Sediment Transport

Sediment Transport

- **Sediment** is any particulate matter that can be **transported** by fluid flow and which eventually is deposited as a layer of solid particles on the bed or bottom of a body of water or other liquid.

- The generic categories of sediments is as follows
  - **Gravel**
  - **Sand**
  - **Silt**
  - **Clay**

Contents

Reference: SEDIMENT TRANSPORT Theory and Practice

By: Chih Ted Yang

1. Properties of Water and Sediment

1. Properties of Water and Sediment

1.1 Introduction

- The science of sediment transport deals with the interrelationship between flowing water and sediment particles and therefore understanding of the physical properties of water and sediment is essential to our study of sediment transport.

1.2 Terminology

- **Density:** Mass per unit volume
- **Specific weight:** Weight per unit volume
  - $\gamma = \rho g$ (1.1)
- **Specific gravity:** It is the ratio of specific weight of given material to the specific weight of water at 4°C or 32.2°F. The average specific gravity of sediment is 2.65
- **Nominal Diameter:** It is the diameter of sphere having the same volume as the particle.
- **Sieve Diameter:** It is the diameter of the sphere equal to length of side of a square sieve opening through which the particle can just pass. As an approximation, the sieve diameter is equal to nominal diameter.

1. 2 Terminology

- **Fall Diameter:** It is the diameter of a sphere that has a specific gravity of 2.65 and has the same terminal fall velocity as the particle when each is allowed to settle alone is a quiescent, distilled water. The Standard fall diameter is determined at a water temperature of 24^{o}C.
- **Fall Velocity:** It is the average terminal settling velocity of particle falling alone in a quiescent distilled water of infinite extent. When the fall velocity is measure at 24oC, it is called Standard Fall Velocity.
- **Angle of Repose:** It is the angle of slope formed by a given material under the condition of incipient sliding.
- **Porosity:** This the measure of volume of voids per unit volume of sediment

$$p = \frac{V_v}{V_t} = \frac{V_t - V_s}{V_t} \qquad (1.2)$$

Where: p= Porosity, V_v =Volume of Voids, V_t = Total Volume of Sediment

V_s = Volume of Sediment excluding that due to Voids

1. 2 Terminology

- **Viscosity:** It is the degree to which a fluid resist flow under an applied force. Dynamic viscosity is the constant of proportionality relating the shear stress and velocity gradient.

$$\tau = \mu \frac{du}{dy} \qquad (1.3)$$

Where: $\tau = Shear\ Stress,\ \mu = Dynamic\ \text{Viscosity},\ \frac{du}{dy} = Velocity\ Gradient$

- **Kinematic Viscosity:** is the ratio between dynamic viscosity and fluid density

$$\nu = \frac{\mu}{\rho} \qquad (1.4)$$

Where: $\upsilon = Kinematic\ \text{Viscosity},\ \rho = Fluid\ Density$

1.3 Properties of Water

- The Basic properties of water that are important to the study of sediment transport are summarized in the following table:

Properties of water

Temperature (°F)	Specific weight (lb/ft^3)	Mass density (lb-s^2/ft^4)	Dynamic viscosity $\times 10^5$ (lb-s/ft^2)	Kinematic viscosity $\times 10^5$ (ft^2/s)
32	62.42	1.940	3.746	1.931
40	62.43	1.940	3.229	1.664
50	62.41	1.940	2.735	1.410
60	62.37	1.938	2.359	1.217
70	62.30	1.936	2.050	1.059
80	62.22	1.934	1.799	0.930
90	62.11	1.931	1.595	0.826
100	62.00	1.927	1.424	0.739
110	61.86	1.923	1.284	0.667
120	61.71	1.918	1.168	0.609
130	61.55	1.913	1.069	0.558
140	61.38	1.908	0.981	0.514
150	61.20	1.902	0.905	0.476
160	61.00	1.896	0.838	0.442
170	60.80	1.890	0.780	0.413
180	60.58	1.883	0.726	0.385
190	60.36	1.876	0.678	0.362
200	60.12	1.868	0.637	0.341
212	59.83	1.860	0.593	0.319

1.4 Properties of a Single Sediment Particle

- **Size:** Size is the most basic and readily measurable property of sediment. Size has been found to sufficiently describe the physical property of a sediment particle for many practical purposes. The size of particle can be determined by sieve size analysis or Visual-accumulation tube analysis. The US Standard sieve series is shown in table.

U.S. Standard sieve series

Sieve No. (mesh)	Opening (mm)	Opening (in)
3-1/2	5.66	0.233
4	4.76	0.187
5	4.00	0.157
6	3.36	0.132
7	2.83	0.111
8	2.38	0.0937
10	2.00	0.0787
12	1.68	0.0661
14	1.41	0.0555
16	1.19	0.0469
18	1.00	0.0394
20	0.84	0.0331
25	0.71	0.0280
30	0.59	0.0232
35	0.50	0.0197
40	0.42	0.0165
45	0.35	0.0138
50	0.297	0.0117
60	0.250	0.0098
70	0.210	0.0083
80	0.177	0.0070
100	0.149	0.0059
120	0.125	0.0049
140	0.105	0.0041
170	0.088	0.0035
200	0.074	0.0029
230	0.062	0.0024

1.4 Properties of a Single Sediment Particle ……(Cont.)

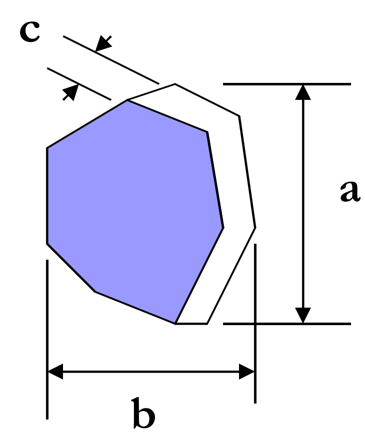
- The sediment grade scale suggested by Lane et al. (1947), as shown in Table below has generally been adopted as a description of particle size

Sediment grade size (Lane *et al.*, 1947)

Millimeters		Micrometers	Inches	Tyler Standard	U.S. Standard	Class
4000–2000			160–80			Very large boulders
2000–1000			80–40			Large boulders
1000–500			40–20			Medium boulders
500–250			20–10			Small boulders
250–130			10–5			Large cobbles
130–64			5–2.5			Small cobbles
64–32			2.5–1.3			Very coarse gravel
32–16			1.3–0.6			Coarse gravel
16–8			0.6–0.3	2-1/2		Medium gravel
8–4			0.3–0.16	5	5	Fine gravel
4–2			0.16–0.08	9	10	Very fine gravel
2–1	2.00–1.00	2000–1000		16	18	Very coarse sand
1–1/2	1.00–0.50	1000–500		32	35	Coarse sand
1/2–1/4	0.50–0.25	500–250		60	60	Medium sand
1/4–1/8	0.25–0.125	250–125		115	120	Fine sand
1/8–1/16	0.125–0.062	125–62		250	230	Very fine sand
1/16–1/32	0.062–0.031	62–31				Coarse silt
1/32–1/64	0.031–0.016	31–16				Medium silt
1/64–1/128	0.016–0.008	16–8				Very fine silt
1/128–1/256	0.008–0.004	8–4				
1/256–1/512	0.004–0.0020	4–2				Coarse clay
1/512–1/1024	0.0020–0.0010	2–1				Medium clay
1/1024–1/2048	0.0010–0.0005	1–0.5				Fine clay
1/2048–1/4096	0.0005–0.00024	0.5–0.24				Very fine clay

1.4 Properties of a Single Sediment Particle ……(Cont.)

- **Shape:** Shape refers to the form or configuration of a particle regardless of its size or composition. Corey shape factor is commonly used to describe the shape. i.e.



$$S_p = \frac{c}{\sqrt{ab}} \qquad (1.5)$$

where : $a, b \,\&\, c$ are lengths of longest, intermediate and short mutually perpendicular axes through the particle respectively

- **The shape factor is 1 for sphere. Naturally worn quartz particles have an average shape factor of 0.7**

1.4 Properties of a Single Sediment Particle ……(Cont.)

- **Density:** The density of sediment particle refers to its mineral composition, usually, specific gravity. Waterborne sediment particles are primarily composed of quartz with a specific gravity of 2.65.
- **Fall Velocity:** The fall velocity of sediment particle in quiescent column of water is directly related to relative flow conditions between sediment particle and water during conditions of sediment entrainment, transportation and deposition.
- It reflect the integrated result of size, shape, surface roughness, specific gravity, and viscosity of fluid.
- Fall velocity of particle can be calculated from a balance of buoyant weight and the resisting force resulting from fluid drag.

1.4 Properties of a Single Sediment Particle …….(Cont.)

- The general Drag Force (F_D) equation is

$$F_D = C_D \rho A \frac{\omega^2}{2} \qquad (1.6)$$

Where: F_D = Drag Force, C_D = Drag Coefficient

ρ=Density of water, A= Projected area of particle in the direction of fall and

ω= Fall velocity

- The buoyant or submerged weight (W_s) of spherical sediment particle is

$$W_s = \frac{4}{3} r^3 \pi (\rho_s - \rho) g \qquad (1.7)$$

Where: r = Particle radius, ρ_s =Density of Sediment.

- Fall velocity can be solved from above two equations(1.6 & 1.7), once the drag coefficient (C_D) has been determined. The Drag Coefficient is a function of Reynolds Number (Re) and Shape Factor.

1.4 Properties of a Single Sediment Particle ……(Cont.)

- **Theoretical Consideration of Drag Coefficient**: For a very slow and steady moving sphere in an infinite liquid at a very small Reynolds Number, the drag coefficient can be expressed as

$$\mathbf{F_D=6}\ \mu\ \pi\ \mathbf{r}\ \omega \qquad (1.8)$$

Eq.(1.8) obtained by Stoke in solving the general **Navier-Stoke Equation**, with the aid of shear function and neglecting inertia term completely. The C_D is thus

$$\mathbf{C_D=24/Re} \qquad (1.9)$$

Equation is acceptable for Reynolds Number Re < 1

From Eq. (1.6) and (1.9), Stoke's (1851) equation can be obtained. i.e

$$\mathbf{F_D=3}\ \pi\ \mathbf{d\ v}\ \omega \qquad (1.10)$$

Now from eqs (1.7) and (1.10), the terminal fall velocity of sediment particle is

$$\omega=\frac{1}{18}\left(\frac{\gamma s-\gamma}{\gamma}\right)\frac{gd^2}{\upsilon} \qquad (1.11)$$

d= Sediment diameter

Eq. (1.11) is applicable if diameter of sediment is less than equal to 0.1mm

1.4 Properties of a Single Sediment Particle ……(Cont.)

- The value of kinematic viscosity in eq. (1.11) is a function of water temperature and can be computed by

$$\upsilon = 1.79^2 x10^{-6} /(1.0 + 0.0337T + 0.000221T^2) \qquad (1.12)$$

Where: T is temperature in °C

- **<u>Onseen (1927)</u>** included inertia term in his solution of Navier-Stoke Eq. The solution thus obtained is

$$CD = \frac{24}{\mathrm{Re}}\left(1 + \frac{3}{16}\mathrm{Re}\right) \qquad (1.13)$$

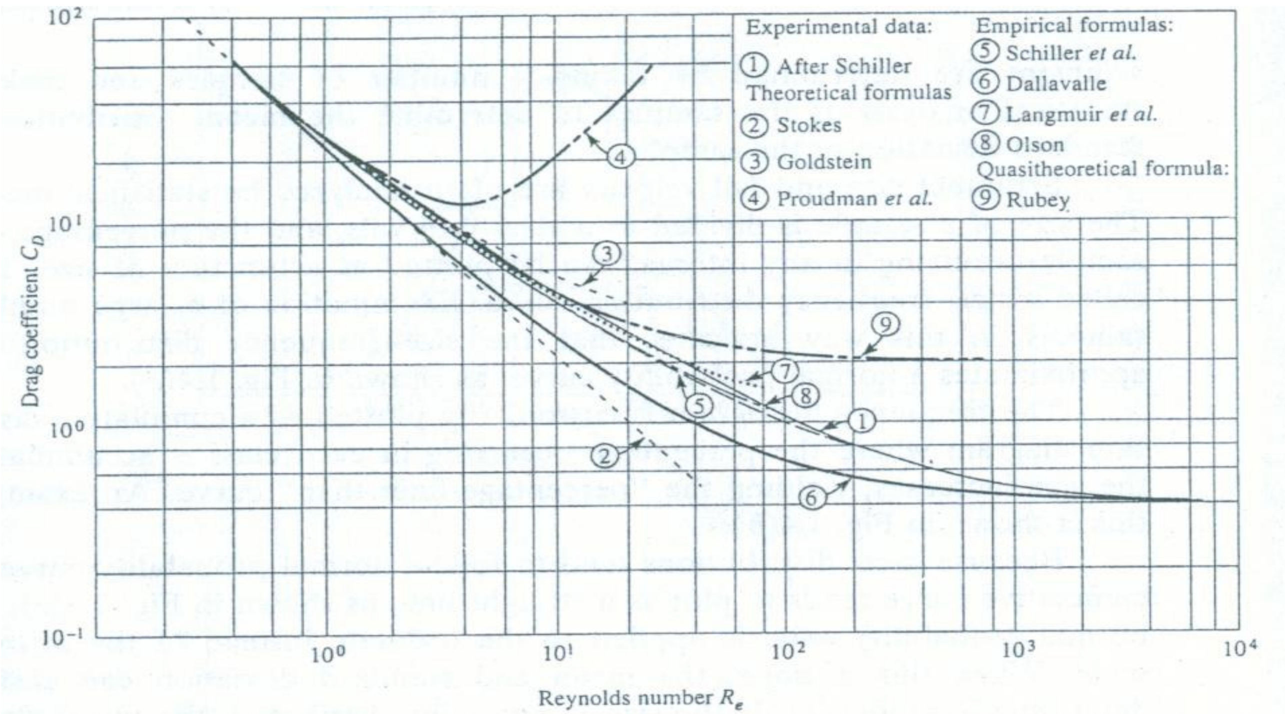
- **<u>Goldstein (1929)</u>** provided a more complete solution of the Onseen approximation, and the drag coefficient becomes

$$C_D = \frac{24}{\mathrm{Re}}\left(1 + \frac{3}{16}\mathrm{Re} - \frac{19}{1280}\mathrm{Re}^2 + \frac{71}{20480}\mathrm{Re}^3 + ...\right) \qquad 1.14$$

- Eq. (1.14) is valid for Reynolds Number up to 2

1.4 Properties of a Single Sediment Particle ……(Cont.)

- The Relationship between drag coefficient and Reynolds Number determined by several investigators is show in figure below



Relationship between drag coefficient and Reynolds number for sphere (Graf and Acaroglu, 1966).

