

Heat Transfer

In the majority of chemical processes heat is either given out or absorbed, and fluids must often be either heated or cooled in a wide range of plant, such as furnaces, evaporators, distillation units, dryers, and reaction vessels where one of the major problems is that of transferring heat at the desired rate. In addition, it may be necessary to prevent the loss of heat from a hot vessel or pipe system. The control of the flow of heat at the desired rate forms one of the most important areas of chemical engineering. Provided that a temperature difference exists between two parts of a system, heat transfer will take place in one or more of three different ways.

Conduction. In a solid, the flow of heat by conduction is the result of the transfer of vibrational energy from one molecule to another, and in fluids it occurs in addition as a result of the transfer of kinetic energy. Heat transfer by conduction may also arise from the movement of free electrons, a process which is particularly important with metals and accounts for their high thermal conductivities.

Convection. Heat transfer by convection arises from the mixing of elements of fluid. If this mixing occurs as a result of density differences as, for example, when a pool of liquid is heated from below, the process is known as *natural convection*. If the mixing results from eddy movement in the fluid, for example when a fluid flows through a pipe heated on the outside, it is called *forced convection*. It is important to note that convection requires mixing of fluid elements, and is not governed by temperature difference alone as is the case in conduction and radiation.

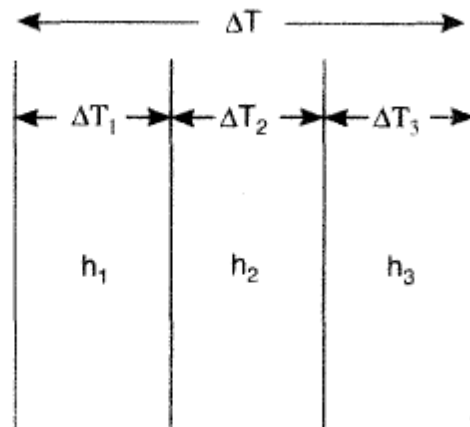
Radiation. All materials radiate thermal energy in the form of electromagnetic waves. When this radiation falls on a second body it may be partially reflected, transmitted, or absorbed. It is only the fraction that is absorbed that appears as heat in the body.

BASIC CONSIDERATIONS

Individual and overall coefficients of heat transfer

In many of the applications of heat transfer in process plants, one or more of the mechanisms of heat transfer may be involved. In the majority of heat exchangers heat passes through a series of different intervening layers before reaching the second fluid. These layers may be of different thicknesses and of different thermal conductivities.

The problem of transferring heat to crude oil in the primary furnace before it enters the first distillation column may be considered as an example. The heat from the flames passes by radiation and convection to the pipes in the furnace, by conduction through the



Heat transfer through a composite wall

pipe walls, and by forced convection from the inside of the pipe to the oil. Here all three modes of transfer are involved. After prolonged usage, solid deposits may form on both the inner and outer walls of the pipes, and these will then contribute additional resistance to the transfer of heat. The simplest form of equation which represents this heat transfer operation may be written as:

$$Q=UA\Delta T.....eq.1$$

where Q is the heat transferred per unit time, A the area available for the flow of heat, ΔT the difference in temperature between the flame and the boiling oil, and U is known as the overall heat transfer coefficient ($W/m^2 K$ in SI units).

At first sight, equation 1 implies that the relationship between Q and ΔT is linear. Whereas this is approximately so over limited ranges of temperature difference for which U is nearly constant, in practice U may well be influenced both by the temperature difference and by the absolute value of the temperatures.

If it is required to know the area needed for the transfer of heat at a specified rate, the temperature difference ΔT , and the value of the overall heat-transfer coefficient must be known. Thus the calculation of the value of U is a key requirement in any design problem in which heating or cooling is involved. A large part of the study of heat transfer is therefore devoted to the evaluation of this coefficient.

The value of the coefficient will depend on the mechanism by which heat is transferred, on the fluid dynamics of both the heated and the cooled fluids, on the properties of the

materials through which the heat must pass, and on the geometry of the fluid paths. In solids, heat is normally transferred by conduction; some materials such as metals have a high thermal conductivity, whilst others such as ceramics have a low conductivity. Transparent solids like glass also transmit radiant energy particularly in the visible part of the spectrum.

