PTE 519 Lecture Note 4

Formulation of Simulator Equations, Grid Calculations and Data Preparation

There are three main methods used in the formulation of simulator equations for solution. These are IMPES (Implicit Pressure, Explicit Saturation) method, the Sequential method, and the Implicit method. The IMPES method was first applied to black oil systems by Sheldon et al., Stone and Garder, and Fagin and Stewart. For the IMPES formulation, the flow equations are combined to eliminate unknown saturations, thereby generating a single pressure equation. The pressure equation is solved implicitly for pressures at each gridblock at the current timestep designated as n+1, using parameters at the previous timestep designated as n. This is then followed by the explicit substitution of the pressures into the corresponding flow equations, to calculate saturations at the current timestep, (n+1), for each gridblock. The advantages of the IMPES method are that it requires less computing time per time step and less computer memory for storage. The main disadvantage of the IMPES method is that it is relatively unstable at large time steps. There are techniques for improving the stability of the IMPES method as reported by Coats K.H., 2003. To improve the stability of IMPES formulations, Macdonald and Coats, 1970 introduced a modification to the IMPES formulation which was later named the Sequential method by Spillete et al. The Sequential method is a two step solution process. In the first step, the pressure equation is solved implicitly for pressures at each gridblock as was done in the IMPES method. This is then followed by a second step in which the saturation equations derived with the aid of fractional equations are also solved implicitly. However, the Sequential method may have material balance problems in the areas of the gridblocks with large saturation or composition changes). The formulation method that is used in many simulators (black oil, compositional, thermal, chemical etc) is the Implicit formulation. In the Implicit method, the flow equations are formed to be solved simultaneously at the current time step, (n + 1).

Grid Calculations

Well Model

A well model is used in simulation to represent the flow between the wellbore and the reservoir gridblock in which the well is located. Assuming the well is producing from a single gridblock, the flow rate of a phase, p (oil, water, or gas), into the well is given by:

$$q_{p,j} = \frac{WI_j}{\mu_p} (P_j - P_{wf}) \qquad \text{.................................... 4.1}$$

In Equation 4.1, $q_{p,j}$ = the flow rate of the phase, p , in the gridblock, j, WI_j = well index; μ_p = viscosity of phase, p ; P_j = gridblock pressure; and P_{wf} = flowing bottomhole pressure of the well. For the sign convention, the flow is considered positive from the gridblock into the well, and negative from the well into the gridblock. For Cartesian grids, the well index, WI_j (also called connection transmissibility factor), was derived by Peaceman, D.W, 2003. As:

$$WI_j = \frac{2\pi\sqrt{k_x k_{yh}}}{\ln\frac{r_o}{r_w} + s} \qquad \text{....................................... 4.2}$$

In Equation 4.2, $WI_j = \mathit{well\ index\ for\ gridblock}, j$;

$k_x, k_y = permeabilities\ in\ the\ x\ and\ y\ directions, respectively$

$h = net\ thickness\ of\ the\ gridblock$

$r_o = equivalent\ radius\ of\ gridblock$

$r_w = wellbore\ radius$

$s = skin\ factor$

$The\ equivalent\ radius\ of\ the\ gridblock, r_o,$ is defined as the distance from the well at which the flowing wellbore pressure is equal the average pressure of the gridblock. Based on Peaceman, D.W. 2003, equivalent wellbore radius, r_o , for an anisotropic reservoir in Cartesian grids is given by:

$$r_o = 0.28 \frac{\sqrt{(k_y/k_x)^{1/2}\Delta x^2 + (k_x/k_y)^{1/2}\Delta y^2}}{(k_y/k_x)^{1/4} + (k_x/k_y)^{1/4}} \quad \text{.................................. 4.3}$$

In Equation 4.3, r_o = equivalent wellbore radius;
$k_x, k_y = permeabilities\ in\ the\ x\ and\ y\ directions, respectively$

$\Delta x, \Delta y = x\ and\ y\ dimensions\ of\ the\ gridblock\ respectively.$

For an isotropic reservoir, the equivalent wellbore radius, r_o for an isotropic reservoir in Cartesian grids is given by:

$$r_o = 0.14\sqrt{(\Delta x^2 + \Delta x^2)} \quad \text{.. 4.4}$$

It is important to note that the well index, WI_j, for a gridblock is not the same as the productivity (or injectivity) index of the well. The productivity index, j, of a well is defined as:

$$J = \frac{q_p}{\bar{p} - p_{wf}} \quad \text{..4.5}$$

In Equation 4.5,

$q_p = production\ rate\ of\ phase\ p\ (oil, water, or\ gas)$;