Acaroglu(1966), when Reynolds Number is greater than 2, the relationship should be determined experimentally.

1.4 Properties of a Single Sediment Particle ……(Cont.)

- **Rubey's Formula:** Rubey(1933) introduced a formula for the computation of fall velocity of gravel, sand, and silt particles. For quartz particles with diameter greater than 1.0mm, the fall velocity can be computed by,

$$\omega = F\left[dg\left(\frac{\gamma_s - \gamma}{\gamma}\right)\right]^{1/2} \qquad (1.15)$$

Where : F= 0.79 for particles >1mm settling in water with temperature between 10^{o} and 25^{o}C

d= Diameter of particle

For smaller grains

$$F = \left[\frac{2}{3} + \frac{36\upsilon^2}{gd^3\left(\frac{\gamma_s - \gamma}{\gamma}\right)}\right]^{1/2} - \left[\frac{36\upsilon^2}{gd^3\left(\frac{\gamma_s - \gamma}{\gamma}\right)}\right]^{1/2} \qquad (1.16)$$

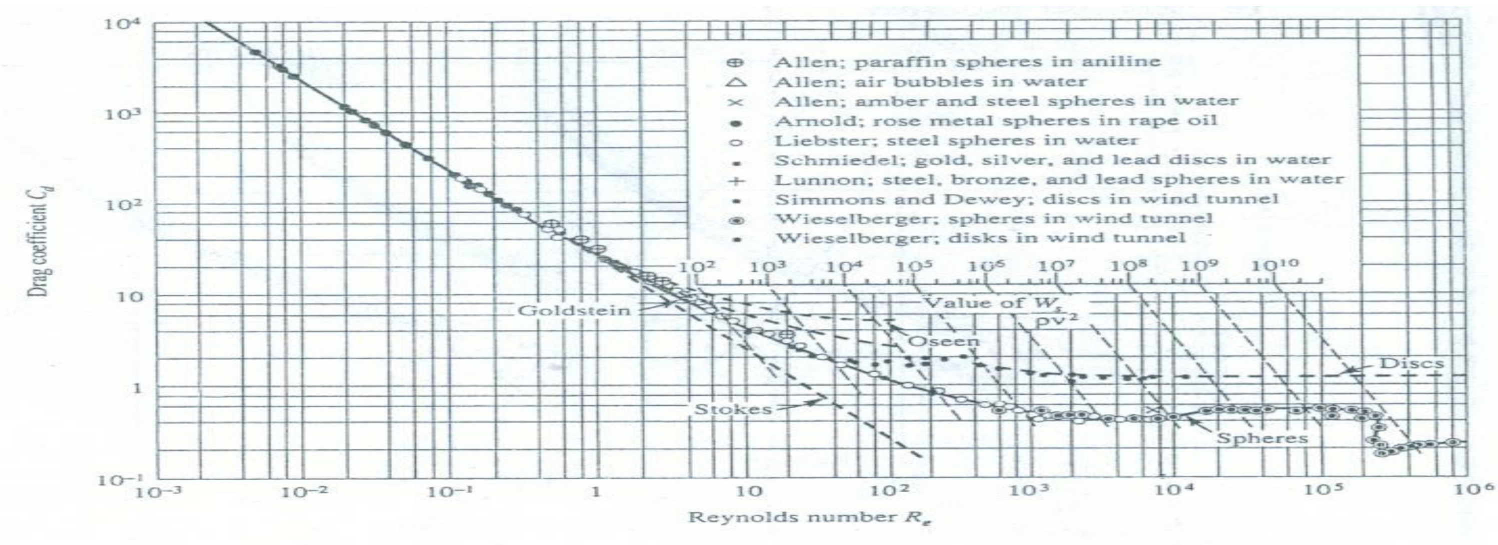
For d >2mm, the fall velocity in 16^{o}C water can be approximated by

ω=6.01d$^{1/2}$ (ω in ft/sec, d in ft) (1.17a)

ω=3.32d$^{1/2}$ (ω in m/sec, d in m) (1.17b)

1.4 Properties of a Single Sediment Particle ……(Cont.)

- **Experimental Determination of Drag Coeffienct and Fall Velocity.**
- The drag coefficient cannot be found analytically when Reynolds Number is greater than 2.0. Therefore it has to be determined experimentally by observing the fall velocities in still water. These relationships are summarized by Rouse (1937), as shown in figure below

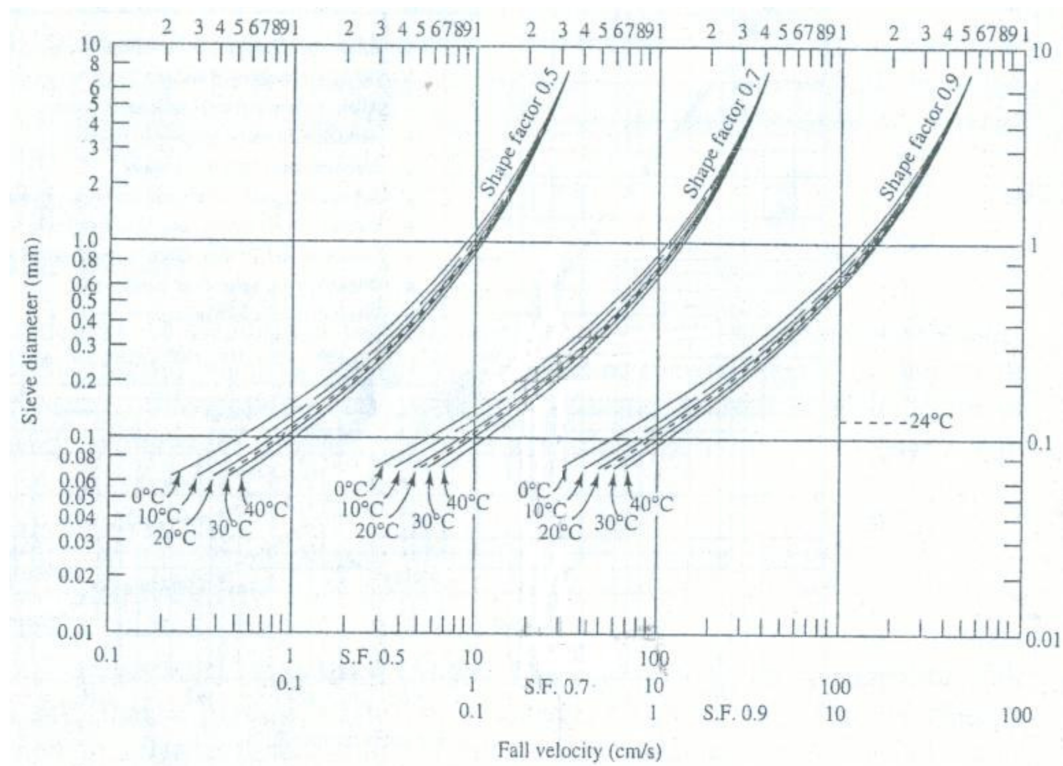


Drag coefficients as functions of Reynolds number (Rouse, 1937).

After the C_D has been determined, ω can be computed by solving eqs (1.6) and (1.7)

1.4 Properties of a Single Sediment Particle ……(Cont.)

- **<u>Factors Affecting Fall Velocity:</u>**
  - Relative density between fluid and sediment, Fluid Viscosity, Sediment Surface Roughness, Sediment Size and Shape, Suspended sediment Concentration and Strength of Turbulence.
  - Most practical approach is the application of figure below when the particle size shape factor and water temperature are given.



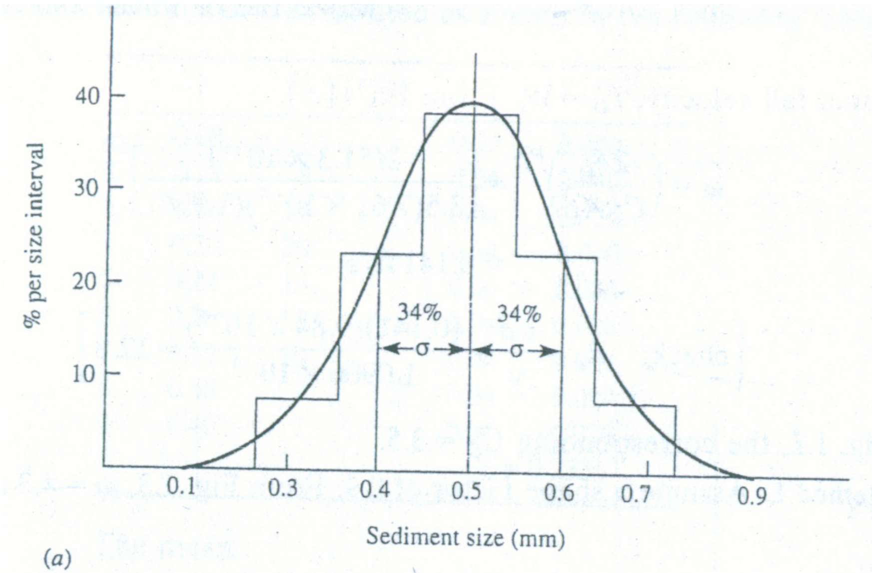
Relation between sieve diameter and fall velocity for naturally worn quartz particles falling alone in quiescent distilled water of infinite extent (U.S. Inter-Agency Committee on Water Resources, Subcommittee on Sedimentation, 1957).

1.5 Bulk Properties of Sediment

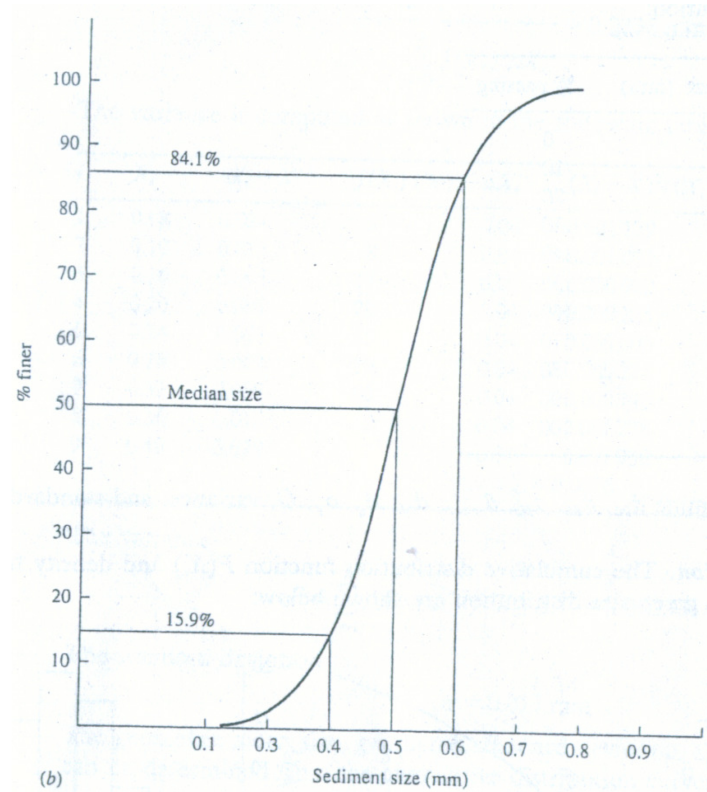
- **<u>Particle Size Distribution:</u>**
  - While the properties and behavior of individual sediment particles are of fundamental concern, the greatest interest is in groups of sediment particles. Various sediment particles moving at any time may have different sizes, shapes, specific gravities and fall velocities. The characteristic properties of the sediment are determined by taking a number of samples and making a statistical analysis of the samples to determine the mean, distributed and standard deviation.
  - The two methods are commonly used
    - **<u>1. Size-Frequency Distribution Curve</u>**
  - The size of sample is divided into class intervals , and the percentage of the sample occurring in any interval is can be plotted as a function of size.
    - **<u>2. Percentage finer than Curve</u>**
  - The frequency histogram is customarily plotted as a cumulative distribution diagram where the percentage occurring in each class is accumulated as size increases.