Liquids also transmit heat readily by conduction, though circulating currents are frequently set up and the resulting convective transfer may be considerably greater than the transfer by conduction. Many liquids also transmit radiant energy. Gases are poor conductors of heat and circulating currents are difficult to suppress; convection is therefore much more important than conduction in a gas. Radiant energy is transmitted with only limited absorption in gases and, of course, without any absorption *in vacua*. Radiation is the only mode of heat transfer which does not require the presence of an intervening medium.

If the heat is being transmitted through a number of media in series, the overall heat transfer coefficient may be broken down into individual coefficients h each relating to a single medium. This is as shown in Figure 9.1. It is assumed that there is good contact between each pair of elements so that the temperature is the same on the two sides of each junction.

If heat is being transferred through three media, each of area A , and individual coefficients for each of the media are h_1 , h_2 , and h_3 , and the corresponding temperature changes are ΔT_1 , ΔT_2 , and ΔT_3 then, provided that there is no accumulation of heat in the media, the heat transfer rate Q will be the same through each. Three equations, analogous to equation 1, can therefore be written as:

$$\left. \begin{aligned} Q &= h_1 A \Delta T_1 \\ Q &= h_2 A \Delta T_2 \\ Q &= h_3 A \Delta T_3 \end{aligned} \right\}$$

Rearranging:

$$\Delta T_1 = \frac{Q}{A} \frac{1}{h_1}$$

$$\Delta T_2 = \frac{Q}{A} \frac{1}{h_2}$$

$$\Delta T_3 = \frac{Q}{A} \frac{1}{h_3}$$

Adding:
$$\Delta T_1 + \Delta T_2 + \Delta T_3 = \frac{Q}{A} \left(\frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} \right)$$

Noting that $(\Delta T_1 + \Delta T_2 + \Delta T_3) = \text{total temperature difference } \Delta T$:

then:
$$\Delta T = \frac{Q}{A} \left(\frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} \right)$$

From equation 9.1:

$$\Delta T = \frac{Q}{A} \frac{1}{U}$$

Comparing equations 9.4 and 9.5:

$$\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3}$$

The reciprocals of the heat transfer coefficients are resistances, and equation 9.6 therefore illustrates that the resistances are additive.

In some cases, particularly for the radial flow of heat through a thick pipe wall or cylinder, the area for heat transfer is a function of position. Thus the area for transfer applicable to each of the three media could differ and may be A_1 , A_2 and A_3 . Equation 9.3 then becomes:

$$\Delta T_1 + \Delta T_2 + \Delta T_3 = Q \left(\frac{1}{h_1 A_1} + \frac{1}{h_2 A_2} + \frac{1}{h_3 A_3} \right) \quad (9.7)$$

Equation 9.7 must then be written in terms of one of the area terms A_1 , A_2 , and A_3 , or sometimes in terms of a mean area. Since Q and ΔT must be independent of the particular

area considered, the value of U will vary according to which area is used as the basis. Thus equation 9.7 may be written, for example:

$$Q = U_1 A_1 \Delta T \quad \text{or} \quad \Delta T = \frac{Q}{U_1 A_1}$$

This will then give U_1 as:

$$\frac{1}{U_1} = \frac{1}{h_1} + \frac{A_1}{A_2} \left(\frac{1}{h_2} \right) + \frac{A_1}{A_3} \left(\frac{1}{h_3} \right) \quad (9.8)$$

9.2.2. Mean temperature difference

Where heat is being transferred from one fluid to a second fluid through the wall of a vessel and the temperature is the same throughout the bulk of each of the fluids, there is no difficulty in specifying the overall temperature difference ΔT . Frequently, however, each fluid is flowing through a heat exchanger such as a pipe or a series of pipes in parallel, and its temperature changes as it flows, and consequently the temperature difference is continuously changing. If the two fluids are flowing in the same direction (*co-current flow*), the temperatures of the two streams progressively approach one another as shown in Figure 9.2. In these circumstances the outlet temperature of the heating fluid must always be higher than that of the cooling fluid. If the fluids are flowing in opposite directions (*countercurrent flow*), the temperature difference will show less variation throughout the heat exchanger as shown in Figure 9.3. In this case it is possible for the cooling liquid to leave at a higher temperature than the heating liquid, and one of the great advantages of

