Assignment 19/SCA01/035

① State Cayley-Hamilton theorem

Ans: The Cayley-Hamilton theorem states that every square matrix $(n \times n)$ over a commutative ring satisfies its own characteristic equation. If A is a provided as $n \times n$ matrix and I_n is the $n \times n$ identity matrix, then the distinctive polynomial of A is articulated as:

$$p(x) = \det(xI_n - A)$$

② Find the characteristics polynomial of

$$\begin{pmatrix} 2 & 27 & 0 \\ 0 & 4 & 40 \\ 0 & 3 & 30 \end{pmatrix}$$

Solution

$tr(A) = 2 + 4 + 30$

$tr(A) = 36$

$|A| = 2\begin{vmatrix} 4 & 40 \\ 3 & 30 \end{vmatrix} - 27\begin{vmatrix} 0 & 40 \\ 0 & 30 \end{vmatrix} + 0\begin{vmatrix} 0 & 4 \\ 0 & 3 \end{vmatrix}$

$|A| = 2(120 - 120) - 27(0-0) + 0(0-0)$

$|A| = 2(0) - 0 + 0$

$|A| = 0$

$A_{11} = (-1)^2 \begin{vmatrix} 4 & 40 \\ 3 & 30 \end{vmatrix}$

$A_{11} = 1 \times (120-120) = 0$

$A_{22} = (-1)^{4} \begin{vmatrix} 2 & 0 \\ 0 & 30 \end{vmatrix}$

$A_{22} = 1(60-0) = 60$

$A_{33} = (-1)^6 \begin{vmatrix} 2 & 27 \\ 0 & 4 \end{vmatrix}$

$A_{33} = 1 \times (8-0) = 8$

$At = t^3 - tr(A)t^2 + (A_{11} + A_{22} + A_{33})t - \det(A)$

$At = t^3 - 36t^2 + (0+60+8)t - 0$

$At = t^3 - 36t^2 + 68t$