1.5 Bulk Properties of Sediment …(Cont.)

- **Particle Size Distribution:**



Normal size-Frequency distribution curve



Cumulative frequency of normal distribution i.e % finer-than curve

1.5 Bulk Properties of Sediment …(Cont.)

- **<u>Specific Weight:</u>** The specific weight of deposited sediment depends upon the extent of consolidation of the sediment. It increases with time after initial deposition. It also depends upon composition of sediment mixture.
- **<u>Porosity:</u>** It is important in the determination of the volume of sediment deposition. It is also important in the conversion from sediment volume to sediment weight and vice versa. Eq. (1.18) can be used for computation of sediment discharge by volume including that due to voids, once the porosity and sediment discharge by weight is known.

$$V_t=V_s/(1-p) \qquad (1.18)$$

Lecture # 10

2. Incipient Motion Criteria and Application

2.0 Incipient Motion

- **2.1 Introduction:**
- Incipient motion is important in the study of sediment transport, channel degradation, and stable channel design.
- Due to stochastic (random) nature of sediment movement along an alluvial bed, it is difficult to define precisely at what flow conditions a sediment particle will begin to move. Consequently, it depends more or less on an investigator's definition of incipient motion. "initial motion", "several grains moving", "weak movement", and "critical movement" are some of the terms used by different investigators.
- In spite of these differences in definition, significant progress has been made on the study of incipient motion, both theoretically and experimentally.

2.2 General Consideration:

- The forces acting on spherical particle at the bottom of an open channel are shown in figure.

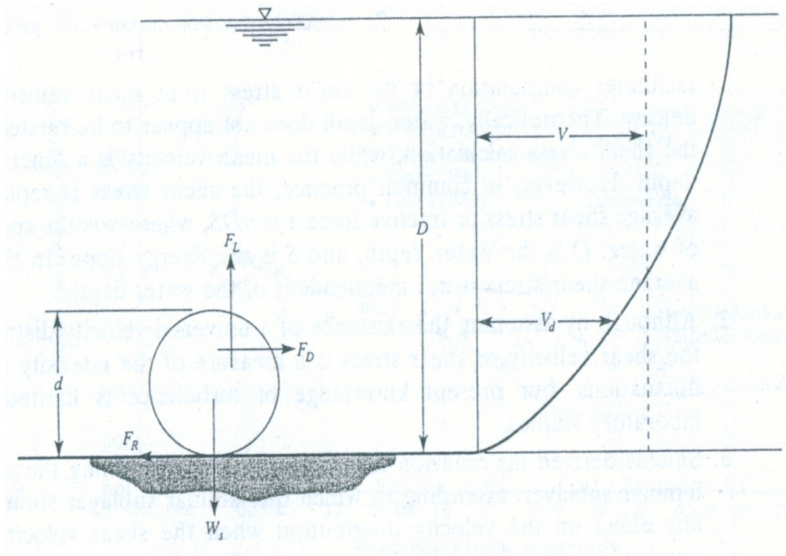


Diagram of forces acting on a sediment particle in open channel flow.

- The forces to be considered are the Drag force F_D, Lift force F_L, Submerged weight Ws, and Resistance force F_R. A sediment particle is in state of incipient motion when one of the following condition is satisfied.
  - $F_L = Ws$ (2.1)
  - $F_D = F_R$ (2.2)
  - $M_o = M_R$ (2.3)

Where: M_o is overturning moment due to F_D & F_R.

M_R is resisting moment due to F_L & Ws

2.3 Incipient Motion Criteria

- Most incipient motion criteria are derived from either a
  - **Shear stress** or
  - **Velocity approach**

Because of stochastic nature of bed load movement,

- **Probabilistic approaches** have also been used.
- **Other Criterion**

SHEAR STRESS APPROACH

- **White's Analysis**

White (1940) assumed that the slope and life force have insignificant influence on incipient motion, and hence can be neglected compared to other factors. According to White a particle will start to move when shear stress is such that $M_o=M_R$.

$$\tau_c = C_5\left(\gamma_s - \gamma\right)d \text{ Where; } C_5 = \text{ Constant,}$$

$$\tau_c = \text{Critical shear stress incipient motion}$$

- Thus the critical shear stress is proportional to sediment diameter.
- The factor C_5 is a function of density and shape of the particle, the fluid properties, and the arrangement of the sediment particles on the bed surface.
- Values of $C_5(\gamma_s-\gamma)$ for sand in water rages from 0.013 to 0.04 when the British system is used.

SHEAR STRESS APPROACH …(Cont.)

- **<u>Shield's Diagram</u>**

  Shield (1936) applied dimensional analysis to determine some dimensionless parameters and established his well know diagram for incipient motion. The factors that are important in the determination of incipient motion are the shear stress, difference in density between sediment and fluid, diameter, kinematic viscosity, and gravitation acceleration.

$$d\frac{\left(\tau_c/\rho_f\right)^{1/2}}{\upsilon} = \frac{dU_*}{\upsilon} \quad \text{and}$$

$$\frac{\tau_c}{d(\rho_s-\rho_f)g} = \frac{\tau_c}{d\gamma(\rho_s/\rho_f-1)}$$

where:

ρs and $\rho =$ Density of sediment and fluid

$\gamma =$ specific weight of water

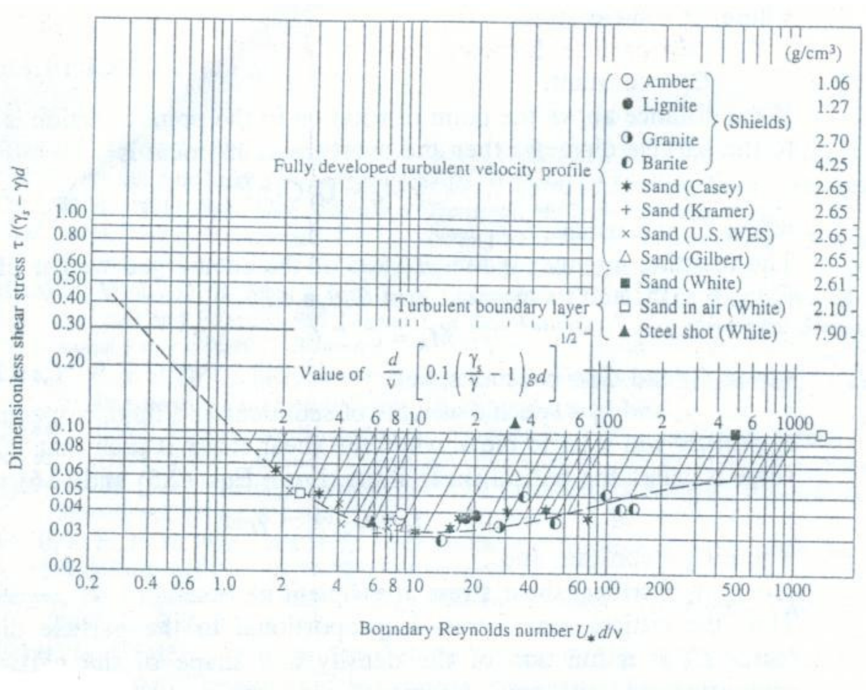
$U_* =$ Shear Velocity

$\tau_c =$ Critical shear stress at intial motion

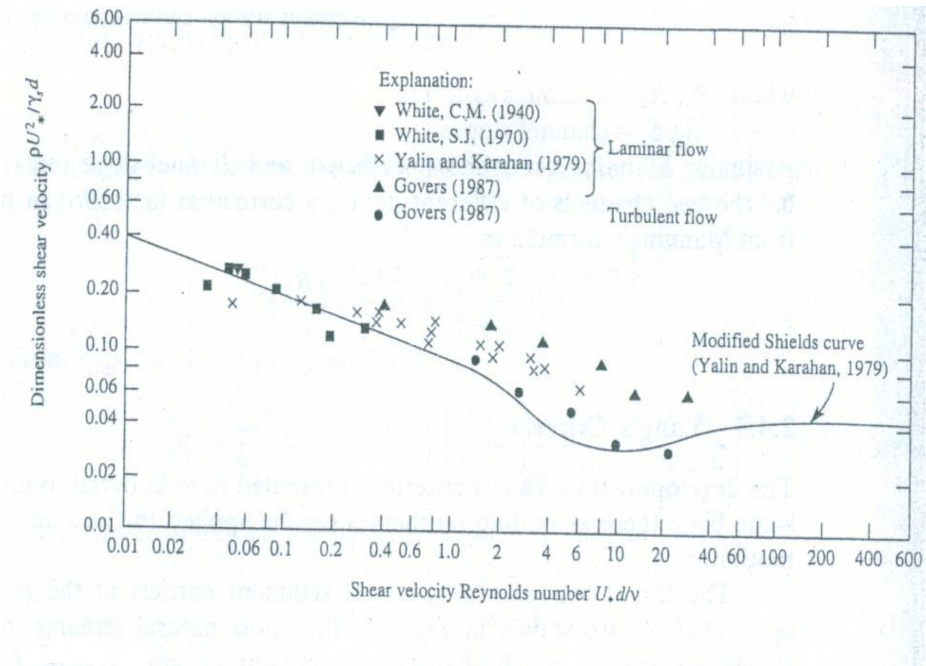
The Relationship between these two parameter is then determined experimentally.

SHEAR STRESS APPROACH …(Cont.)

- **Shield's Diagram**



Shields diagram for incipient motion (Vanoni, 1975).



Modified Shields diagram (Govers, 1987).

- Figure shows the experimental results obtained by Shield and others at incipient motion. At points above the curve, particles will move and at points below the curve, the flow will be unable to move the particles.

VELOCITY APPROACH

- **Frontier and Scobey's Study.**
- Frontier and Scobey (1926) made an extensive field survey of maximum permissible values of mean velocities in canals. The permissible velocities for canal of different materials are summarized in table below.

TABLE 2.1
Permissible canal velocities (Fortier and Scobey, 1926)

Original material excavated for canal (1)	Velocity† (ft/s) Clear water, no detritus (2)	Water-transporting colloidal silts (3)	Water-transporting noncolloidal silts, sands, gravels, or rock fragments (4)
Fine sand (noncolloidal)	1.50	2.50	1.50
Sandy loam (noncolloidal)	1.75	2.50	2.00
Silt loam (noncolloidal)	2.00	3.00	2.00
Alluvial silts when noncolloidal	2.00	3.50	2.00
Ordinary firm loam	2.50	3.50	2.25
Volcanic ash	2.50	3.50	2.00
Fine gravel	2.50	5.00	3.75
Stiff clay (very colloidal)	3.75	5.00	3.00
Graded, loam to cobbles, when noncolloidal	3.75	5.00	5.00
Alluvial silts when colloidal	3.75	5.00	3.00
Graded, silt to cobbles, when colloidal	4.00	5.50	5.00
Coarse gravel (noncolloidal)	4.00	6.00	6.50
Cobbles and shingles	5.00	5.50	6.50
Shales and hard pans	6.00	6.00	5.00

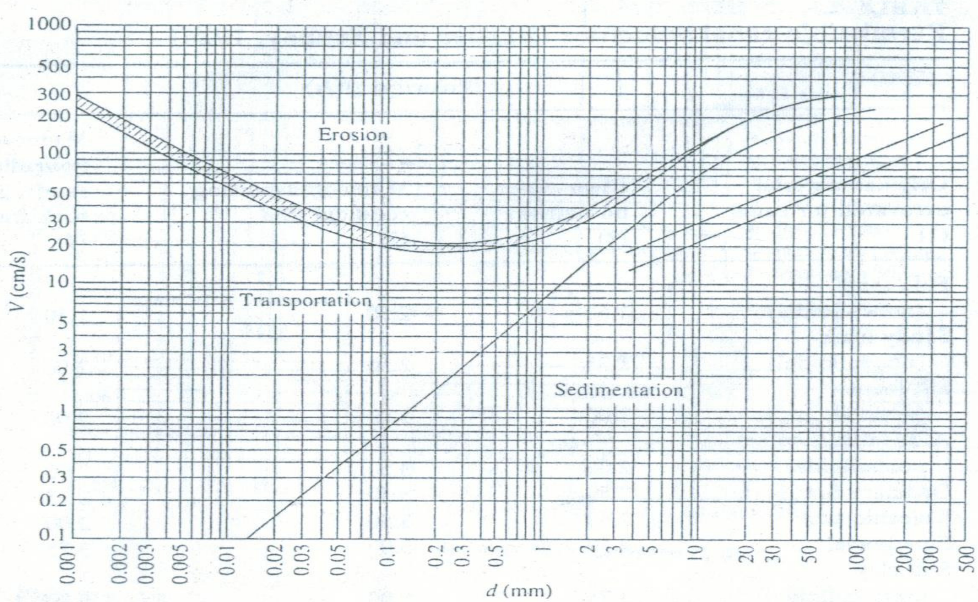
† For channels with depth of 3 ft or less after aging.