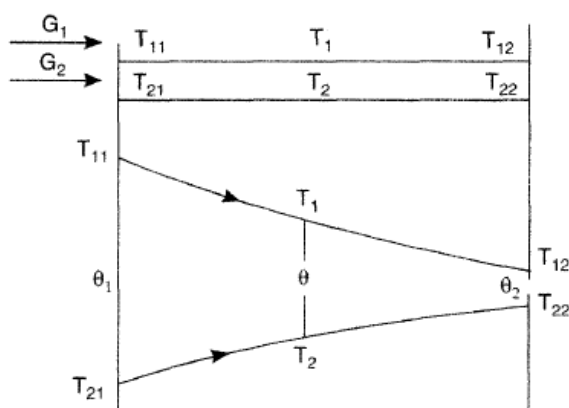


Figure 9.2. Mean temperature difference for co-current flow

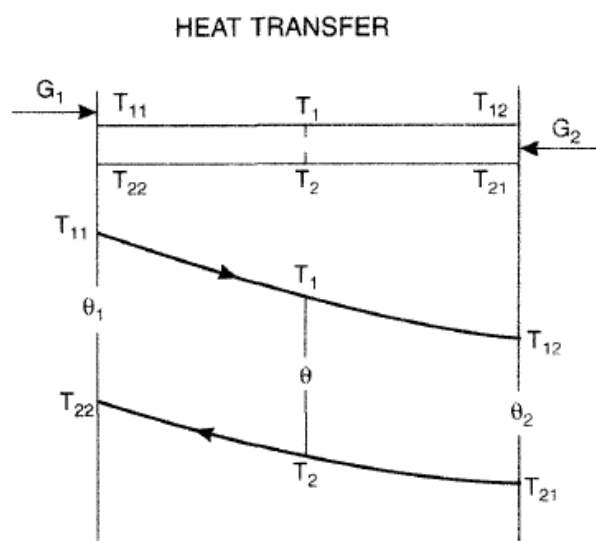


Figure 9.3. Mean temperature difference for countercurrent flow

countercurrent flow is that it is possible to extract a higher proportion of the heat content of the heating fluid. The calculation of the appropriate value of the temperature difference for co-current and for countercurrent flow is now considered. It is assumed that the overall heat transfer coefficient U remains constant throughout the heat exchanger.

It is necessary to find the average value of the temperature difference θ_m to be used in the general equation:

$$Q = UA\theta_m \quad (\text{equation 9.1})$$

Figure 9.3 shows the temperature conditions for the fluids flowing in opposite directions, a condition known as countercurrent flow.

The outside stream specific heat C_{p1} and mass flow rate G_1 falls in temperature from T_{11} to T_{12} .

The inside stream specific heat C_{p2} and mass flow rate G_2 rises in temperature from T_{21} to T_{22} .

Over a small element of area dA where the temperatures of the streams are T_1 and T_2 . The temperature difference:

$$\theta = T_1 - T_2$$

$$d\theta = dT_1 - dT_2$$

$$\text{Heat given out by the hot stream} = dQ = -G_1 C_{p1} dT_1$$

$$\text{Heat taken up by the cold stream} = dQ = G_2 C_{p2} dT_2$$

$$\therefore d\theta = -\frac{dQ}{G_1 C_{p1}} - \frac{dQ}{G_2 C_{p2}} = -dQ \left(\frac{G_1 C_{p1} + G_2 C_{p2}}{G_1 C_{p1} \times G_2 C_{p2}} \right) = -\psi dQ \quad (\text{say})$$

$$\therefore \theta_1 - \theta_2 = \psi Q$$

$$\text{Over this element:} \quad U dA\theta = dQ$$

$$\therefore U dA\theta = -\frac{d\theta}{\psi}$$

If U may be taken as constant:

$$-\psi U \int_0^A dA = \int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta}$$

$$\therefore -\psi UA = -\ln \frac{\theta_1}{\theta_2}$$

From the definition of θ_m , $Q = UA\theta_m$.

$$\therefore \theta_1 - \theta_2 = \psi Q = \psi UA\theta_m = \ln \frac{\theta_1}{\theta_2} (\theta_m)$$

$$\text{and:} \quad \theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} \quad (9.9)$$

where θ_m is known as the *logarithmic mean temperature difference*.

UNDERWOOD⁽¹⁾ proposed the following approximation for the logarithmic mean temperature difference:

$$(\theta_m)^{1/3} = \frac{1}{2}(\theta_1^{1/3} + \theta_2^{1/3}) \quad (9.10)$$

Example

A heat exchanger is required to cool 20 kg/s of water from 360 K to 340 K by means of 25 kg/s water entering at 300 K. If the overall coefficient of heat transfer is constant at 2 kW/m²K, calculate the surface area required in (a) a countercurrent concentric tube exchanger, and (b) a co-current flow concentric tube exchanger.

