MASS TRANSFER

Mass transfer is the flow of molecules from one body to another when these bodies are in contact or within a system consisting of two components when the distribution of materials is not uniform. When a copper plate is placed on a steel plate, some molecules from either side will diffuse into the other side. When salt is placed in a glass and water poured over it, after sufficient time the salt molecules will diffuse into the water body. A more common example is drying of clothes or the evaporation of water spilled on the floor when water molecules diffuse into the air surrounding it. Usually mass transfer takes place from a location where the particular component is proportionately high to a location where the component is proportionately low. Mass transfer may also take place due to potentials other than concentration difference.

1. PROPERTIES OF MIXTURE

In a mixture consisting of two or more materials the mass per unit volume of any component is called mass concentration of that component. If there are two components *A* and *B*, then the mass concentration of *A* is;

$$m_a = \frac{\text{mass of } A \text{ in the mixture}}{\text{volume of the mixture}}$$

and concentration of *B*,

$$m_b = \frac{\text{mass of } B \text{ in the mixture}}{\text{volume of the mixture}}$$

The total mass concentration is $ma + mb$, which is also the density of the mixture. Mass concentration can also be expressed in terms of individual and total densities of the mixture *i.e.*,

$$m_a = \frac{\rho_A}{\rho}$$

where ρa is the density of *A* in the mixture and ρ is the density of the mixture. It is more convenient to express the concentration in terms of the molecular weight of the component.

Mole fraction Na can be expressed as

$$N_a = \frac{\text{Number of moles of component } A}{\text{Total number of moles in the mixture}}$$

Number of Mole = mass/molecular weight

For gases as ρi =

$$\rho_i = \frac{P}{R_i T}$$

$$N_i = \frac{P_i}{\Re T}$$

where $\Re$ is universal gas constant. At the temperature T of the mixture then

$$N_i \propto P_i$$

$$C_a = \frac{N_a}{Nt} = \frac{P_a}{P_T}$$

where Pa is the partial pressure of A in the mixture and PT is the total pressure of the mixture. Ca is the mole concentration of A in the mixture. Also Ca + Cb = 1 for a two component mixture.

2. DIFFUSION MASS TRANSFER

Diffusion mass transfer occurs without macroscopic mass motion or mixing. A lump of sugar dropped into a cup of tea will dissolve by diffusion even if left unstirred. But it will take a long time for the sugar to reach all of the volume in the cup. However it will diffuse into the volume by and by. Consider a chamber in which two different gases at the same pressure and temperature are kept separated by a thin barrier. When the barrier is removed, the gases will begin to diffuse into each other's volume. After some time a steady condition of uniform mixture would be reached. This type of diffusion can occur in solids also. The rate in solids will be extremely slow. Diffusion in these situations occurs at the molecular level and the governing equations are similar to those in heat conduction where energy transfer occurs at the molecular level.

The basic law governing mass transfer at the molecular diffusion level is known as Fick's law. This is similar to the Fourier heat conduction law.

In Mass transfer, molar quantities are more convenient to use as compared to mass units, because mass transfer is due to the movement of molecules as discrete quantities. Hence it is convenient to use number of moles, or molar concentration instead of density etc.

2.1. FICK'S LAW OF DIFFUSION

The Fick's law can be stated as

$$N_a = -D_{ab}\frac{dC_a}{dx} \tag{1}$$

Where N_a —> number of moles of 'a' diffusing perpendicular to area A, moles/m^2 sec

D_{ab} —> Diffusion coefficient or mass diffusivity, m^2/s, a into b

C_a —> mole concentration of 'a' moles/m^3

x —> diffusion direction

The diffusion coefficient is similar to thermal diffusivity, α and momentum diffusivity v. Number of moles multiplied by the molecular mass (or more popularly known as molecular weight) will provide the value of mass transfer in kg/s.