- **Although there is no theoretical study to support or verify the values shown in table, these results are based on inputs from experienced irrigation engineers and should be useful for preliminary designs**

VELOCITY APPROACH …(Cont.)

■ Hjulstrom and ASCE Studies.

- Hjulstrom (1935) made a detailed analysis of data obtained from movement of uniform materials. His study was based on average flow velocity due to ease of measurement instead of channel bottom velocity. Figure (a) gives the relationship between sediment size and average flow velocity for erosion, transportation and sedimentation.



Erosion–deposition criteria for uniform particles (Hjulstrom, 1935).

Figure (a)

VELOCITY APPROACH …(Cont.)

■ Hjulstrom and ASCE Studies.

- Figure (b) summarizes the relationship between critical velocity proposed by different investigators and mean particle size. It was suggested by ASCE Sediment Task Committee. For stable channel desigr

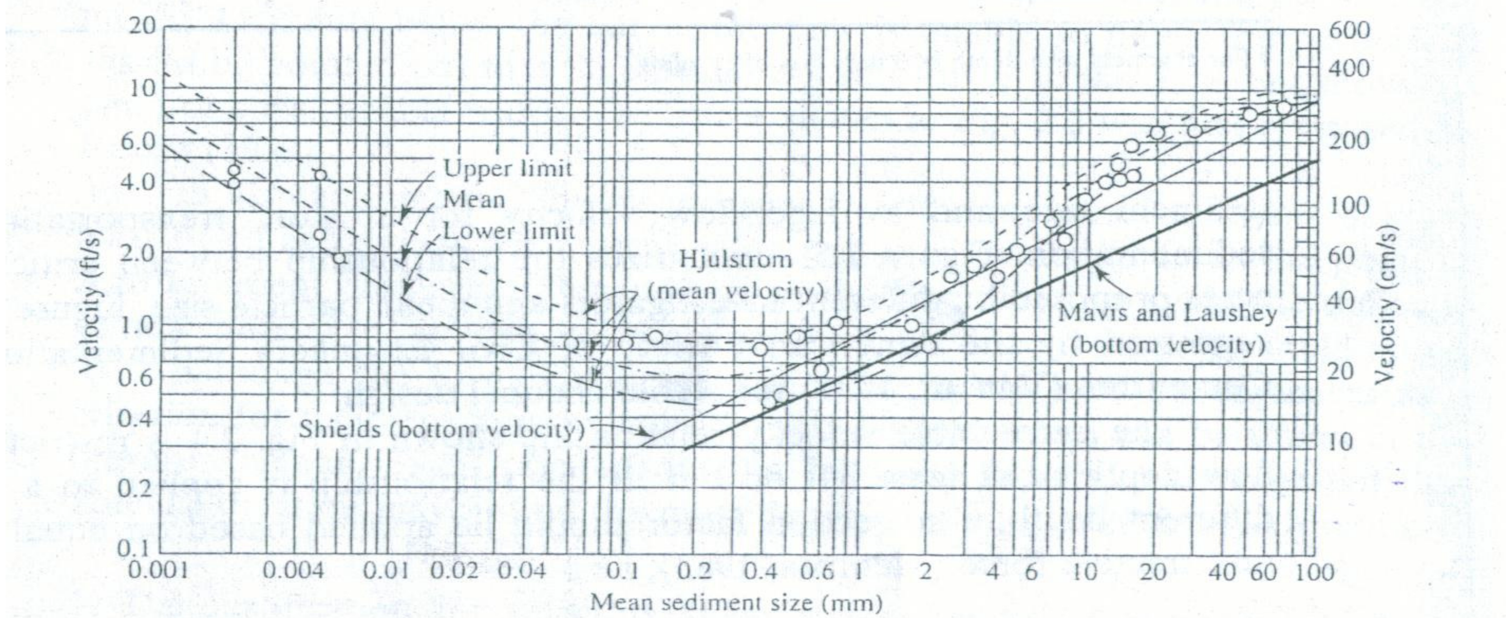


Figure (b) Critical water velocities for quartz sediment as a function of mean grain size (Vanoni, 1977).

VELOCITY APPROACH

…(Cont.)

Hjulstrom and ASCE Studies.

- The permissible velocity relationship shown in figure above is restricted to a flow depth of at least 3ft or 1m. If the relationship is applied to a flow of different depth, a correction factor should be applied based on equal tractive unit force.

$$\tau_c = \gamma R_1 S_1 = \gamma R_2 S_2$$

Where: R_1 & R_2 are hydraulic radii

S_1 & S_2 are Channel Slopes

- Assuming Manning's Roughness coefficient and channel slopes remain same for the two channels of different depths, a correction factor *k* can be obtained from Manning's formula as

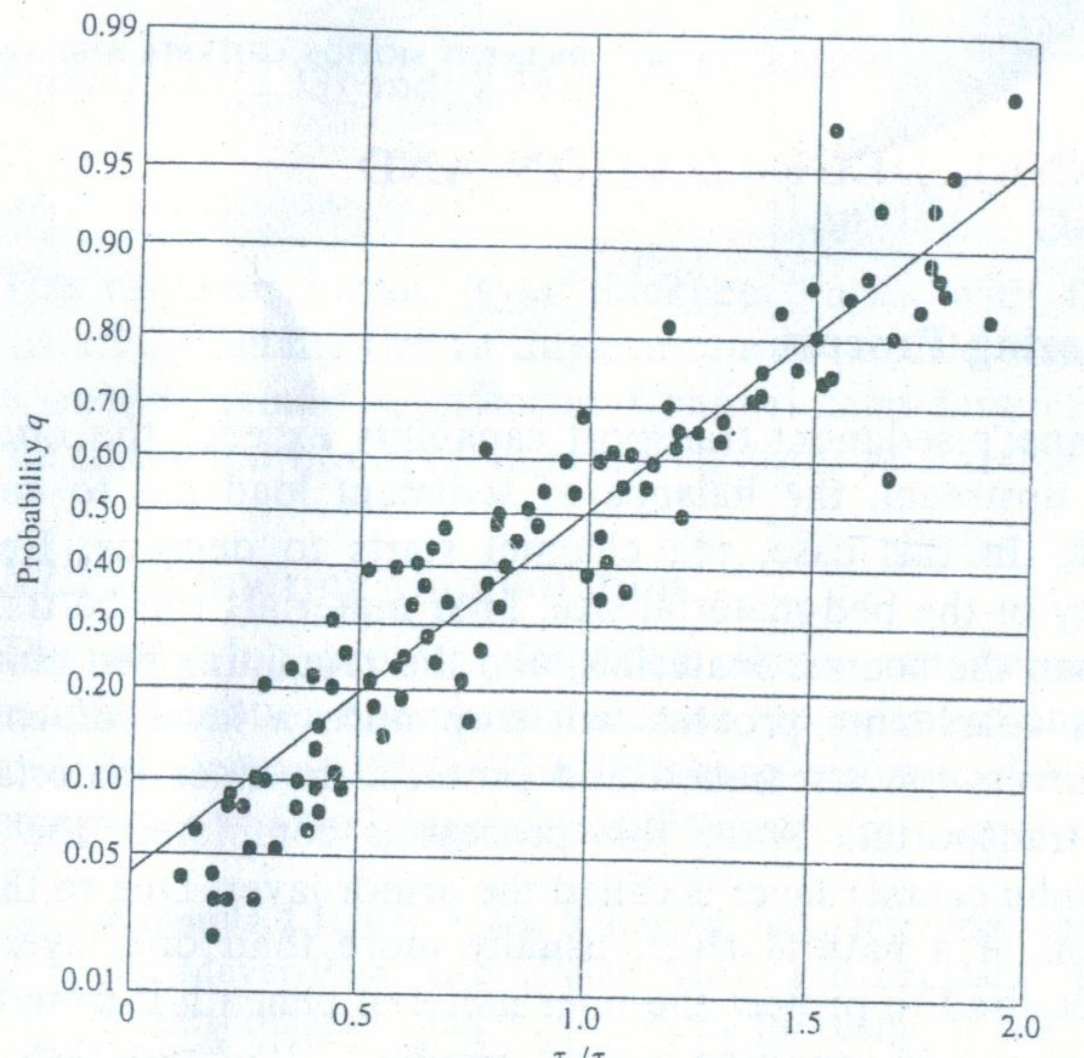
$$k = \frac{V_2}{V_1} = \left(\frac{R_2}{R_1}\right)^{1/6}$$

PROBABILISTIC CONSIDERATION

- The incipient motion of a single sediment particle along an alluvial bed is probabilistic in nature.
- It depends on
  - The location of a given particle with respect to particles of different sizes
  - Position on a bed form, such as ripples and dunes.
  - Instantaneous strength of turbulence
  - The orientation of sediment particle.
- Gessler (1965, 1970) measured the probability that the grains of specific size will stay. It was shown that the probability of a given grain to stay on bed depends mainly upon Shields parameter and slightly on the grain Reynolds Number. The ratio between critical shear stress, determined from Shields diagram and the bottom shear stress is directly related to the probability that a sediment particle will stay. The relationship is shown in figure.

PROBABILISTIC CONSIDERATION

…(Cont.)



Probability for grains to stay: q versus τ_c/τ (Gessler, 1965).

OTHER INCIPIENT MOTION CRITERIA

- **Meyer-Peter and Muller Criterion:**
- From Meyer-Peter and Muller (1948) bed load equation, the sediment size at incipient motion can be obtained as,

$$d = \frac{SD}{K_1\left(n / d_{90}^{1/6}\right)^{3/2}}$$

- **Where;**
  - d= Sediment size in armor layer
  - S= Channel slope
  - K_1= Constant (=0.19 when D is in ft and 0.058 when D is in m)
  - n= Channel bottom roughness or Manning's roughness coefficient
  - d_{90}=Bed material size where 90% of the material is finer

OTHER INCIPIENT MOTION CRITERIA

…(Cont.)

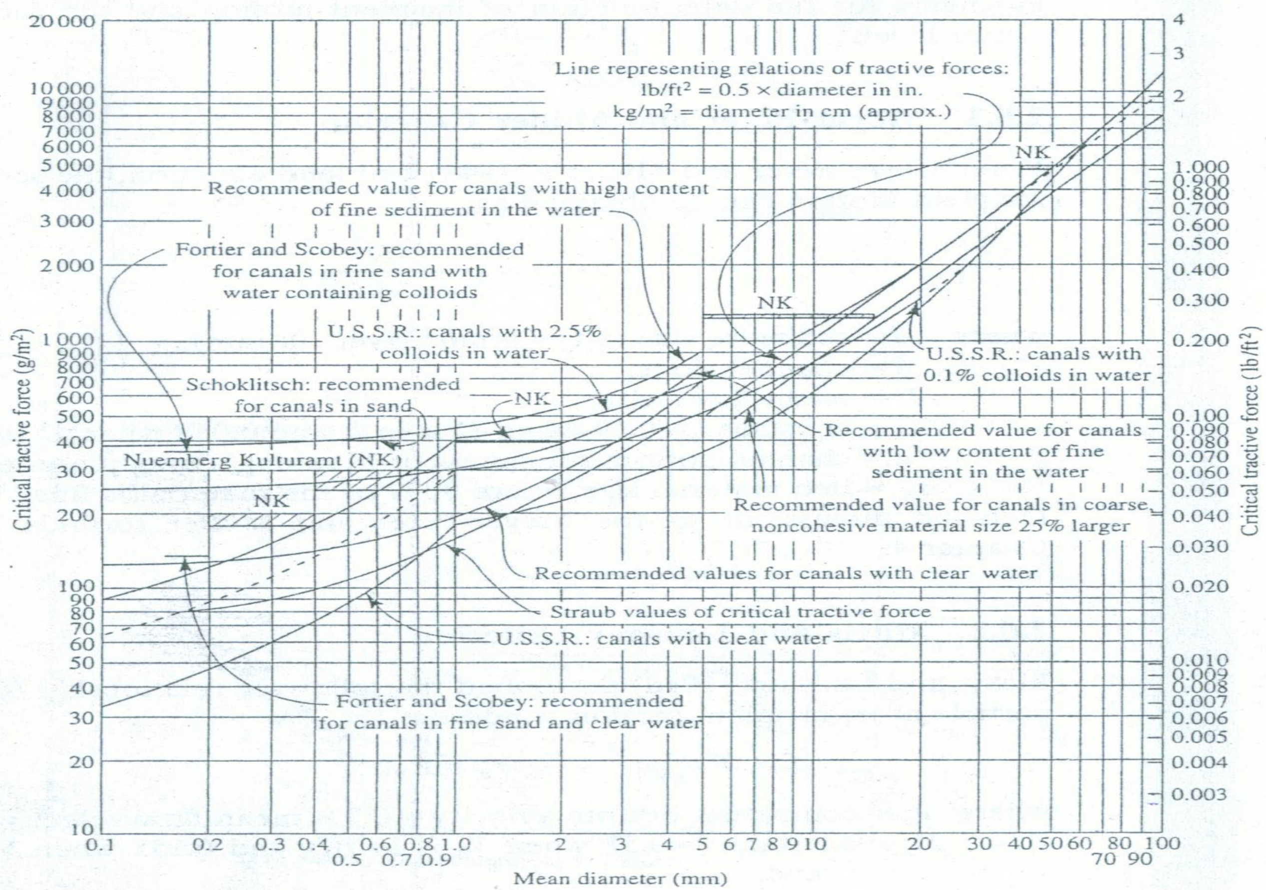
- **Mavis and Laushey Criterion:**
- Mavis and Laushey (1948) developed the following relationship for a sediment particle at its incipient motion condition:

$$V_b = K_2 d^{1/2}$$

- **Where;**
  - V_b=Competent bottom velocity = 0.7 x Mean flow velocity
  - K_2= Constant(=0.51 when V_b is in ft/sec and 0.155 when V_b is in m/sec)
  - d= Sediment size in armor layer

OTHER INCIPIENT MOTION CRITERIA

- **US Bureau of Reclamation:** …(Cont.)