Solution

Heat load: $Q = 20 \times 4.18(360 - 340) = 1672 \text{ kW}$

The cooling water outlet temperature is given by:

$$1672 = 25 \times 4.18(\theta_2 - 300) \quad \text{or} \quad \theta_2 = 316 \text{ K}$$

(a) *Counterflow*

In equation 9.9: $\theta_m = \frac{44 - 40}{\ln(44/40)} = 41.9 \text{ K}$

Heat transfer area: $A = \frac{Q}{U\theta_m}$

$$\begin{aligned} &= \frac{1672}{2 \times 41.9} \\ &= \underline{\underline{19.95 \text{ m}^2}} \end{aligned}$$

(b) *Co-current flow*

In equation 9.9: $\theta_m = \frac{60 - 24}{\ln(60/24)} = 39.3 \text{ K}$

Heat transfer area: $A = \frac{1672}{2 \times 39.3} = \underline{\underline{21.27 \text{ m}^2}}$

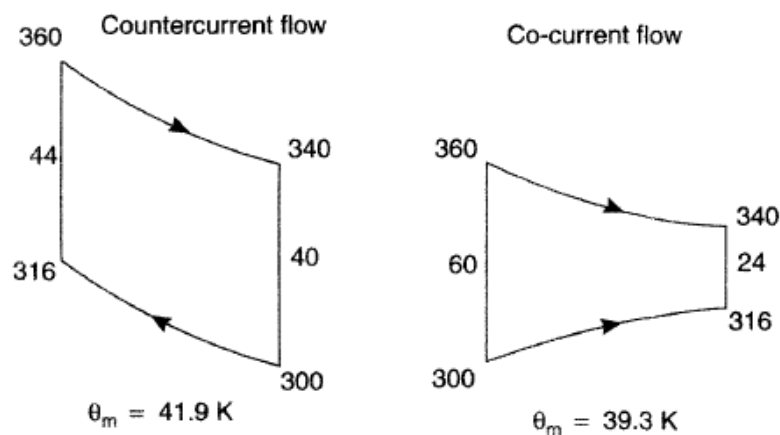


Figure 9.4. Data for Example 9.1

9.3. HEAT TRANSFER BY CONDUCTION

9.3.1. Conduction through a plane wall

This important mechanism of heat transfer is now considered in more detail for the flow of heat through a plane wall of thickness x as shown in Figure 9.5.

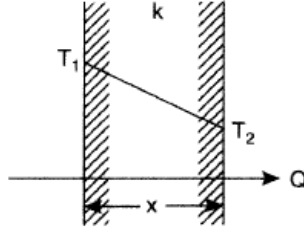


Figure 9.5. Conduction of heat through a plane wall

The rate of heat flow Q over the area A and a small distance dx may be written as:

$$Q = -kA \left(\frac{dT}{dx} \right) \quad (9.11)$$

which is often known as *Fourier's equation*, where the negative sign indicates that the temperature gradient is in the opposite direction to the flow of heat and k is the thermal conductivity of the material. Integrating for a wall of thickness x with boundary temperatures T_1 and T_2 , as shown in Figure 9.5:

$$Q = \frac{kA(T_1 - T_2)}{x} \quad (9.12)$$

Thermal conductivity is a function of temperature and experimental data may often be expressed by a linear relationship of the form:

$$k = k_0(1 + k'T) \quad (9.13)$$

where k is the thermal conductivity at the temperature T and k_0 and k' are constants. Combining equations 9.11 and 9.13:

$$-k dT = -k_0(1 + k'T)dT = \frac{Q dx}{A}$$

Integrating between the temperature limits T_1 and T_2 ,

$$-\int_{T_1}^{T_2} k dT = (T_1 - T_2)k_0 \left\{ 1 + k' \left(\frac{T_1 + T_2}{2} \right) \right\} = Q \int_{x_1}^{x_2} \frac{dx}{A} \quad (9.14)$$

Where k is a linear function of T , the following equation may therefore be used:

$$k_a(T_1 - T_2) = Q \int_{x_1}^{x_2} \frac{dx}{A} \quad (9.15)$$

where k_a is the arithmetic mean of k_1 and k_2 at T_1 and T_2 respectively or the thermal conductivity at the arithmetic mean of T_1 and T_2 .

