Assignment David Onuoha chukwu Favour

19/SC101/037

Computer Science

Assignment

1 State Cayley-Hamilton theorem

The Cayley Hamilton theorem states that every square matrix (n×n) over a commutative ring satisfies its own character equation. If A is provided as n×n matrix and In is the n×n identity matrix, then the distinct polynomial of A is attributed as

$$P(x) = \det(xI_n - A)$$

Find the characteristic polynomial of

$$\begin{pmatrix} 2 & 27 & 0 \\ 0 & 4 & 40 \\ 0 & 3 & 30 \end{pmatrix}$$

Solution

$tr(A) = 2+4+30$

$tr(A) = 36$

$|A| = 2\begin{vmatrix} 4 & 40 \\ 3 & 30 \end{vmatrix} - 27\begin{vmatrix} 0 & 40 \\ 0 & 30 \end{vmatrix} + 0\begin{vmatrix} 0 & 4 \\ 0 & 3 \end{vmatrix}$

$|A| = 2(120-120) - 27(0-0) + 0$

$|A| = 2(0) - 27(0) + 0$

$|A| = 0 - 0 + 0 = 0$

$$A_{11} = (-1)^2 \begin{vmatrix} 4 & 40 \\ 3 & 30 \end{vmatrix}$$

$$A_{11} = 1\,(120 - 120) = 0$$

$$A_{22} = (-1)^4 \begin{vmatrix} 2 & 0 \\ 0 & 30 \end{vmatrix}$$

$$= 1\,(60 - 0) = +60$$

$$A_{33} = (-1)^6 \begin{vmatrix} 2 & 27 \\ 0 & 4 \end{vmatrix}$$

$$= 1\,(8 - 0) = 8$$

$$\Delta t = t^3 - tr(A)t^2 + (A_{11} + A_{22} + A_{33})t - det(A)$$

$$\Delta t = t^3 - 36t^2 + (0 + 60 + 8)t - 0$$

$$\Delta t = t^3 - 36t^2 + 68t$$