Tractive force versus transportable sediment size (U.S. Bureau of Reclamation, 1987).

Lecture # 11

3. Resistance to Flow and Bed Forms

3. Resistance to Flow and Bed Forms

- **3.1 Introduction:** In the study of open channel hydraulics with rigid boundaries, the roughness coefficient can be treated as a constant. After the roughness coefficient has been determined, a roughness formula can be applied directly for the computation of velocity, slope or depth.
- In fluvial hydraulics, the boundary is movable and the resistance to flow or roughness coefficient is variable and resistance formula cannot be applied directly without the knowledge of how the resistance coefficient will change under different flow and sediment conditions. Extensive studies has been made by different investigators for the determination of roughness coefficient of alluvial beds. The results investigators often differ from each other leaving uncertainties regarding applicability and accuracy of their results.
- The lack of reliable and consistent method for the prediction of the variation of the roughness coefficient makes the study of fluvial hydraulics a difficult and challenging task.

3.2 Resistance to Flow with Rigid Boundary

- **<u>Velocity Distribution Approach:</u>**
- According to Prandtle's (1924) mixing length theory,

$$u = \left(8.5 + 5.75 \log \frac{y}{K_s}\right) U_* \quad and \quad u = \left(5.5 + 5.75 \log \frac{y U_*}{\upsilon}\right) U_*$$

- **Where:**
  - u= Velocity at a distance y above bed
  - U_*= $(gDS)^{1/2}$ = Shear Velocity
  - S= Slope
  - v= Kinematic Viscosity
  - K_s= Equivalent Roughness
- The above equations can be integrated to obtain the relationship between mean flow velocity V, and shear velocity U_*, or roughness *Ks*.

3.2 Resistance to Flow with Rigid Boundary

The Darcy-Weisbach Formula …(Cont.)

- The Darcy-Weisbach formula originally developed for pipe flow is

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

- **Where:**
  - h_f= Friction loss, f= Dary-Weisbach friction factor, L= Pipe length, D= Pipe Diameter, V= Average Flow velocity, g= Gravitation acceleration

- **For open channel flow,** D=4R and S=h_f/L. The value of f can be expressed as

$$f = \frac{8gRS}{V^2}, \quad \textit{Where}: \text{ R= Hydraulic Radius, S = Energy Slope}$$

Because $U_*^2 = gRS$ so above eq. can be rewritten as

$$\frac{V}{U_*} = \left(\frac{8}{f}\right)^{1/2}$$

3.2 Resistance to Flow with Rigid Boundary

- **Chezy's Formula** …(Cont.)
- The Chezy's Formula can be written as

$$V = C\sqrt{RS}$$

- **Where:**
  - V= Average Flow velocity, C= Chezy's Constant, R= Hydraulic Radius,, S= Channel Slope

- **Manning's Formula**
- Most common used resistance equation for open channel flows is Manning's Equation

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

- **Where:**
  - V= Average Flow velocity, n= Manning's Constant, R= Hydraulic Radius,, S= Channel Slope

3.3 Bed Forms

- There is strong interrelationship between resistance to flow, bed configuration, and rate of sediment transport. In order to understand the variation of resistance to flow under different flow and sediment conditions, it is necessary to know the definitions and the conditions under which different bed forms exits.
- **Terminology:**
- **Plane Bed:** This is a plane bed surface without elevations or depressions larger than the largest grains of bed material
- **Ripples:** These are small bed forms with wave lengths less than 30cm and height less than 5cm. Ripple profiles are approx. triangular with long gentle upstream slopes and short, steep downstream slopes.

3.3 Bed Forms …(Cont.)

- **Bars:** These are bed forms having lengths of the same order as the channel width or greater, and heights comparable to the mean depth of the generating flow. There are *point bars, alternate bars, middle bars and tributary bars* as shown in figure below.

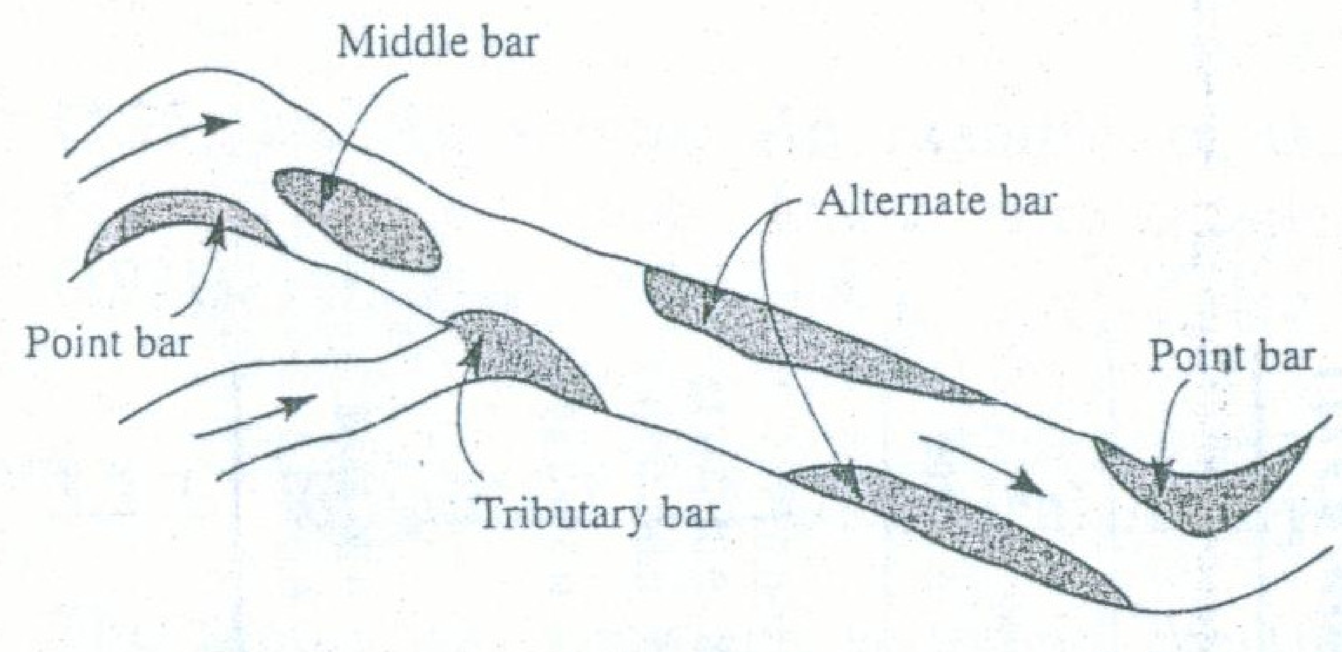
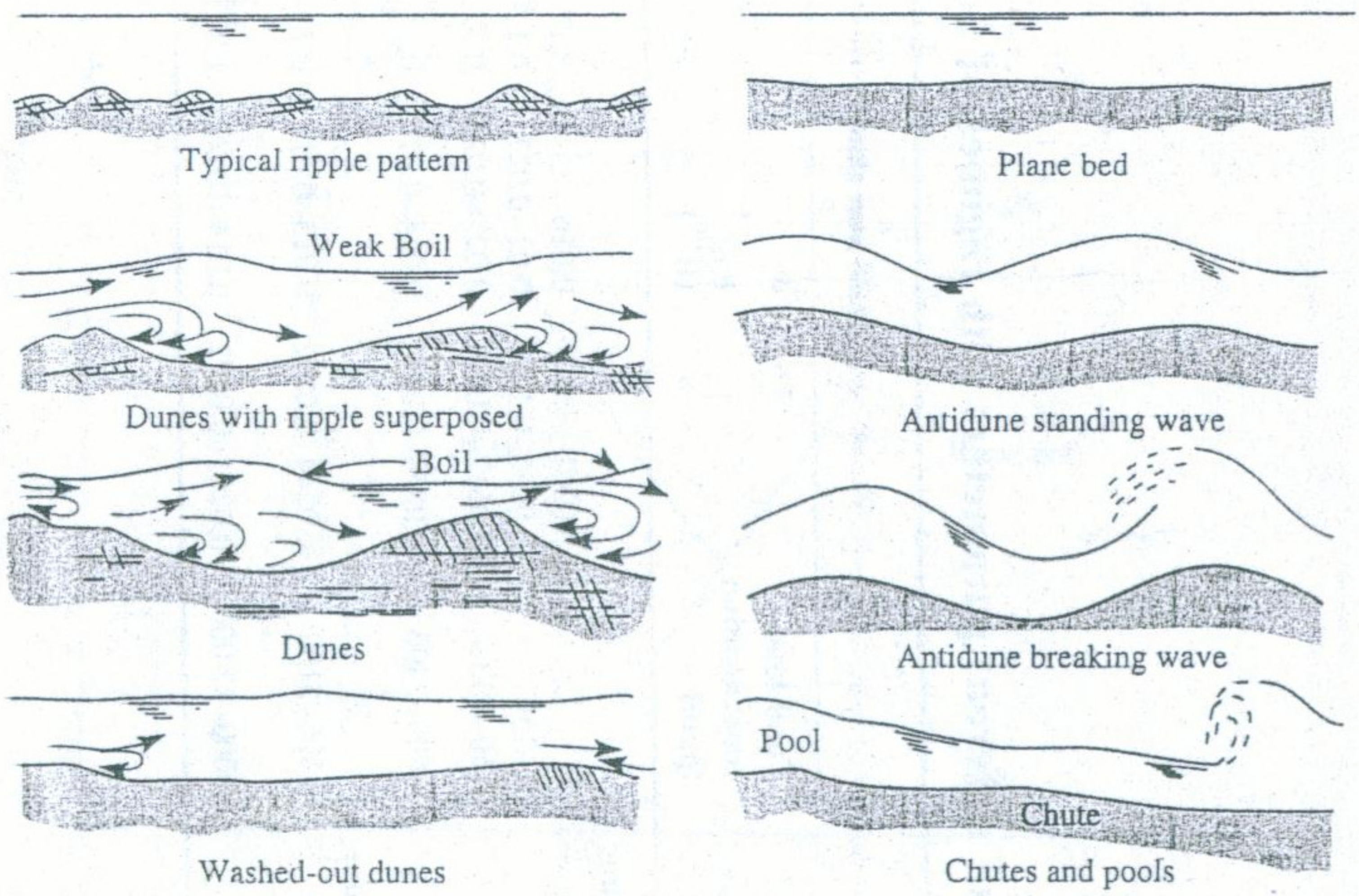


Illustration of different types of bars.

3.3 Bed Forms …(Cont.)

- **Dunes:** These are bed forms smaller than bars but larger than ripples. Their profile is out of phase with water surface profile.
- **Transitions:** The transitional bed configuration is generated by flow conditions intermediate between those producing dunes and plane bed. In many cases, part of the bed is covered with dunes while a plane bed covers the remaining.
- **Antidunes:** these are also called standing waves. The bed and water surface profiles are in phase. While the flow is moving in the downstream direction, the sand waves and water surface waves are actually moving in the upstream direction.
- **Chutes and Pools:** These occur at relatively large slopes with high velocities and sediment concentrations.

3.3 Bed Forms …(Cont.)



Bed forms of sand bed channels (Simons and Richardson, 1966).