Where k is a non-linear function of T , some mean value, k_m will apply, where:

$$k_m = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} k dT \quad (9.16)$$

It is convenient to rearrange equation 9.12 to give:

$$Q = \frac{(T_1 - T_2)A}{(x/k)}$$

where x/k is known as the *thermal resistance* and k/x is the *transfer coefficient*.

9.3.2. Thermal resistances in series

It has been noted earlier that thermal resistances may be added together for the case of heat transfer through a complete section formed from different media in series.

Figure 9.6 shows a composite wall made up of three materials with thermal conductivities k_1 , k_2 , and k_3 , with thicknesses as shown and with the temperatures T_1 , T_2 , T_3 , and T_4 at the faces. Applying equation 9.12 to each section in turn, and noting that the same quantity of heat Q must pass through each area A :

$$T_1 - T_2 = \frac{x_1}{k_1 A} Q, \quad T_2 - T_3 = \frac{x_2}{k_2 A} Q \quad \text{and} \quad T_3 - T_4 = \frac{x_3}{k_3 A} Q$$

On addition: $(T_1 - T_4) = \left(\frac{x_1}{k_1 A} + \frac{x_2}{k_2 A} + \frac{x_3}{k_3 A} \right) Q$ (9.18)

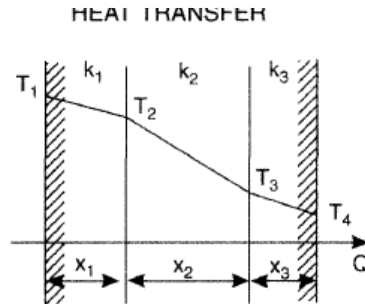


Figure 9.6. Conduction of heat through a composite wall

or:

$$Q = \frac{T_1 - T_4}{\Sigma(x_i/k_i A)}$$

$$= \frac{\text{Total driving force}}{\text{Total (thermal resistance/area)}} \quad (9.19)$$

A furnace is constructed with 0.20 m of firebrick, 0.10 m of insulating brick, and 0.20 m of building brick. The inside temperature is 1200 K and the outside temperature is 330 K. If the thermal conductivities are as shown in Figure 9.7, estimate the heat loss per unit area and the temperature at the junction of the firebrick and the insulating brick.

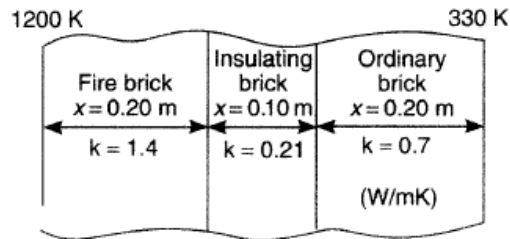


Figure 9.7. Data for Example 9.3

Solution

From equation 9.19:

$$\begin{aligned}
 Q &= (1200 - 330) / \left[\left(\frac{0.20}{1.4 \times 1} \right) + \left(\frac{0.10}{0.21 \times 1} \right) + \left(\frac{0.20}{0.7 \times 1} \right) \right] \\
 &= \frac{870}{(0.143 + 0.476 + 0.286)} = \frac{870}{0.905} \\
 &= \underline{\underline{961 \text{ W/m}^2}}
 \end{aligned}$$

The ratio (Temperature drop over firebrick)/(Total temperature drop) = $(0.143/0.905)$

$$\therefore \text{Temperature drop over firebrick} = \left(\frac{870 \times 0.143}{0.905} \right) = 137 \text{ deg K}$$

Hence the temperature at the firebrick-insulating brick interface = $(1200 - 137) = \underline{\underline{1063 \text{ K}}}$

9.3.3. Conduction through a thick-walled tube

The conditions for heat flow through a thick-walled tube when the temperatures on the inside and outside are held constant are shown in Figure 9.8. Here the area for heat flow is proportional to the radius and hence the temperature gradient is inversely proportional to the radius.

