MBE (Error/Convergence) – Lecture Note 5 Prepared by Okereke, N.U. (PhD)

General Material Balannce Equation (GMBE):

The GMBE can be represented volumetrically in reservoir barrels as:

Net change in oil and water production = Gas cap volume change + Released solution gas + Oil volume change + Connate water expansion + Rock expansion

Material Balance Error/Convergence Criteria:

One key criterion used in all simulators to check the accuracy and stability of the solution at each time-step is the material balance error (MBE). A low value of material balance error by itself is not sufficient measure of the accuracy of the solution. Material balance error is used in conjunction with other criteria; to determine convergence of the solution at each timestep.

Different commercial simulators use different methods to calculate and normalize material balance errors. Irrespective of the method used, the criterion applied in most simulators is that material balance error for each gridblock at each timestep should be close to zero.

One method for calculating material balance error is the summation of mass accumulation for each phase at the beginning and at the end of the timestep for all gridblocks. The net mass accumulation for each phase should be equal to net influx through the wells during the timestep.

For blackoil simulator, the material balance error is calculated as follows:

For Oil Phase:

$$MBE_o = \frac{\sum_{j=1}^{m}\left[PV\left(\frac{S_o}{B_o}\right)\right]_j^n - \sum_{j=1}^{m}\left[PV\left(\frac{S_o}{B_o}\right)\right]_j^{n+1}}{q_o\,\Delta t} - 1 \qquad \text{..... Equation 5.1}$$

For Water Phase:

$$MBE_w = \frac{\sum_{j=1}^{m}\left[PV\left(\frac{S_w}{B_w}\right)\right]_j^n - \sum_{j=1}^{m}\left[PV\left(\frac{S_w}{B_w}\right)\right]_j^{n+1}}{q_w\,\Delta t} - 1 \qquad \text{..... Equation 5.2}$$

For Gas Phase:

$$MBE_g = \frac{\sum_{j=1}^{m}\left[PV\left(\frac{S_g}{B_g}\right)\right]_j^n - \sum_{j=1}^{m}\left[PV\left(\frac{S_g}{B_g}\right)\right]_j^{n+1}}{q_g\,\Delta t} - 1 \qquad \text{..... Equation 5.3}$$

In Equations 5.1 – 5.3, $PV = Pore\ Volume\ of\ the\ gridblock, j$ s_o, s_w and s_g are saturations of oil, water and gas respectively, in gridblock, ; Δt = Length of timestep; m = total number of gridblocks in reservoir model; n = previous timestep and n+1 = current timestep.

N/B:

Many commercial simulators consider material balance errors to be sufficiently small for all the phases, if less than 1×10^{-7}. Other key measures that are used to check stability and accuracy of the solutions are changes in saturation of the phases in iteration at each timestep. The iteration is considered to have converged, if maximum changes in saturation of each phase for each gridblock is less than a preset maximum saturation change for the timestep.

Many commercial simulators set the largest convergence error for any phase in any grid block at less than 0.001. If the conditions for the material balance error for each phase and the convergence error for phase saturations are not met for the timestep size, the calculations and iterations are repeated with a reduction in the size of the timestep until convergence is achieved.

The amount of reduction of timestep size applied automatically due to convergence failure varies among commercial simulators. Some commercial simulators automatically reduce the timestep size by half after a convergence failure. This can drastically slow down the speed of simulation run. Note that the amount of cut applied to timestep size due to convergence failure can be specified by the user in most commercial simulators, in addition to the changes on the convergence criteria. However, these options should be used with caution to avoid unstable and errorneous results.

Numerical Dispersion:

Numerical dispersion is a term generally used to describe the false appearance of smeared spatial gradients of saturation or concentration arising from time and space discretizations of the flow equations in simulators based on finite differences. In simpler terms, numerical dispersion describes the smearing of saturation or concentration of the displacing phase ahead of the actual front.

Numerical dispersion generally increases with increase in the areal sizes of the model gridblocks, $(\Delta x, \Delta y)$, and increases in timestep sizes, Δt . Consequently for the same areal grid sizes, numerical dispersion is likely to be larger with implicit solution methods that use larger timestep sizes than IMPES solution methods that use smaller timestep sizes. For both IMPES and implicit solution techniques, numerical dispersion can be reduced by increasing the number of areal gridblocks. This approach is expected to create larger simulation models. However, if accurate prediction of water, steam or solvent breakthrough is an objective for simulation of the displacement process, then it is necessary to use as many areal gridblocks as necessary in order to minimize the effects of numerical dispersion.

Model Initialization:

The initial conditions in the reservoir based on the input data are established during model initialization. These initial conditions are distributions of phase pressures and saturations for each gridblock in the reservoir model.

Generally, two methods are used for initialization. These are simulator generated initial conditions or user specified initial conditions. Under simulator generated initial conditions, the simulator uses the input data to generate equilibrated distributions of phase pressures and saturations and initial conditions. The input data consists of specifications of pressure at a given datum depth, depth of fluid contacts (water-oil, gas-oil or gas-water contacts), capillary pressure at the contacts, fluid densities at datum, and saturation end points and capillary pressure data in the relative permeability tables. For the

user-specified initial conditions, the distributions of pressure and saturation at initial conditions for each gridblock are provided and set by the user.

N/B:

User-specified initial conditions may cause non-equilibrated model at initial conditions. Hence, it is important for the user to verify that these conditions exist in the reservoir at initial conditions, as non-equilibrated results could lead to erroneous modelling/post-simulation results.

After initializing the model, one key role is to verify the quality of the model. One key method to the verify the quality of the model output is the compare the in-place volumes of fluid calculated by the simulator against the volume in-place of fluid calculated from the geologic model. The fluid-in place computed by both methods should be in a close agreement. A good rule of thumb is that the variation between calculation of the volume of fluid-in place between the two methods should be less than 2 percent.

References:

Petroleum Reservoir Engineering Practice by Nnaemeka Ezekwe

Principles of Applied Reservoir Simulation by John R. Fanchi