3.3 Bed Forms …(Cont.)

- **Flow Regimes:** The flow in sand-bed channel can be classified as lower and upper flow regimes, with a transition in between. The bed forms associated with these flow regimes are as follows;
- **Lower flow regime:**
  - Ripples
  - Dunes
- **Transition Zone:**
  - Bed configuration ranges from dunes to plane beds or to antidunes
- **Upper flow regime:**
  - Plane bed with sediment movement
  - Antidunes
  - Breaking antidunes
  - Standing waves
  - Chutes and pools

3.3 Bed Forms …(Cont.)

- **<u>Factors Affecting Bed forms:</u>**
- **Depth**
- **Slope**
- **Density**
- **Size of bed material**
- **Gradation of bed material**
- **Fall velocity**
- **Channel cross sectional shape**
- **Seepage flow**

3.4 Resistance to Flow with Movable Boundary

- The total roughness of alluvial channel consists of two parts.
- **1. Grain roughness or Skin roughness**, that is directly related to grain size
- **2. Form roughness**, that is due to existence of bed forms and that changes with change of bed forms.
- If the Manning's roughness coefficient is used, the total coefficient n can be expressed as

$$n = n' + n''$$

Where: n'= Manning's roughness coefficient due to grain roughness

n"= Manning's roughness coefficient due to form roughness

The Value of n' is proportional to the sediment diameter to the sixth power. However there is no reliable method for computation of n". Our instability to determine or predict variation of form roughness poses a major problem in the study of alluvial hydraulics.

3.4 Resistance to Flow with Movable Boundary

...(Cont.)

- **<u>Surface Drag and Form Drag:</u>** Similar to roughness the shear stress or drag force acting along an alluvial bed can be divided into two parts

$$\tau = \tau' + \tau''$$

$$\tau = \gamma S \left(R' + R'' \right)$$

Where : τ=Total Drag Force acting along an alluvial bed

τ' & τ'' = Drag force due to grain and form roughness

R' & R'' = Hydraulic radii due to grain and form roughness

4. Bed Load Transport

Bed Load Transport

- **Introduction:**
- When the flow conditions satisfy or exceed the criteria for incipient motion, sediment particles along the alluvial bed will start move.
- If the motion of sediment is “rolling”, “sliding” or “jumping” along the bed, it is called bed load transport
- Generally the bed load transport of a river is about 5-25% of that in suspension. However for coarser material higher percentage of sediment may be transported as bed load.

4.2 SHEAR STRESS APPROACH

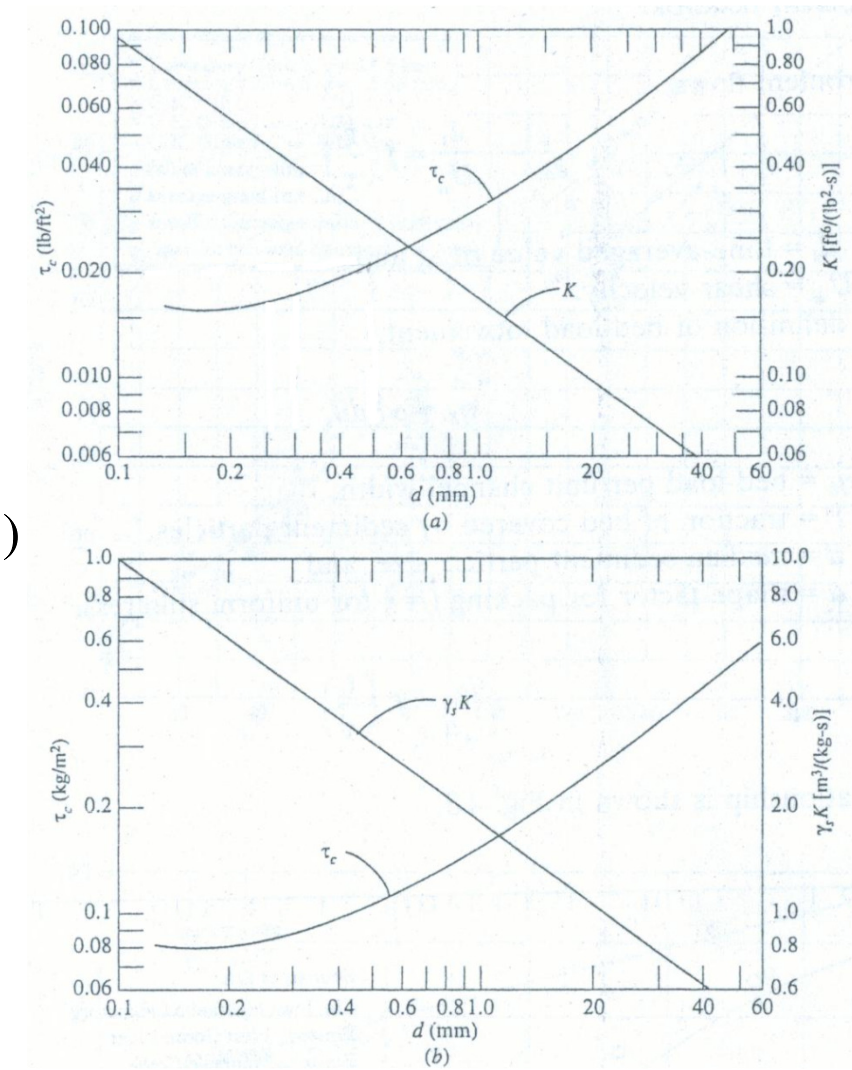
- **<u>DuBoys' Approach:</u>**
- Duboys (1879) assume that sediment particles move in layers along the bed and presented following relationship

$$q_b = K\tau(\tau - \tau_c) = (ft^3/s)/ft$$

$$K = \frac{0.173}{d^{3/4}} = ft^6(lb^2 - s) \quad \text{(Straub, 1935)}$$

The relationship between τ_c, k and d are shown in figure below.

τ_c can be determined from shields diagram



Duboys' Eqution was criticized mainly due to two reasons

1. **All data was obtained from small laboratory flume with a small range of particle size.**
2. **It is not clear that eq. (4.2) is applicable to field condition,**

4.2 SHEAR STRESS APPROACH …(Cont.)

Shields' Approach:

- In his study of incipient motion, Shield obtain semi empirical equation for bed-load which is given as

$$\frac{q_b \gamma_s}{q \gamma S} = 10 \frac{\tau - \tau c}{\gamma_s - \gamma}$$

$Where:$ qb and q = bed load and water discharge per unit width

$\tau = \gamma DS$

$d =$ Sediment particle diameter

$\gamma_s \;\&\; \gamma =$ Specific weights of sediment and water

- **<u>Note:</u> The above equation is dimensionally homogenous, and can be used for any system of units. The critical shear stress can be estimated from shields' diagram.**

4.3 ENERGY SLOPE APPROACH

Meyer-Peter's Approach:

- Meyer-Peter et al. (1934) conducted extensive laboratory studies on sediment transport. His formula for bed-load using the metric system is

$$\frac{0.4 q_b^{2/3}}{d} = \frac{q^{2/3} S}{D} - 17$$

$$Where:\ \mathrm{q_b} = \text{Bed load (kg/s/m)}$$

$$\mathrm{q} = \text{Water discharge (kg/s/m)}$$

$$S = \text{Slope and, } d = \text{ Particle diameter}$$

- **Note:**
  - **The Constants 17 and 0.4 are valid for sand with Sp. Gr =2.65**
  - **Above formula can be applied only to coarse material have d>3mm**
  - **For non-uniform material $d=d_{35}$,**

4.3 ENERGY SLOPE APPROACH …(Cont.)

- **Meyer-Peter and Muller's Approach:**
  - After 14 years of research and analysis, Meyer-Peter and Muller (1948) transformed the Meyer-Peter formula into Meyer-Peter and Mullers' Formula

$$\gamma\left(\frac{k_s}{K_r}\right)^{3/2} RS = 0.047\left(\gamma_s - \gamma\right)d + 0.25\rho^{1/3}q_b^{2/3}$$

$Where:\ \gamma_s$ & γ = Specific weights of sediment and water [Metric Tons/m^3]

R= Hydraulic Radius [m]

ρ= Specific mass of water [Metric tons-s/m^4]

S = Energy Slope and,

d = Mean particle diameter

q_b = Bed load rate in underwater weight per unit time and width [(Metric tons/s)/m]

$\left(\frac{k_s}{K_r}\right)S$ = the kind of slope, which is adjusted such that only a portion of the total energy loss ,namely that due to the grain resistance Sr, is responsible for bed load motion.

4.3 ENERGY SLOPE APPROACH …(Cont.)

Meyer-Peter and Muller's Approach:

- The slope energy can be found by Stricker's Formula

$$S = \frac{V^2}{K_s^2 R^{4/3}} \quad \& \quad S_r = \frac{V^2}{K_r^2 R^{4/3}} \quad then$$

$$\frac{Ks}{Kr} = \left(\frac{Sr}{S}\right)^{1/2}$$

However test results showed the relationship to be of form

$$\left(\frac{K_s}{K_r}\right)^{3/2} = \left(\frac{Sr}{S}\right),$$

The coefficient K_r was determined by Muller as,

$$K_r = \frac{26}{d_{90}^{1/6}},$$

$where: d_{90} =$ Size of sediment for which 90% of the material is finer

4.4 DISCHARGE APPROACH

- **<u>Schoklistch's Approach:</u>**
- Schoklistch pioneered the use of discharge for determination of bed load. There are two Schoklistch formulas:

$$Schoklistch(1934)$$

$$q_b = 7000 \frac{S^{3/2}}{d^{1/2}} (q - q_c)$$

$$q_c = \frac{0.00001944 d}{S^{4/3}}$$

where: q_b =Bed load [kg/s/m]

d= Particle size [mm]

q & q_c = Water discharge and critical discharge at incipient motion.[m^3/s/m]

$$Schoklistch(1943)$$

$$q_b = 2500 S^{3/2} (q - q_c)$$

$$q_c = \frac{0.6 d^{3/2}}{S^{7/6}}$$

<u>Note:</u> q_c formulas are applicable for sediment with specific gravity 2.65

4.5 REGRESSION APPROACH

Rottner's Approach:

- Rottner(1959) applied a regression analysis on laboratory data and developed dimensionally homogenous formula give below

$$q_b = \gamma_s\left[\left(\xi_s - 1\right) gD^3\right]^{1/2}\left[\frac{V}{\sqrt{\left(\xi_s - 1\right) gD}}\left[0.667\left(\frac{d_{50}}{D}\right)^{2/3} + 0.14\right] - 0.778\left(\frac{d_{50}}{D}\right)^{2/3}\right]^3$$

where: q_b =Bed load discharge

γ_s = Specific weight of sediment

ξ_s = Specific gravity of sediment

g= Acceleration of gravity

D= Mean depth

V = Mean velocity

d_{50} = Particle size at 50% passing

Note: q_c **formulas are applicable for sediment with specific gravity 2.65**

4.6 OTHER APPROACHES

- **Velocity Approach**
  - Duboy's Approach
- **Bed Form Approach**
- **Probabilistic Approach**
  - Einstein Approach
  - The Einstein-Brown Approach
- **Stochastic Approach**
  - Yang and Syre Approach
- Etc etc

Note: **Consult reference book for details**

Numerical Problem

- **Q. The sediment load was measured from a river, with average depth *D=1.44 ft* and average width *W=71 ft*. The bed load is fairly uniform, with medium size d_{50}*=0.283 mm*, average velocity *V=3.20f t/sec*, slope *S=0.00144*, water temperature *T=5.6°C* and measured bed load concentration is *Cm=1610 ppm by weight*. Calculate the bed load proposed by Duboys, shields, Schoklitsch, Meyer-Peter, Meyer-Peter and Mullers and Rottners, and compare the results with measurement**

- **Solution**
  - Depth of river, D= 1.44 ft
  - Width, W= 71 ft
  - Sediment size, d_{50} = 0.283 mm
  - Flow velocity, V=3.2 ft/sec
  - Slope, S= 0.00144
  - Water temp, T=5.6°C
  - Measured bed load concentration, Cm =1610 ppm by weight
  - Discharge= AV=(71x1.44)x3.2= 327 ft^3/sec
  - Discharge/width , q=327/71=4.61 ft^3/sec/ft
  - Measured bed load =q_m=(Q/W)Cm(6.24x10^{-5})=0.46 lb/sec/ft