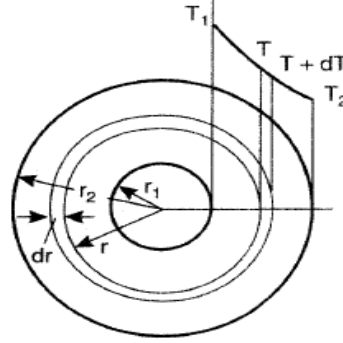


Figure 9.8. Conduction through thick-walled tube or spherical shell

The heat flow at any radius r is given by:

$$Q = -k2\pi rl \frac{dT}{dr} \quad (9.20)$$

where l is the length of tube.

Integrating between the limits r_1 and r_2 :

$$Q \int_{r_1}^{r_2} \frac{dr}{r} = -2\pi lk \int_{T_1}^{T_2} dT$$

or:

$$Q = \frac{2\pi lk(T_1 - T_2)}{\ln(r_2/r_1)} \quad (9.21)$$

This equation may be put into the form of equation 9.12 to give:

$$Q = \frac{k(2\pi r_m l)(T_1 - T_2)}{r_2 - r_1} \quad (9.22)$$

where $r_m = (r_2 - r_1) / \ln(r_2/r_1)$, is known as the *logarithmic mean radius*. For thin-walled tubes the arithmetic mean radius r_a may be used, giving:

$$Q = \frac{k(2\pi r_a l)(T_1 - T_2)}{r_2 - r_1} \quad (9.23)$$

9.4. HEAT TRANSFER BY CONVECTION

9.4.1. Natural and forced convection

Heat transfer by convection occurs as a result of the movement of fluid on a macroscopic scale in the form of eddies or circulating currents. If the currents arise from the heat transfer process itself, *natural convection* occurs, such as in the heating of a vessel containing liquid by means of a heat source situated beneath it. The liquid at the bottom of the vessel becomes heated and expands and rises because its density has become less than that of the remaining liquid. Cold liquid of higher density takes its place and a circulating current is thus set up.

In *forced convection*, circulating currents are produced by an external agency such as an agitator in a reaction vessel or as a result of turbulent flow in a pipe. In general, the magnitude of the circulation in forced convection is greater, and higher rates of heat transfer are obtained than in natural convection.

In most cases where convective heat transfer is taking place from a surface to a fluid, the circulating currents die out in the immediate vicinity of the surface and a film of fluid, free of turbulence, covers the surface. In this film, heat transfer is by thermal conduction and, as the thermal conductivity of most fluids is low, the main resistance to transfer lies there. Thus an increase in the velocity of the fluid over the surface gives rise to improved heat transfer mainly because the thickness of the film is reduced. As a guide, the film coefficient increases as (fluid velocity)^{*n*}, where 0.6 < *n* < 0.8, depending upon the geometry.

If the resistance to transfer is regarded as lying within the film covering the surface, the rate of heat transfer *Q* is given by equation 9.11 as:

$$Q = kA \frac{(T_1 - T_2)}{x}$$

The effective thickness *x* is not generally known and therefore the equation is usually rewritten in the form:

$$Q = hA(T_1 - T_2) \quad (9.54)$$

where *h* is the heat transfer coefficient for the film and (1/*h*) is the thermal resistance.

9.4.2. Application of dimensional analysis to convection

So many factors influence the value of h that it is almost impossible to determine their individual effects by direct experimental methods. By arranging the variables in a series of dimensionless groups, however, the problem is made more manageable in that the number of groups is significantly less than the number of parameters. It is found that the heat transfer rate per unit area q is dependent on those physical properties which affect flow pattern (viscosity μ and density ρ), the thermal properties of the fluid (the specific heat capacity C_p and the thermal conductivity k) a linear dimension of the surface l , the velocity of flow u of the fluid over the surface, the temperature difference ΔT and a factor determining the natural circulation effect caused by the expansion of the fluid on heating (the product of the coefficient of cubical expansion β and the acceleration due to gravity g). Writing this as a functional relationship:

$$q = \phi[u, l, \rho, \mu, C_p, \Delta T, \beta g, k] \quad (9.55)$$

Noting the dimensions of the variables in terms of length **L**, mass **M**, time **T**, temperature θ , heat **H**:

q	Heat transferred/unit area and unit time	$\mathbf{HL}^{-2}\mathbf{T}^{-1}$
u	Velocity	\mathbf{LT}^{-1}
l	Linear dimension	\mathbf{L}
μ	Viscosity	$\mathbf{ML}^{-1}\mathbf{T}^{-1}$
ρ	Density	\mathbf{ML}^{-3}
k	Thermal conductivity	$\mathbf{HT}^{-1}\mathbf{L}^{-1}\theta^{-1}$
C_p	Specific heat capacity at constant pressure	$\mathbf{HM}^{-1}\theta^{-1}$
ΔT	Temperature difference	θ
(βg)	The product of the coefficient of thermal expansion and the acceleration due to gravity	$\mathbf{LT}^{-2}\theta^{-1}$

It may be noted that both temperature and heat are taken as fundamental units as heat is not expressed here in terms of **M**, **L**, **T**.

