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1) Eigen value and eigen vectors of

$$\begin{bmatrix} -3 & 0 & 6 \\ 4 & 5 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

Solution

$$\left| \begin{pmatrix} -3 & 0 & 6 \\ 4 & 5 & 3 \\ 1 & 2 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right|$$

$$\begin{vmatrix} -3-\lambda & 0 & 6 \\ 4 & 5-\lambda & 3 \\ 1 & 3 & 1-\lambda \end{vmatrix} = 0$$

$$-3-\lambda \begin{vmatrix} 5-\lambda & 3 \\ 2 & 1-\lambda \end{vmatrix} - 0 + 6 \begin{vmatrix} 4 & 5-\lambda \\ 1 & 2 \end{vmatrix} = 0$$

$$-3-\lambda((5-\lambda)(1-\lambda)-6] - 0 + 6(8-(5-\lambda)) = 0$$

$$[(3-\lambda)(5-\lambda)(1-\lambda)] + 18 + 6\lambda + 48 - 6(5-\lambda) = 0$$

$$[(-3-\lambda)(5-\lambda)(1-\lambda)] + 36 + 12\lambda = 0$$

$$[(-3-\lambda)(5-\lambda)(1-\lambda)] - 12(-3-\lambda) = 0$$

$$(-3-\lambda)[(5-\lambda)(1-\lambda) - 12] = 0$$

$$(-3-\lambda)[\lambda^2 - 6\lambda - 7] = 0$$

$$(-\lambda-3)[(\lambda-7)(\lambda+1)] = 0$$

$$\lambda = -3 \text{ or } \lambda = 7 \text{ or } \lambda = -1$$

eigen vectors

$\lambda = -3$

$$\begin{bmatrix} 0 & 0 & 6 \\ 4 & 8 & 3 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$6x_3 = 0 \quad - I$

$4x_1 + 8x_2 + 3x_3 = 0 \quad - ii$

$x_1 + 2x_2 + 5x_3 = 0 \quad - iii$

eqn iii

$x_1 + 2x_2 + 5x_3 = 0$

$x_1 + 2x_2 + 5(0) = 0$

$x_1 + 2x_2 = 0$

$x_1 / x_2 = -2/1$

$x_1 = -2, \quad x_2 = 1$

$$\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda = 7$

$$\begin{bmatrix} -10 & 0 & 6 \\ 4 & -2 & 3 \\ 1 & 2 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-10x_1 + 6x_3 = 0$ — i

$4x_1 - 2x_2 + 3x_3 = 0$ — ii

$x_1 + 2x_2 - 6x_3 = 0$ — iii

eqn i

$-10x_1 + 6x_3 = 0$

$-10x_1 = 6x_3$

$x_1/x_3 = 6/-10$

$x_1 = 3 \quad x_3 = 5$

eqn iii

$x_1 + 2x_2 - 6x_3$

$3 + 2x_2 - 30 = 0$

$\frac{2x_2}{2} = \frac{27}{2}$

$x_2 = 13.5$

$$\begin{pmatrix} 3 \\ 13.5 \\ 5 \end{pmatrix}$$

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Question 1

Find the eigen values and eigen vectors of

$$\begin{bmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{bmatrix}$$

eigen value

$$\begin{bmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{bmatrix}$$

$$2-\lambda \begin{vmatrix} 3-\lambda & 1 \\ 0 & 8-\lambda \end{vmatrix} - 7 \begin{vmatrix} 1 & 1 \\ 5 & 8-\lambda \end{vmatrix} + 0 \begin{vmatrix} 1 & 3-\lambda \\ 5 & 0 \end{vmatrix}$$

$$2-\lambda\,((3-\lambda)(8-\lambda)] - 7((8-\lambda)-5]$$

$$2-\lambda\,[24 - 3\lambda - 8\lambda + \lambda^2] - 7(8-\lambda] + 35$$

$$-\lambda^3 + 2\lambda^2 + 3\lambda^2 + 8\lambda^2 - 6\lambda - 16\lambda + 7\lambda - 24\lambda + 48 - 56$$

35

$$-\lambda^3 + 13\lambda^2 - 39 + 27 = 0$$

Factorise by cubic polynomial

$x(\lambda-1)(\lambda^2-12\lambda+27)=0$

$(\lambda-1)(\lambda^2-9\lambda-3\lambda+27)=0$

$(\lambda-1)(\lambda(\lambda-9)-3(\lambda-9))=0$

$(\lambda-1)(\lambda-3)(\lambda-9)$

$\lambda_1=1 \quad \lambda_2=3 \quad \lambda_3=9$

eigen vector

$(A-\lambda I)x=0$

$$\begin{bmatrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{bmatrix}$$ when $\lambda=1$

$$\begin{pmatrix} 1 & 7 & 0 \\ 1 & 2 & 1 \\ 5 & 0 & 7 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1+7x_2=0$ - i

$x_1+2x_2+x_3=0$ - ii

$5x_1+7x_3=0$ - iii

from equ (i)

$x_1=-7x_2$

$x_1/x_2=-7/1$

$x_1=-7,\ x_2=1$

from equ (iv)

$-7 + 2(1) + x_3 = 0$

$x_3 = 5$

Hence $x_1 = \begin{pmatrix} -7 \\ 1 \\ 5 \end{pmatrix}$

When $\lambda = 3$

$$\begin{matrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{matrix}$$

$$\begin{pmatrix} -1 & 7 & 0 \\ 1 & 0 & 1 \\ 5 & 0 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-x_1 + 7x_2 = 0$

$x_1 + x_3 = 0$

$5x_1 + 5x_3 = 0$

from eq (i)

$\frac{x_1}{x_2} = \frac{-7}{-1}$

$x_1 = 7, \quad x_2 = 1$

from eq (iii)

$5(7) + 5x_3 = 0$

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$$\frac{5x_3}{5} = \frac{-35}{5}$$

$x_3 = -7$ hence $x_2 = \begin{pmatrix} 7 \\ 1 \\ -7 \end{pmatrix}$

When $\lambda = 9$

$$\begin{matrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{matrix}$$

$$\begin{pmatrix} -7 & 7 & 0 \\ 1 & -6 & 1 \\ 5 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-7x_1 + 7x_2 = 0$ - i

$x_1 - 6x_2 + x_3 = 0$ - ii

$5x_1 - x_3 = 0$ - iii

from equ (iii)

$$\frac{x_1}{x_3} = \frac{1}{5}$$

$x_1 = 1$; $x_3 = 5$

from equ i

$-7(1) + 7x_2 = 0$ Hence $x_3 = \begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix}$

$$\frac{7x_2}{7} = \frac{7}{7}$$

$x_2 = 1$

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