Numerical Problem

- **Duboys Formula**

$$q_b = \frac{0.173}{d^{3/4}} \tau(\tau - \tau_c) = (ft^3/s)/ft$$

$$\tau = \gamma DS = 62.4 \times 1.44 \times 0.00144$$

$$= 0.129\, lb/ft^2$$

From Figure with d_{50}=0.283,

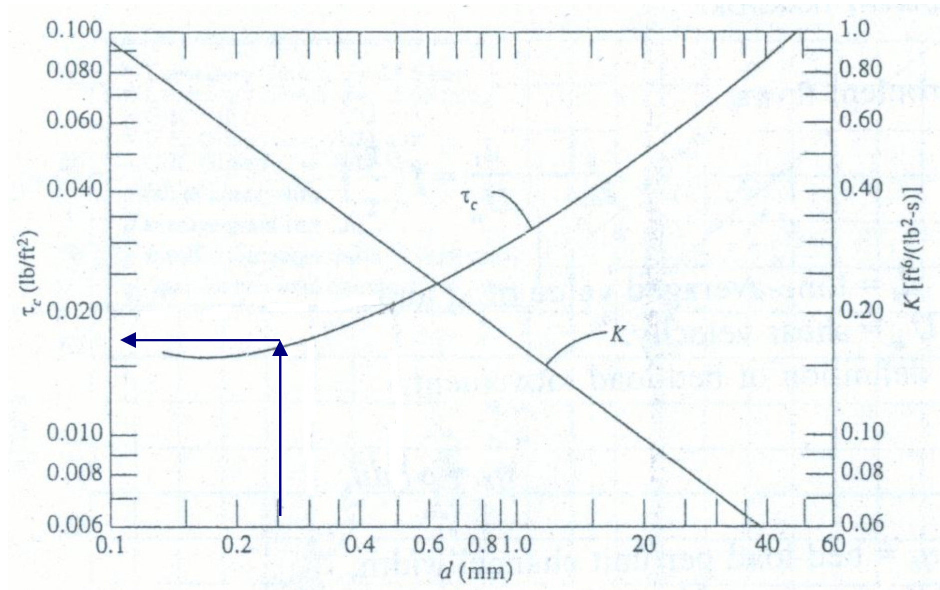
$$\tau_c = 0.018\, lb/ft^2$$

$$q_b = \frac{0.173}{0.283^{3/4}} 0.129(0.129 - 0.018)$$

$$= 0.0064\left(ft^3/s\right)/ft$$

$$q_b = 0.0064\gamma_s = 0.0064 \times 2.65 \times 62.4$$

$$= 1.06\left(lb/s\right)/ft$$



Numerical Problem

- **Shields Formula**

$$\frac{q_b \gamma_s}{q S \gamma} = \frac{10(\tau - \tau_c)}{(\gamma_s - \gamma)\, d_{50}}$$

$$\tau = \gamma D S = 62.4 \times 1.44 \times 0.00144$$

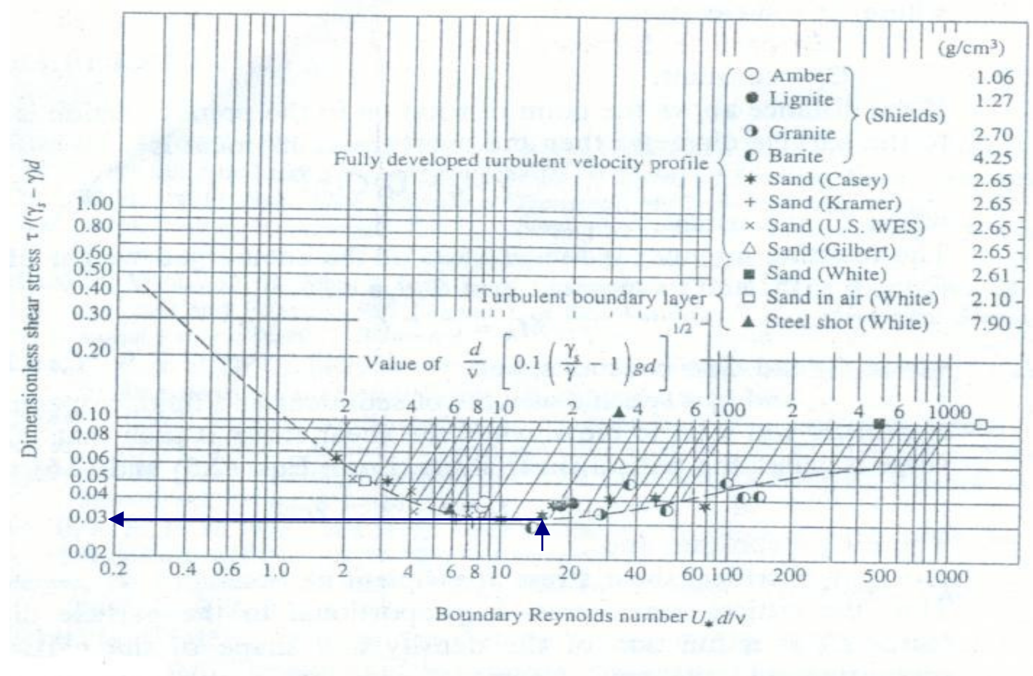
$$= 0.129\, lb / ft^2$$

Using Figure to calculate τ_c

$$U_* = \sqrt{gDS} = \sqrt{32.2 \times 1.44 \times 0.00144}$$

$$= 0.26\, ft / s$$

$$\mathrm{Re} = \frac{U_* d}{\upsilon} = \frac{0.26 \times 9.29 \times 10^{-4}}{1.613 \times 10^{-5}} = 15.1$$



Shields diagram for incipient motion (Vanoni, 1975).

From figure Dimensionless shear stress

$$\frac{\tau_c}{(\gamma_s - \gamma)\mathrm{d}} = 0.031 \Rightarrow \tau_c = 0.031\left[(2.65 \times 62.4) - 62.4\right] 9.29 \times 10^{-4} = 0.003 lb / ft^2$$

$$q_b = \frac{10(0.130 - 0.003)}{\left[2.65 \times 62.4 - 62.4\right] 9.29 \times 10^{-4}} \frac{0.00144 \times 62.4}{2.65 \times 62.4} = 0.033 \quad \left(ft^3 / s\right) / ft$$

$$q_b = 0.033 \gamma_s = 5.46 \left(lb / s\right) / ft$$

Numerical Problem

- **Schoklistch's Formula**

$$q_b = \frac{7000 S^{3/2}}{d^{0.5}}(q - q_s) \Rightarrow (Metric \quad Units)$$

$$q_c = 1.94\times10^{-5}\frac{d}{S^{4/3}} = 1.94\times10^{-5}\frac{0.283}{0.00144^{4/3}} = 0.033\left(m^3/\mathrm{s}\right)/m$$

$$q = 4.61\left(ft^3/s\right)/ft = 0.43\left(m^3/s\right)/m$$

$$q_b = \frac{7000\times0.00144^{3/2}}{0.283^{0.5}}(0.43 - 0.033) = 0.287\left(kg/s\right)/m$$

$$= 0.193\ \left(\mathrm{lb/s/}\right)\mathrm{ft}$$

Numerical Problem

- **Meyer-Peter Formula**

$$\frac{0.4}{d} q_b^{2/3} = \frac{q^{2/3} S}{d} - 17 \Rightarrow (Metric\ Units)$$

$$q = 4.61 \left(ft^3 / s \right) / ft = 428 \left(kg / s \right) / m$$

$$d = 0.283mm = 0.000283m$$

$$q_b^{2/3} = \left(\frac{q^{2/3} S}{d} - 17 \right) \frac{d}{0.4} = 0.20$$

$$q_b = 0.089 \left(kg / s \right) / m$$

$$q_b = 0.059 \left(lb / s \right) / ft$$

Numerical Problem

- **Meyer-Peter and Muller Formula**

$$\gamma\left(\frac{k_s}{K_r}\right)^{3/2} RS = 0.047\left(\gamma_s - \gamma\right)d + 0.25\rho^{1/3}q_b^{2/3} \Rightarrow \left(Metric\ Units\right)$$

$$\gamma = 62.4lb/ft^3 = \frac{62.4\times 0.454\times 0.0001}{0.3048^3} = 1 \quad metric\ ton/m^3$$

$$\gamma_s = 2.65(1) = 2.65 \quad metric\ ton/m^3$$

$$R = D = 1.44 \quad ft = 0.44 \quad m$$

$$d = 0.000283m$$

$$\rho = \gamma/g = 1/9.81 = 0.102 metric\ ton\ S^{-2}/m^4$$

$$K_r = \frac{26}{d_{90}^{1/6}} = \frac{26}{d_{50}^{1/6}} = \frac{26}{0.000286^{1/6}} = 101.7$$

$$S_r = \frac{V^2}{K_r^2 R^{4/3}} = \frac{(3.2\times 0.3048)^2}{101.7^2\times 0.44^{4/3}} = 0.00027$$

$$Since\left(\frac{K_s}{K_r}\right)^{3/2} = \left(\frac{S_r}{S}\right) \Rightarrow \left(\frac{K_s}{K_r}\right)^{3/2} S = S_r = 0.00027$$

Numerical Problem

- **Meyer-Peter and Muller Formula** **…(Cont.)**

$$\gamma\left(\frac{k_s}{K_r}\right)^{3/2} SR = 0.047\left(\gamma_s - \gamma\right)d + 0.25\rho^{1/3}q_b^{2/3} \Rightarrow \left(Metric\,Units\right)$$

$$1(0.00027)0.44 = 0.047(2.65-1)62.4\times0.000283 + 0.25\times0.102^{1/3}q_b^{2/3}$$

$$q_b^{2/3} = 0.0008376 \Rightarrow q_b = 0.0000242 \quad \left(Metric\ ton/s\right)/m$$

$$q_b = 0.0000242\times2205\times0.3048 = 0.01626\ \left(lb/s\right)/ft$$

For Dry weight of sediment

$$\mathrm{q_b'} = \frac{\gamma_s}{\gamma_s - \gamma}q_b = 0.026\left(lb/s\right)/ft$$

Numerical Problem

- **Rottner Formula**

$$q_b = \gamma_s \left[(\xi_s - 1) g D^3 \right]^{1/2} \left[\frac{V}{\sqrt{(\xi_s - 1) g D}} \left[0.667 \left(\frac{d_{50}}{D} \right)^{2/3} + 0.14 \right] - 0.778 \left(\frac{d_{50}}{D} \right)^{2/3} \right]^3$$

$$= 62.4 \times 2.65 \left[(2.65 - 1) 32.2 \times 1.44^3 \right]^{1/2}$$

$$\times \left[\frac{3.2}{\sqrt{(2.65 - 1) 32.2 \times 1.44}} \left[0.667 \left(\frac{2.283}{1000 \times 0.3048 \times 1.44} \right)^{2/3} + 0.14 \right] - 0.778 \left(\frac{2.283}{1000 \times 0.3048 \times 1.44} \right)^{2/3} \right]$$

$$= 0.293 \quad (lb/s)/ft$$

Comparison

Formula	q_b	q_b/q_m
Duboys	1.06	2.30
Shields	5.46	11.87
Schoklistch	0.193	0.42
Meyer-Peter	0.059	0.13
Meyer-Peter and Muller	0.026	0.057
Rottner	0.293	0.64
Mean Value	1.18	2.57

5. Suspended Load Transport

5.0 Suspended Load

- **Introduction:**
- Suspended sediment refers to sediment that is supported by the upward component of turbulent currents and stays in suspension for an appreciable length of time.
- In natural rivers sediments are mainly transported as suspended load.

Note: **Consult reference book for details**

6. Total Load Transport

6.0 Total Load Transport

6.1 Introduction:

- **Based on Mode of Transportation:**
  - Total load = Bed-load + Suspended load

- **Based on Source of Material Being Transported:**
  - Total load = Bed-load Material + Wash load

- **Wash load consists of material that are finer than those found on bed. It amount mainly depends upon the supply from watershed, not on the hydraulics of river.**

Note: Most total load equations are actually total bed material load equations. In the comparison between computed and measured total bed-material, wash load should be subtracted from measurement before comparison in most of cases.

6.0 Total Load Transport

- **Engelund & Hansen's Approach:**
  - **They applied Bagnold's stream power concept and similarity principle to obtain a sediment transport function.**

$$q_s = 0.05\gamma_s V^2 \left[\frac{d^2}{g\left(\gamma_s/\gamma - 1\right)}\right]^{1/2} \left[\frac{\tau_o}{\left(\gamma_s - \gamma\right)d_{50}}\right]^{3/2}$$

Where: q_s = Total sediment load (lb/s)/ft

τ_o = Bed shear stress = γDS

γ & γ_s = Specific weight of water and sediment

V = Velocity of flow

6.0 Total Load Transport

- **Ackers and White's Approach:**
  - **Based on Bagnold's stream power concept they applied dimensional analysis and obtain the following relation ship.**

$$F_{gr} = U_*^n \left[gd\left(\frac{\gamma s}{\gamma} - 1\right)\right]^{-1/2} \left[\frac{V}{\sqrt{32}\log\left(\alpha D/d\right)}\right]^{1-n}$$

Where: F_{gr} = Mobility number for sediment,

U_*=Shear Velocity,

n=Transition component, depending on sediment size

d=Sediment particle size and D=Water depth,

$$\text{dgr}=\text{d}\left[\frac{\left(\gamma s/\gamma\text{-}1\right)}{\upsilon^2}\right]^{1/3}$$

Where:υ=Kinematic Viscosity

6.0 Total Load Transport

Ackers and White's Approach: …(Cont.)

A general dimensionless sediment transport function can be expressed as

$$G_{gr} = f\left(F_{gr}, D_{gr}\right);$$

$$G_{gr} = \frac{XD}{d\gamma_s/\gamma}\left(\frac{U_*}{V}\right) \quad also$$

$$G_{gr} = C\left(\frac{F_{gr}}{A} - 1\right)^m$$

Where: X = rate of sediment transport in terms of mass flow per unit mass flow rate i.e Concentration by weight of fluid flux

For Transition zone with $1 < dgr \le 60$

$$n = 1.00 - 0.56 \log d_{gr}$$

$$A = 0.23 d_{gr}^{-1/2} + 0.14$$

$$m = \frac{9.66}{d_{gr}} + 1.34$$

$$\log C = 2.86 \log d_{gr} - \left(\log d_{gr}\right)^2 - 3.53$$

For Coarse Sediment, $d_{gr} > 60$

n=0.00

A=0.17

m=1.5

C=0.025

The Values of n, A, m, and C were determined by Ackers and White based on best curves of laboratory data with sediment size greater than 0.04mm and Froud Number less than 0.8.