With nine parameters and five dimensions, equation 9.55 may be rearranged in four dimensionless groups.

Using the Π -theorem for solution of the equation, and taking as the recurring set: $l, \rho, \mu, \Delta T, k$

The non-recurring variables are: $q, u, (\beta g), C_p$

Then:

$$\begin{aligned}
 l &\equiv \mathbf{L} & \mathbf{L} &= l \\
 \rho &\equiv \mathbf{ML}^{-3} & \mathbf{M} &= \rho l^3 = \rho l^3 \\
 \mu &\equiv \mathbf{ML}^{-1}\mathbf{T}^{-1} & \mathbf{T} &= \mathbf{ML}^{-1}\mu^{-1} = \rho l^3 l^{-1} \mu^{-1} = \rho l^2 \mu^{-1} \\
 \Delta T &\equiv \theta & \theta &= \Delta T \\
 k &\equiv \mathbf{HL}^{-1}\mathbf{T}^{-1}\theta^{-1} & \mathbf{H} &= k l \mathbf{LT} \theta = k l \rho l^2 \mu^{-1} \Delta T = k l^3 \rho \mu^{-1} \Delta T
 \end{aligned}$$

The Π groups are then:

$$\Pi_1 = q\mathbf{H}^{-1}\mathbf{L}^2\mathbf{T} = qk^{-1}l^{-3}\rho^{-1}\mu\Delta T^{-1}l^2\rho l^2\mu^{-1} = qk^{-1}l\Delta T^{-1}$$

$$\Pi_2 = u\mathbf{L}^{-1}\mathbf{T} = ul^{-1}\rho l^2\mu^{-1} = u\rho l\mu^{-1}$$

$$\Pi_3 = C_p\mathbf{H}^{-1}\mathbf{M}\mathbf{\theta} = C_pk^{-1}l^{-3}\rho^{-1}\mu\Delta T^{-1}\rho l^3\Delta T = C_pk^{-1}\mu$$

$$\Pi_4 = \beta g\mathbf{L}^{-1}\mathbf{T}^2\mathbf{\theta} = \beta gl^{-1}\rho^2 l^4\mu^{-2}\Delta T = \beta g\Delta T\rho^2\mu^{-2}l^3$$

The relation in equation 9.55 becomes:

$$\frac{ql}{k\Delta T} = \frac{hl}{k} = \phi \left[\left(\frac{l\rho\mu}{\mu} \right) \left(\frac{C_p\mu}{k} \right) \left(\frac{\beta g\Delta T l^3 \rho^2}{\mu^2} \right) \right] \quad (9.56)$$

or:

$$Nu = \phi[Re, Pr, Gr]$$

This general equation involves the use of four dimensionless groups, although it may frequently be simplified for design purposes. In equation 9.56:

hl/k	is known as the <i>Nusselt</i> group Nu (already referred to in equation 9.46),
$l\rho\mu/\mu$	the <i>Reynolds</i> group Re ,
$C_p\mu/k$	the <i>Prandtl</i> group Pr , and
$\beta g\Delta T l^3 \rho^2/\mu^2$	the <i>Grashof</i> group Gr

It is convenient to define other dimensionless groups which are also used in the analysis of heat transfer. These are:

$l\rho C_p/k$	the <i>Peclet</i> group, $Pe = RePr$,
GC_p/kl	the <i>Graetz</i> group Gz , and
$h/C_p\rho u$	the <i>Stanton</i> group, $St = Nu/(RePr)$

It may be noted that many of these dimensionless groups are ratios. For example, the Nusselt group $h/(k/l)$ is the ratio of the actual heat transfer to that by conduction over a thickness l , whilst the Prandtl group, $(\mu/\rho)/(k/C_p\rho)$ is the ratio of the kinematic viscosity to the thermal diffusivity.