$\bar{p} = average\ reservoir\ pressure\ or\ pressure\ at\ the\ drainage\ radius\ of\ well$; and

$p_{wf} = flowing\ bottomhole\ pressure\ of\ the\ well.$

Assuming steady-state, radial Darcy flow within the drainage radius of the well, a relationship between the well index and the productivity index of the well is expressed as:

$$J = \sum_j \left[\frac{WI_j}{\mu_p}\left\{\frac{\ln(r_o/r_w) + s}{\ln(r_d/r_w) + s}\right\}\right] \quad \text{.. 4.6}$$

In Equation 4.6, WI_j = well index for gridblock, j; μ_p = viscosity of phase, p; r_o = equivalent radius of the gridblock; r_w = wellbore radius; s = skin factor ; r_d = drainage radius of the well. The summation is over all the gridblocks connected to the well. If the drainage radius of the well is assumed equal to equivalent wellbore radius, Equation 4.6 reduces to:

$$J = \sum_j \left[\frac{WI_j}{\mu_p}\right] \quad \text{...} \; 4.7$$

Note: All the equations developed in this section of the lecture note for well models apply only to vertical wells located in rectangular gridblocks in Cartesian grids. They are not applicable to wells in radial gridblocks or advanced wells. Advanced wells are described as wells with complex trajectories such as horizontal wells, and multilateral wells, and wells with downhole control devices (sensors, flow control valves, etc) called “smart” wells.

Example 4.1 Calculation of Well Index:

Problem:

Calculate the well index for a vertical well located completely in a single rectangular gridblock in an areal Cartesian Grid system. The dimensions and properties of the gridblock are given as follows:

Gridblock dimension, Δx = 150 ft

Gridblock dimension, Δy = 100 ft

Gridblock dimension, Δz = 5 ft

Gridblock permeability, k_x = 350 md

Gridblock permeability, k_y = 225 md

Wellbore radius, r_w = 0.5 ft

Skin factor, s = 2.3

Assume the net thickness of gridblock, h, is equal to gridblock dimension, Δ_z .

Solution:

Step 1: Calculate the equivalent radius of the gridblock, r_o using equation 4.3:

$$r_o = 0.28 \frac{\sqrt{(k_y/k_x)^{1/2} \Delta x^2 + (k_x/k_y)^{1/2} \Delta y^2}}{(k_y/k_x)^{1/4} + (k_x/k_y)^{1/4}} = 0.28 \frac{\sqrt{(225/350)^{1/2} (150)^2 + (350/225)^{1/2} (100)^2}}{(225/350)^{1/4} + (350/225)^{1/4}}$$

$$= 24.3065 \text{ ft}$$

Step 1: Calculate the well index for the gridblock: Using Equation 4.2

$$WI_j = \frac{2\pi \sqrt{k_x k_{yh}}}{\ln\frac{r_o}{r_w} + s} = \frac{2\pi \sqrt{350 \; X \; 225 \; X \; 5}}{\ln(\frac{24.3065}{0.5}) + 2.3} = 637.65 \text{ md-ft}$$

Data Preparation

In a typical reservoir simulation study, it is necessary to first specify project objectives. The objectives help define the level of detail that will be incorporated in the reservoir model. Once the objectives have been defined, it is important to think of the study as proceeding in three phases: the history match phase, a calibration phase, which provides a smooth transition between the first and third phase; and the prediction phase. The first step towards a history match is the collection and analysis of data.

Data must be acquired and evaluated with a focus on it's quality and the identification of relevant drive mechanisms that should be included in the model. With the earlier mentioned information obtained, it is possible to select the type of model that will be needed for the study: conceptual, window area or full field study. In many cases, all three models may need to be used. Data must be acquired for each. Some of the data that is required for a model study can be found in existing reports. It is expedient for the modelling team to find as many reports as possible from as many disciplines as possible. Table 1 lists the types of data that are needed in a model study. A review of geophysical, petrophysical and engineering reports provides a background on how the project has been developed and what preconceived interpretations have been established. In the course of the study, it may be essential to develope not just a new view of the reservoir, but to also prepare an explanation of why the new view is superior to a previously approved interpretation. In the event that significant gaps exist in the reports, particularly historical performance of the field, it is important to update them.

