David Oluomachukwu favour
19/sci01/037
Computer Science

Find the eigen values and the corresponding eigen vectors of the following matrices

1. $\begin{pmatrix} -3 & 0 & 6 \\ 4 & 5 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ 2. $\begin{pmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{pmatrix}$

Solution

1. For eigen values, $|A - \lambda I| = 0$

$$\left| \begin{pmatrix} -3 & 0 & 6 \\ 4 & 5 & 3 \\ 1 & 2 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} -3-\lambda & 0 & 6 \\ 4 & 5-\lambda & 3 \\ 1 & 2 & 1-\lambda \end{vmatrix} = 0$$

$(-3-\lambda)[(5-\lambda)(1-\lambda) - 6] - 0 + 6[8 - (5-\lambda)] = 0$

$(-3-\lambda)[(5-\lambda)(1-\lambda) - 6] + 6[+8 - (5-\lambda)] = 0$

~~$(5-\lambda)(-3-\lambda)(1-\lambda) - 6$~~

$[(-3-\lambda)(5-\lambda)(1-\lambda)] + 18 + 6\lambda + 48 - 6(5-\lambda) = 0$

$[(-3-\lambda)(5-\lambda)(1-\lambda)] + 66 + 6\lambda - 30 + 6\lambda = 0$

$[(-3-\lambda)(5-\lambda)(1-\lambda)] + 36 + 12\lambda = 0$

$[(-3-\lambda)(5-\lambda)(1-\lambda)] - 12(-3-\lambda) = 0$

$(-3-\lambda)[(5-\lambda)(1-\lambda) + 12] = 0$

$(-3-\lambda)[\lambda^2 - 6\lambda - 7] = 0$

$(-3-\lambda)[(\lambda-7)(\lambda+1)] = 0$

$\therefore \lambda = -3, \lambda = 7, \lambda = -1$

∴ the eigen values are $-3, 7, -1$

for eigen vectors

$(A - \lambda I)x = 0$

for $\lambda = -3$

$$\begin{pmatrix} -3+3 & 0 & 6 \\ 4 & 5+3 & 3 \\ 1 & 2 & 1+3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 6 \\ 4 & 8 & 3 \\ 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$6x_3 = 0$

$x_3 = 0 \quad \ldots\ldots (i)$

$4x_1 + 8x_2 + 3x_3 = 0 \ldots\ldots (ii)$

$x_1 + 2x_2 + 4x_3 = 0 \ldots\ldots (iii)$

From equation III

$$x_1 + 2x_2 + 4(0) = 0$$

$$x_1 + 2x_2 = 0$$

$$x_1 = -2x_2$$

$$x_1 : x_2 = -2 : 1$$

$$x_1 = -2,\ x_2 = 1 \qquad X_1 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

For $\lambda = 7$

$$\begin{pmatrix} -3-7 & 0 & 6 \\ 4 & 5-7 & 3 \\ 1 & 2 & 1-7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -10 & 0 & 6 \\ 4 & -2 & 3 \\ 1 & 2 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-10x_1 + 6x_3 = 0 \quad \cdots \text{I}$$

$$4x_1 - 2x_2 + 3x_3 = 0 \quad \cdots \text{II}$$

$$x_1 + 2x_2 + 6x_3 = 0 \quad \cdots \text{III}$$

From equation I

$$-10x_1 = -6x_3$$

$$x_1/x_3 = 6/10 = 3/5$$

$$x_1 = 3 \qquad x_3 = 5$$

from equation III $\quad x_1 + 2x_2 - 6x_3 = 0$

$$3 + 2x_2 - 6(5) = 0$$

$$\frac{2x_2}{2} = \frac{27}{2}$$

$$x_2 = 13.5 \qquad X_2 = \begin{pmatrix} 3 \\ 13.5 \\ 5 \end{pmatrix}$$

for $\lambda = -1$

$$\begin{pmatrix} -3+1 & 0 & 6 \\ 4 & 5+1 & 3 \\ 1 & 2 & 1+1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 & 6 \\ 4 & 6 & 3 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-2x_1 + 6x_3 = 0$ --- I

