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Matric: 19/SCI01/088

2) $\begin{pmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{pmatrix}$

Solution

To find the eigenvalues = $|A - \lambda I| = 0$

$$\left|\begin{pmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}\right| = 0$$

$$\left|\begin{pmatrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{pmatrix}\right| = 0$$

$(2-\lambda)(0-(3-\lambda)(8-\lambda)) - 7((8-\lambda) - 5) + 0 = 0$

$(2-\lambda)(24 - 11\lambda + \lambda^2) - 7(3-\lambda) = 0$

$(48 - 46\lambda + 13\lambda^2 - \lambda^3) - (21 - 7\lambda) + 0 = 0$

$(-\lambda^3 + 13\lambda^2 - 39\lambda + 27) = 0$

$\lambda^3 \div 13\lambda^2 + 39\lambda - 27 = 0$

$\lambda = 1$ or 3 or 9.

For eigenvectors $(A - \lambda I)x = 0$

$$\begin{pmatrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{pmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

When $\lambda = 1$

$$\begin{pmatrix} 2-1 & 7 & 0 \\ 1 & 3-1 & 1 \\ 5 & 0 & 8-1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 7 & 0 \\ 1 & 2 & 1 \\ 5 & 0 & 7 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$x_1 + 7x_2 = 0$ — (1)

$x_1 + 2x_2 + x_3 = 0$ — (2)

$5x_1 + x_2 + 7x_3 = 0$ — (3)

From

$x_1 + 7x_2 = 0$

$x_1 = -7x_2$

$x_1 : x_2 = -7 : 1$

equation(ii) becomes

$\cancel{-7 + 2(1) +}$

$-7 + 2(1) + x_3 = 0$

$\cancel{-7+2}\ -5 + x_3 = 0$

$\cancel{x_3 = +5} = -4$

$X_1 = \begin{pmatrix} -7 \\ 1 \\ -4 \end{pmatrix}$

$X_3 = \begin{pmatrix}1\\1\\3\end{pmatrix}$.

$\lambda = 3 = \begin{pmatrix}-1 & 7 & 0\\1 & 0 & 1\\5 & 0 & 5\end{pmatrix}$

$-x_1 + 7x_2 = 0$ —(1)

$x_1 + x_3 = 0$ —(2)

$5x_1 + 5x_3 = 0$ — 3

From (1)

$-x_1 + 7x_2 = 0$

$-x_1 = -7x_2$

$x_1 = 7x_2$

$x_1 : x_2 = 7 : 1$

equ (ii) becomes

$7 + x_3 = 0$

$x_3 = -7$

$X_2 = \begin{pmatrix}7\\1\\-7\end{pmatrix}$

When $\lambda = 9$

$\begin{pmatrix}-7 & 7 & 0\\1 & -6 & 1\\5 & 0 & -1\end{pmatrix}$

$-7x + 7x_2 = 0$

$x_1 - 6x_2 + x_3 = 0$

$5x_1 + 0x_2 - x_3 = 0$

from (1)

$-7x_1 + 7x_2 = 0$

$-7x_1 = -7x_2$

$x_1 = x_2$

$x_1 : x_2 = 1 : 1$

equ (iii) becomes

$5(1) - x_3 = 0$

$-x_3 = -5$

$x_3 = 5$

(2) Properties of diagonal Matrices

(1) If addition or multiplication is being applied on diagonal matrices, than the matrices should be of the same order.

(2) When transpose a diagonal matrix, it is just the same as the orginal because all the diagonal numbers are 0.