Table 1: Data Required for Simulation Study

S/N	Property	Sources
1.	Permeability	Pressure transient testing, Core analyses, Correlations, Well performance
2.	Porosity, Rock compressibility	Core analyses, Well logs
3.	Relative permeability and capillary pressure	Laboratory core flow tests
4.	Saturations	Well logs, Core analyses, Pressure cores, Single-well tracer tests
5.	Fluid property (PVT) Data	Laboratory analyses of reservoir fluid samples
6.	Faults, boundaries, fluid contacts	Seismic, Pressure transient testing
7.	Aquifers	Seismic, Material balance calculations, Regional exploration studies
8.	Fracture spacing, orientation, connectivity	Core analyses, Well logs, Seismic, Pressure transient tests, Interference testing, Wellbore performance
9.	Rate and pressure data, completion and workover data	Field performance history

A review of rock and fluid property may show that the amount of available data is limited. If so, additional data should be obtained when possible. This may often require special laboratory tests, depending on the objectives of the study. If measured data cannot be obtained during the scope of the study, then correlations or data from analogous fields will have to be used. Values must be entered into the simulator and it is prudent to select values that can be justified.

Typically, well data should be reviewed. If additional field tests are needed, they should be requested and incorporated into the study schedule. Due to the costs and operating constraints of a project, it may be difficult to justify the expense of acquiring more data or delaying the study while additional data is acquired. The modelling team must take care to avoid underestimating the amount of work that would be needed in preparing an input data set. It can take as long as to collect the data, as it does to do the study.

Geologic Model Data:

All the structural and petrophysical data in the geologic model are typically assembled as data input for the gridblocks in a section of the simulator. The structure of the geologic model is represented by the geometrical data on the gridblocks in terms of location and dimensions. This is usually accompanied with separate specifications of the petrophysical data for each gridblock. Typically, the petrophysical data usually specified for each gridblock include porosity, permeability, and net sand thickness or net-to-gross ratio data. Initial fluid saturations for each gridblock may also be specified in some models. These data are then followed with modifications to the grid system (such as local grid refinement).

Fluid Properties Data:

This section of model data set contains data that represent the PVT properties of the fluids present in the reservoir. The PVT data are usually presented in a tabular form for black oil models. For compositional simulators, the PVT data are represented in compatible form as output generated with an equation of state.

Rock/Fluid Properties Data:

Rock/fluid properties data in the form of relative permeability data and capillary pressure data are represented in the model as functions of fluid saturations. These data are usually presented in the simulator in a tabular form. Note that the data in these tables are sometimes used by the simulator to establish initial conditions in the reservoir model, if the option for simulator generated initial conditions is selected.

Model Equilibration Data:

Model equilibration data include fluid contact depths (oil-water contact, gas-oil contact or gas-water contact), capillary pressures at the fluid contacts, and reservoir pressure at a selected datum depth. The model equilibration data are in some cases used by the simulator to establish initial reservoir conditions.

Well Data:

In this section, the locations of wells in the grid system are specified. Also, the gridblocks in which the wells are completed are specified. The production or injection rates of the wells including the type of fluid produced or injected are specified. The progression of the simulation in terms of time is defined in this section in the form of time steps, cumulative time, or dates. These time-based data are very important, because the speed and duration of the simulation are controlled by these data. For reservoirs with production history, the production data are provided at specific time intervals, which could be daily, monthly, quarterly, semi-annually, or annually. The frequency of production data entry is totally at the discretion of the user of the simulator.

Simulator Data Output:

All simulators are capable of generating large amounts of output data, especially for large models. This should be controlled by the user to avoid being overwhelmed by the simulator data input/output. Most simulators are equipped with powerful graphical software that can assist in the evaluation and presentation of the output from the simulator.

References:

Coats, K.H.: “IMPES Stability: Selection of Stable Timesteps,” SPEJ (June, 2003) 181 – 187.

Macdonald, R.C. and Coats, K.H.: “Methods for Numerical Simulation of Water and Gas Coning” SPEJ (December, 1970) 573 – 592.

Peaceman, D.W. : A New Method for Calculating Well Indexes for Multiple Wellblocks with Arbitrary Rates in Numerical Reservoir Simulation,” SPE 79687, February 3 – 5, 2003.

Principles of Applied Reservoir Simulation by John R. Fanchi

Petroleum Reservoir Engineering Practice by Nnaemeka Ezekwe