -(1)$

$4x_1 + 8x_2 + 3x_3 = 0 \to (2)$

$x_1 + 2x_2 + 4x_3 = 0 \quad -(3)$

From (1)

$6x_3 = 0$

$x_3 = 0$

equ (iii) becomes

$x_1 + 2x_2 + 0 = 0$

$x_1 = -2x_2$

$x_1 : x_2 = -2 : 1$

$$X = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

When $\lambda = 1$

$$\begin{pmatrix} -2 & 0 & 6 \\ 4 & 6 & 3 \\ 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-2x_1 + 6x_3 = 0 \quad -(1)$

$\cancel{4x_1} + 4x_1 + 6x_2 + 3x_3 = 0 \quad -(2)$

$x_1 + 2x_2 + 2x_3 = 0 \quad -(3)$

From (1)

$-2x_1 + 6x_3 = 0$

$-2x_1 = -6x_3 \quad [-6/2] = 3$

$x_1 : x_3 = 3 : 1$

equ (iii) becomes

$x_1 + 2x_2 + 2x_3 = 0$

$3 + 2x_2 + 2(1) = 0$

$3 + 2x_2 + 2 = 0$

$5 + x_2 = 0$

$x_2 = -5$

$$X_2 = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$$

When $\lambda = 7$

$$\begin{pmatrix} 10 & 0 & 6 \\ 4 & -2 & 3 \\ 1 & 2 & -6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$10x_1 + 6x_3 = 0$

$4x_1 - 2x_2 + 3x_3 = 0$

$x_1 + 2x_2 - 6x_3 = 0$

from (i)

$\frac{10x_1}{2} = -\frac{6x_3}{2}$

$5x_1 = -3x_3$

$x_1 : x_3 = -3 : 5$

$= -0.6 : 1$

equ (iii) becomes

$-0.6 + 2x_2 - 6(1) = 0$

$2x_2 = 6.6$

$x_2 = 3.3$

$$X_3 = \begin{pmatrix} -0.6 \\ 3.3 \\ 1 \end{pmatrix}$$

2) $\begin{pmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{pmatrix}$

Solution

To find the eigenvalues $= |A - \lambda I| = 0$

$$\left| \begin{pmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\left| \begin{pmatrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{pmatrix} \right| = 0$$

$$(2-\lambda)(0-(3-\lambda)(8-\lambda) - 7((8-\lambda) - 5) + 0 = 0$$

$$(2-\lambda)(24 - 11\lambda + \lambda^2) - 7(3-\lambda) = 0$$

$$(48 - 46\lambda + 13\lambda^2 - \lambda^3) - (21 - 7\lambda) + 0 = 0$$

$$(-\lambda^3 + 13\lambda^2 - 39\lambda + 27) = 0$$

$$\lambda^3 + 13\lambda^2 + 39\lambda - 27 = 0$$

$$\lambda = 1 \text{ or } 3 \text{ or } 9$$

For eigevectors $(A - \lambda I)x = 0$

$$\begin{pmatrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

When $\lambda = 1$

$$\begin{pmatrix} 2-1 & 7 & 0 \\ 1 & 3-1 & 1 \\ 5 & 0 & 8-1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 7 & 0 \\ 1 & 2 & 1 \\ 5 & 0 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$x_1 + 7x_2 = 0 \quad -(1)$

$x_1 + 2x_2 + x_3 = 0 \quad -(2)$

$5x_1 + x_2 + 7x_3 = 0 \quad -(3)$

From

$x_1 + 7x_2 = 0$

$x_1 = -7x_2$

$x_1 : x_2 = -7 : 1$

equ (ii) becomes

$\cancel{-7 + 2(1) +}$

$-7 + 2(1) + x_3 = 0$

$\cancel{-7+2} -5 + x_3 = 0$

$\cancel{x_3 = +5} = -4$

$$X_1 = \begin{pmatrix} -7 \\ 1 \\ -4 \end{pmatrix}$$

$\lambda = 3 = \begin{pmatrix} -1 & 7 & 0 \\ 1 & 0 & 1 \\ 5 & 0 & 5 \end{pmatrix}$ $X_0 = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$.

$-x_1 + 7x_2 = 0$ — (1)

$x_1 + x_3 = 0$ — (2)

$5x_1 + 5x_3 = 0$ — 3

From (1):

$-x_1 + 7x_2 = 0$

$-x_1 = -7x_2$

$x_1 = 7x_2$

$x_1 : x_2 = 7 : 1$

equ (ii) becomes

$7 + x_3 = 0$

$x_3 = -7$

$X_2 = \begin{pmatrix} 7 \\ 1 \\ -7 \end{pmatrix}$

When $\lambda = 9$

$\begin{pmatrix} -7 & 7 & 0 \\ 1 & -6 & 1 \\ 5 & 0 & -1 \end{pmatrix}$

$-7x + 7x_2 = 0$

$x_1 - 6x_2 + x_3 = 0$

$5x_1 + 0x_2 - x_3 = 0$

from (1)

$-7x_1 + 7x_2 = 0$

$-7x_1 = -7x_2$

$x_1 = x_2$

$x_1 : x_2 = 1 : 1$

equ (iii) becomes

$5(1) - x_3 = 0$

$-x_3 = -5$

$x_3 = 5$