Equation 1 can also be written as

$$\frac{m_a}{A} = -D_{ab} \cdot \frac{d\rho_a}{dx} \qquad (2)$$

but this form is not as popular as the more convenient equation 1. The conduction equation similar to this is

$$\frac{Q}{A} = -\left(\frac{k}{\rho c}\right) \cdot \frac{d(\rho c T)}{dx} \qquad (3)$$

k/ϱc is thermal diffusivity α and ϱc is the heat capacity (energy density) for unit volume. The derivation of the general mass diffusion equation is similar to that of the general heat conduction equation with Ca replacing T and D replacing k/ϱc. The general mass diffusion equation for the species A under steady state condition is given by equation 4

$$\frac{\partial^2 C_a}{dx^2} + \frac{\partial^2 C_a}{\partial y^2} + \frac{\partial^2 C_a}{\partial z^2} = \frac{1}{D}\frac{\partial C_a}{\partial \tau} \qquad (4)$$

Generation of mass of the species 'A' by chemical reaction is not considered in the equation. However an additive term Na/D on the LHS will take care of this similar to heat generation term q/k.

The solutions for this equation are also similar to the solutions of the general conduction equation. However there exist some differences. These are

(i) While heat flow is in one direction, the mass of one species flows opposite to the flow of the other component of the mixture. (here two component mixture is considered).

(ii) Even while one component alone diffuses under certain circumstances, a bulk flow has to be generated as otherwise a density gradient will be created spontaneously, which is not possible. For example when water evaporates into an air body over water surface, an equal quantity of air cannot enter the water phase. The density gradient created is dispersed by some mixture moving away from the surface maintaining a balance. This is termed as bulk flow.

The value of Dab for certain combinations of components are available in literature. It can be proved that Dab= Dba. When one molecule of 'A' moves in the x direction, one molecule of 'B' has to move in the opposite direction. Otherwise a macroscopic density gradient will develop, which is not sustainable, (A is area)

$$\frac{N_a}{A} = -\frac{N_b}{A} \text{ and so } D_{ab} = D_{ba}$$

$$\frac{N_a}{A} = -D_{ab}\frac{dC_a}{dx}$$

$$\frac{N_b}{A} = -D_{ba}\frac{dC_b}{dx} = -D_{ba}\frac{d(1-C_a)}{dx} = D_{ba}\frac{dC_a}{dx}$$

2.2. EQUIMOLAL COUNTER DIFFUSION

The total pressure is constant all through the mixture. Hence the difference in partial pressures will be equal. The Fick's equation when integrated for a larger plane volume of thickness L will give

$$\frac{N_a}{A} = D_{ab}\,\frac{(C_{a1} - C_{a2})}{L}$$

$$\frac{N_b}{A} = D_{ba}\,\frac{(C_{b2} - C_{b1})}{L}$$

$$\frac{N_b}{A} = -\,\frac{N_a}{A},\ \text{and}\ (C_{a1} - C_{a2}) = (C_{b2} - C_{b1}). \tag{5}$$

Dab equals Dba. Where Ca1 and Cb1 are the mole concentrations at face 1 and Ca2 and Cb2 are mole concentrations at face 2 which is at a distance L from the first face. When applied to gases,

$$\frac{N_a}{A} = \frac{D}{\Re T}\cdot\frac{P_{a1} - P_{a2}}{(x_2 - x_1)} \tag{6}$$

Where Pa1 and Pa2 are partial pressures of component 'A' at x1 and x2 and $\Re$ is the universal gas constant in J/kg mol K. T is the temperature in absolute units. The distance should be expressed in metre.

Example 1: In order to avoid pressure build up ammonia gas at atmospheric pressure in a pipe is vented to atmosphere through a pipe of 3 mm dia and 20 m length. Determine the mass of ammonia diffusing out and mass of air diffusing in per hour. Assume D = 0.28 × 10–4 m2/s, M = 17 kg/kg mole

Solution: P_{NH_3} in pipe = 1 atm.

P_{NH_3} at the outlet = 0

$$m_{NH_3} = \frac{D.A.}{\Re T}\,\frac{P_{NH_3 1} - P_{NH_3 2}}{L} \times M$$

$$= 0.28 \times 10^{-4} \times \frac{\pi}{4}(0.003)^2 \times \frac{(1.013 \times 10^5 - 0)}{20} \times 3600 \times 17/8315$$

$$= \mathbf{7.38 \times 10^{-6}\ kg/hr.}$$

$$m_{air}.\ N_B = -N_A = -\,7.38 \times 10^{-6} \times 28.97/17$$

$$= \mathbf{-\,1.26 \times 10^{-5}\ kg/hr.}$$

$$M_{air} = 28.97\ \text{kg/kg mole.}$$

Reference:

Fundamentals of Heat and Mass Transfer by C. P. Kothandaraman