$4x_1 + 6x_2 + 3x_3 = 0$ --- II

$x_1 + 2x_2 + 2x_3 = 0$ --- III

From eqn I $\quad -2x_1 + 6x_3 = 0$

$$-2x_1 = -6x_3$$

$$x_1/x_3 = \frac{+6}{+2} = \frac{3}{1}$$

$$x_1 = 3, \quad x_3 = 1$$

From eqn III $\quad x_1 + 2x_2 + 2x_3 = 0$

$$3 + 2x_2 + 2 = 0$$

$$2x_2 + 5 = 0$$

$$2x_2 = -5$$

$$x_2 = -5/2 = -2.5 \qquad X_3 = \begin{pmatrix} 3 \\ -2.5 \\ 1 \end{pmatrix}$$

2. Find the eigen values and vectors of $\begin{pmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{pmatrix}$

Solution

For eigen values $|(A - \lambda I)| = 0$

$$\left| \begin{pmatrix} 2 & 7 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 8 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 7 & 0 \\ 1 & 3-\lambda & 1 \\ 5 & 0 & 8-\lambda \end{vmatrix} = 0$$

$$2-\lambda[(8-\lambda)(3-\lambda)] - 7[(8-\lambda)-5] + 0 = 0$$

$$2-\lambda[(8-\lambda)(3-\lambda)] - 7(3-\lambda) = 0$$

$$(3-\lambda)[(8-\lambda)(2-\lambda) - 7] = 0$$

$$(3-\lambda)[(\lambda^2 - 10\lambda + 9] = 0$$

$$(3-\lambda)(\lambda-9)(\lambda-1) = 0$$

$\therefore \lambda = 3, \lambda = 9, \lambda = 1$

$\therefore$ the eigen values are 3, 9, 1

For eigen vectors $(A - \lambda I)x = 0$

for $\lambda = 3$

$$\begin{pmatrix} 2-3 & 7 & 0 \\ 1 & 8-3 & 1 \\ 5 & 0 & 8-3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 7 & 0 \\ 1 & 0 & 1 \\ 5 & 0 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-x_1 + 7x_2 = 0 \quad \ldots (i)$

$x_1 + x_3 = 0 \quad \ldots (ii)$

$5x_1 + 5x_3 = 0 \quad \ldots (iii)$

from eqn iii $\quad 5x_1 + 5x_3 = 0$

$5x_1 = -5x_3$

$x_1/x_3 = -5/5 = -1/1 \quad \therefore x_1 = -1, x_3 = 1$

From eqn i $\quad -x_1 + 7x_2 = 0$

$-x_1 + 7x_2 = 0$

$-x_1 = +7x_2$ $\quad x_2 = -1/7$ $\quad \therefore x_1 = -1, x_2 = -1/7$

$7x_2 = -1$ $\quad x_2 = -1/7$ $\quad X_1 = \begin{pmatrix} -1 \\ -1/7 \\ 1 \end{pmatrix}$

$\lambda = 9$

$$\begin{pmatrix} 2-9 & 7 & 0 \\ 1 & 3-9 & 1 \\ 5 & 0 & 8-9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-7x_1 + 7x_2 = 0 \cdots (1)$

$x_1 - 6x_2 + x_3 = 0 \cdots (II)$

$5x_3 - x_3 = 0 \cdots (III)$

From eqn (I) $-7x_1 + 7x_2 = 0$

$7x_1 = 7x_2$ $\quad x_1/x_2 = 1/1 \therefore x_1 = 1, x_2 = 1$

From eqn (III) $5x_1 - x_3 = 0$

$5(1) = x_3 \quad \therefore x_3 = 5$ $\quad X_2 = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$

$\lambda = 1$

$$\begin{pmatrix} 2-1 & 7 & 0 \\ 1 & 3-1 & 1 \\ 5 & 0 & 8-1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 + 7x_2 = 0 \cdots (I)$

$x_1 + 2x_2 + x_3 = 0 \cdots (II)$

$5x_1 + 7x_3 = 0 \cdots (III)$

From eqn (I) $x_1 + 7x_2 = 0$

$x_1 = -7x_2$ $\quad x_1/x_2 = -7/1, \therefore x_1 = -7, x_2 = 1$

From eqn III $5x_1 + 7x_3 = 0$

$7(-7) + 7x_3 = 0$

$\frac{7}{7}x_3 = 49/7$

$x_3 = 7$ $\quad X_3 = \begin{pmatrix} -7 \\ 1 \\ 7 \end{pmatrix}$