6.0 Total Load Transport

- **Yang Approach:** **He used concept of unit stream power(velocity slope product) and obtained following expression by running regression analysis for 463 sets of laboratory data**

$$\log C_{ts} = 5.435 - 0.286\log\frac{\omega d}{\upsilon} - 0.457\log\frac{U_*}{\omega} +$$

$$\left(1.799 - 0.409\log\frac{\omega d}{\upsilon} - 0.314\log\frac{U_*}{\omega}\right)\log\left(\frac{VS}{\omega} - \frac{V_{cr}S}{\omega}\right)$$

Where: C_{ts}=Total sand concentration (in ppm by weight)

ω=Fall velocity, υ=Kinematic Viscosity, S= Channel slope

$$\frac{V_{cr}S}{\omega} = \frac{2.5}{\log\left(\frac{U_* d}{\upsilon}\right) - 0.06} + 0.66 \quad \text{for } 1.2 < \frac{U_* d}{\upsilon} < 70$$

$$= 2.05 \qquad \text{for } \frac{U_* d}{\upsilon} > 70$$

6.0 Total Load Transport

- **<u>Shen and Hung's Approach:</u>**
- **They recommended a regression equation based on 587 sets of laboratory data in the sand size range.**

  Their regression equation is

$$\log C_t = -107404.45838164 + 324214.74734085Y - 326309.58908739Y^2 + 109503.87232539Y^3$$

$$\text{Where: } Y = \left(VS^{0.57}/\omega^{0.32}\right)^{0.00750189}$$

C_t =Total sand concentration (in ppm by weight)

ω=Fall velocity, υ=Kinematic Viscosity,

S= Channel slope, V= flow velocity

Numerical Problem

- Q. The following data was collected from a river:
  - Median particle size, d_{50}=0.283 mm
  - Velocity, V=3.7 ft/sec
  - Slope, S= 0.00169
  - Water Temperature, T=14.4^{o}C=57.9^{o}F
  - Average Depth, D= 1.73 ft
  - Channel Width, W=71 ft
- The measured total bed-material concentration was 1900 ppm by weight. Assume the bed material is fairly uniform
- Compute the total bed material concentration by the following methods.
  - Yang (1973)
  - Ackers and White
  - Engelund and Hansen
  - Shen and Hung

Numerical Problem

- Yang's (1973) Method

$$\log C_{ts} = 5.435 - 0.286\log\frac{\omega d}{\upsilon} - 0.457\log\frac{U_*}{\omega} +$$
$$\left(1.799 - 0.409\log\frac{\omega d}{\upsilon} - 0.314\log\frac{U_*}{\omega}\right)\log\left(\frac{VS}{\omega} - \frac{V_{cr}S}{\omega}\right)$$

At a temperature $=14.4^{o}\mathrm{C}$, $\upsilon = 1.26\times10^{-5}\ ft^2/s$

From figure, assuming a shape factor of 0.7 the fall velocity is

$$\omega \quad = \quad 3.7\,cm/s \quad = \quad 0.12\,ft/\sec$$

Assuming the river is wide(D=R), the shear velocity is

$$\mathrm{U*} = \sqrt{\mathrm{gDS}} = \sqrt{32.2\times1.73\times0.00169} = 0.307\,ft/s$$

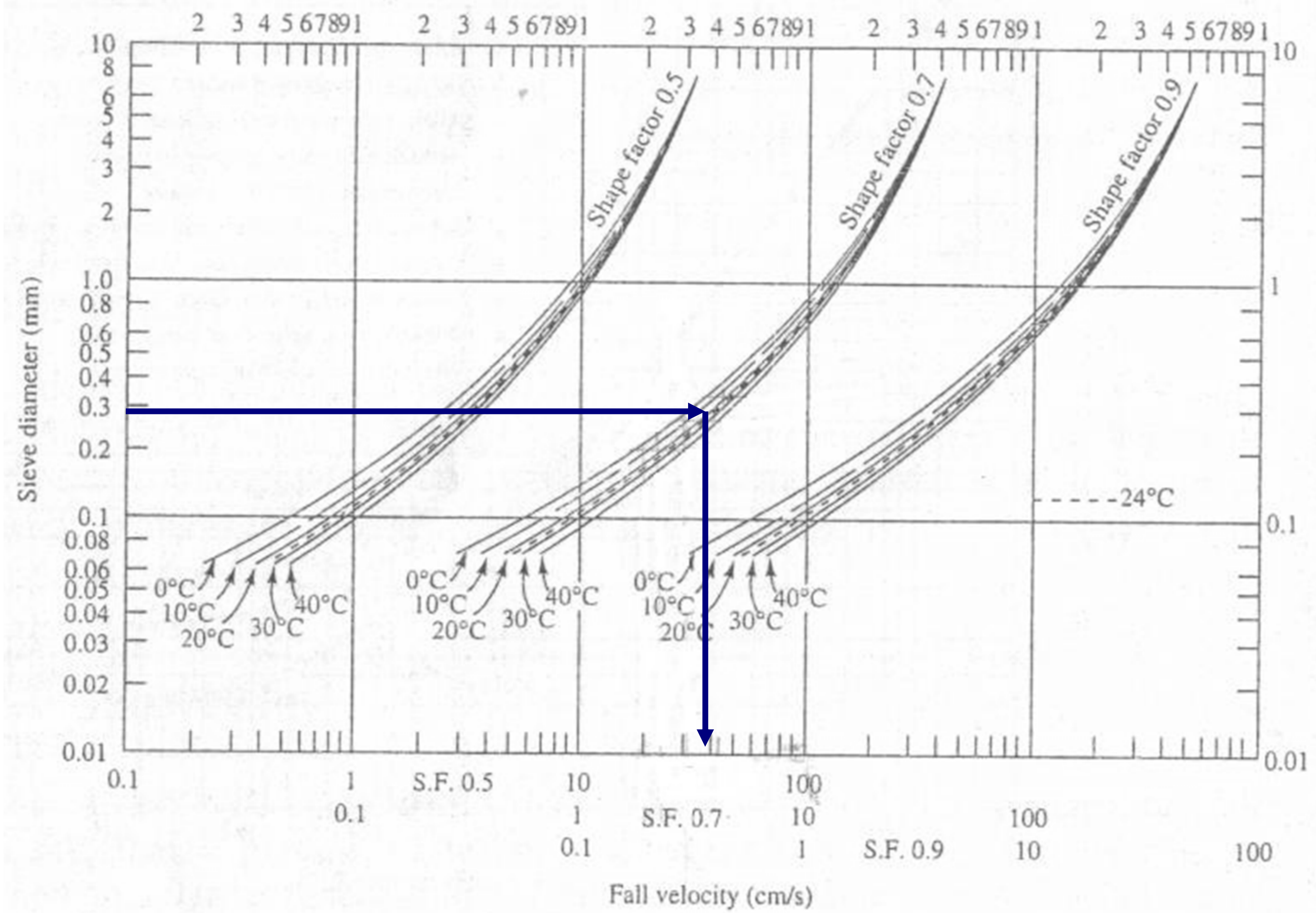
$$\text{Shear Velocity Reynolds No. Re} = \frac{\mathrm{U_*d}}{\upsilon} = \frac{0.307\times9.28\times10^{-4}}{1.26\times10^{-5}} = 22.6$$

$\therefore \dfrac{V_{cr}}{\omega}$ can be determined by following eq.

$$\frac{V_{cr}}{\omega} = \frac{2.5}{\log\left(\dfrac{U_*d}{\upsilon}\right) - 0.06} + 0.66 = \frac{2.5}{\log(22.6) - 0.06} + 0.66 = 2.59$$

Numerical Problem

- Yang's (1973) Method



Relation between sieve diameter and fall velocity for naturally worn quartz particles falling alone in quiescent distilled water of infinite extent (U.S. Inter-Agency Committee on Water Resources, Subcommittee on Sedimentation, 1957).

Numerical Problem

- Yang's (1973) Method

$$\log C_{ts} = 5.435 - 0.286 \log \frac{0.12 \times 9.28 \times 10^{-4}}{1.26 \times 10^{-5}} - 0.457 \log \frac{0.307}{0.12} +$$

$$\left(1.799 - 0.409 \log \frac{0.12 \times 9.28 \times 10^{-4}}{1.26 \times 10^{-5}} - 0.314 \log \frac{0.307}{0.12}\right)$$

$$\times \log \left(\frac{3.7 \times 0.00169}{0.12} - 2.59 \times 0.00169\right)$$

$$\log C_{ts} = 3.282$$

$$C_{ts} = 1910 \quad ppm$$

Numerical Problem

- Ackers and White Method

$$d_{gr}=d\left[\frac{g\left(\gamma_s/\gamma-1\right)}{\upsilon^2}\right]^{1/3}=9.28\times10^{-4}\left[\frac{32.2\left(2.65-1\right)}{\left(1.26\times10^{-5}\right)^2}\right]^{1/3}=6.44$$

The parameter, m, A, n and C are

$$\text{m}=\frac{9.66}{d_{gr}}+1.34=\frac{9.66}{6.44}+1.34=2.84$$

$$A=\frac{0.23}{\sqrt{d_{gr}}}+0.14=\frac{0.23}{\sqrt{6.44}}+0.14=0.23$$

$$n=1-0.56\log d_{gr}=1-0.56\log 6.44=0.55$$

$$C=10^{\left[2.86\log d_{gr}-(\log d_{gr})^2-3.53\right]}=10^{\left[2.86\log 6.44-(\log 6.44)^2-3.53\right]}=0.013$$

Numerical Problem

- Ackers and White Method

Since $U_* = 0.307\,ft/s$ and α=10, the mobility parameter is

$$F_{gr} = \frac{U_*^n}{\left[gd\left(\gamma_s/\gamma-1\right)\right]^{1/2}}\left[\frac{V}{\sqrt{32}\log\left(\alpha D/d\right)}\right]^{1-n}$$

$$F_{gr} = \frac{0.307^n}{\left[32.2\times\left(9.28\times10^{-4}\right)\left(2.65-1\right)\right]^{1/2}}\left[\frac{3.7}{\sqrt{32}\log\left(10\times1.73/9.28\times10^{-4}\right)}\right]^{1-0.55}$$

$$F_{gr} = 1.01$$

The dimensionless sediment transport rate is

$$\mathrm{G_{gr}} = C\left(\frac{F_{gr}}{A}-1\right)^m = 0.013\left(\frac{1.01}{0.23}-1\right)^{2.84} = 0.43$$

and now X:

$$\mathrm{X}=\frac{\mathrm{G_{gr}}d(\gamma_s/\gamma)}{\mathrm{D(U_*/V)^n}} = \frac{0.43\times9.28\times10^{-4}(2.65)}{1.73(0.307/3.7)^{0.55}} = 0.0024$$

$$Hence \qquad Ct = 2400 \quad ppm$$

Numerical Problem

- Engelund and Hansen's Method

$$q_s = 0.05\gamma_s V^2 \left[\frac{d^2}{g(\gamma_s/\gamma - 1)}\right]^{1/2} \left[\frac{\tau_o}{(\gamma_s - \gamma)d_{50}}\right]^{3/2}$$

$$\gamma_s = 165 \quad lb/ft^3$$

$$\gamma \; at \; 14.4^o C = 62.38 \quad lb/ft^3$$

$$\tau_o = \gamma DS = 62.38(1.73)0.00169 = 0.182 \quad lb/ft^2$$

$$q_s = 0.05(165)3.7^2 \left[\frac{(9.28\times10^{-4})^2}{32.2(1.65)}\right]^{1/2} \left[\frac{0.182}{(165 - 62.38)9.28\times10^{-4}}\right]^{3/2}$$

$$q_s = 1.25 \quad (lb/s)/ft$$

$$C_t = \frac{Wq_s}{\gamma WDV} = \frac{71(1.25)}{62.38(71)1.73(3.7)} = \quad 3120 \quad ppm$$

Numerical Problem

- Shen and Hung's Method

$$\log C_t = -107404.45838164 + 324214.74734085Y$$
$$-326309.58908739Y^2 + 109503.87232539Y^3$$

$$Where: Y = \left(VS^{0.57} / \omega^{0.32}\right)^{0.00750189} \quad = \left(3.7(0.00169)^{0.57} / 0.12^{0.32}\right)^{0.00750189}$$

$$Y = 0.98769$$

$$\log C_t = 3.3809$$

$$C_t = 2400\ ppm$$

Comparison

Formula	**Ct**	**Ct/Ctm**
Yang(1973)	1910	
Ackers and White	2400	
Engelund and Hansen	3120	
Shen and Hung	2400	