For conditions in which only natural convection occurs, the velocity is dependent on the buoyancy effects alone, represented by the Grashof number, and the Reynolds group may be omitted. Again, when forced convection occurs the effects of natural convection are usually negligible and the Grashof number may be omitted. Thus:

$$\text{for natural convection:} \quad Nu = f(Gr, Pr) \quad (9.57)$$

$$\text{and for forced convection:} \quad Nu = f(Re, Pr) \quad (9.58)$$

For most gases over a wide range of temperature and pressure, $C_p\mu/k$ is constant and the Prandtl group may often be omitted, simplifying the design equations for the calculation of film coefficients with gases.

9.5. HEAT TRANSFER BY RADIATION

9.5.1. Introduction

It has been seen that heat transfer by conduction takes place through either a solid or a stationary fluid and heat transfer by convection takes place as a result of either forced or natural movement of a hot fluid. The third mechanism of heat transfer, radiation, can take place without either a solid or a fluid being present, that is through a vacuum, although many fluids are transparent to radiation, and it is generally assumed that the emission of thermal radiation is by "waves" of wavelengths in the range $0.1-100\ \mu\text{m}$ which travel in straight lines. This means that direct radiation transfer, which is the result of an interchange between various radiating bodies or surfaces, will take place only if a straight line can be drawn between the two surfaces; a situation which is often expressed in terms of one surface "seeing" another. Having said this, it should be noted that opaque surfaces sometimes cast shadows which inhibit radiation exchange and that indirect transfer by radiation can take place as a result of partial reflection from other surfaces. Although all bodies at temperatures in excess of absolute zero radiate energy in all directions, radiation is of especial importance from bodies at high temperatures such as those encountered in furnaces, boilers and high temperature reactors, where in addition to radiation from hot surfaces, radiation from reacting flame gases may also be a consideration.

9.5.2. Radiation from a black body

In thermal radiation, a so-called *black body* absorbs all the radiation falling upon it, regardless of wavelength and direction, and, for a given temperature and wavelength, no surface can emit more energy than a black body. The radiation emitted by a black body, whilst a function of wavelength and temperature, is regarded as *diffuse*, that is, it is independent of direction. In general, most rough surfaces and indeed most engineering materials may be regarded as being diffuse. A black body, because it is a perfect emitter or absorber, provides a standard against which the radiation properties of real surfaces may be compared.

If the *emissive power* E of a radiation source—that is the energy emitted per unit area per unit time—is expressed in terms of the radiation of a single wavelength λ , then this is known as the *monochromatic* or *spectral emissive power* E_λ , defined as that rate at which radiation of a particular wavelength λ is emitted per unit surface area, per unit wavelength in all directions. For a black body at temperature T , the spectral emissive power of a wavelength λ is given by *Planck's Distribution Law*:

$$E_{\lambda,b} = C_1 / [\lambda^5 (\exp(C_2/\lambda T) - 1)] \quad (9.108)$$

where, in SI units, $E_{\lambda,b}$ is in W/m^3 and $C_1 = 3.742 \times 10^{-16} \text{ W/m}^2$ and $C_2 = 1.439 \times 10^{-2} \text{ mK}$ are the respective radiation constants. Equation 9.108 permits the evaluation of the emissive power from a black body for a given wavelength and absolute temperature and values obtained from the equation are plotted in Figure 9.33 which is based on the work of INCROPERA and DE WITT⁽⁴⁵⁾. It may be noted that, at a given wavelength, the radiation from a black body increases with temperature and that, in general, short wavelengths are associated with high temperature sources.

Example 9.12

What is the temperature of a surface coated with carbon black if the emissive power at a wavelength of $1.0 \times 10^{-6} \text{ m}$ is $1.0 \times 10^9 \text{ W/m}^3$? How would this be affected by a +2 per cent error in the emissive power measurement?

Solution

From equation 9.108 $\exp(C_2/\lambda T) = [C_1/E_{\lambda,b}\lambda^5] + 1]$

$$\text{or: } \exp(1.439 \times 10^{-2}/(1.0 \times 10^{-6}T)) = [3.742 \times 10^{-16}/(1 \times 10^{-9} \times (1.0 \times 10^{-6})^5)] \\ = 3.742 \times 10^5$$

$$\text{Thus: } (1.439 \times 10^4)/T = \ln(3.742 \times 10^5) = 12.83$$

$$\text{and: } T = (1.439 \times 10^4)/12.83 = \underline{\underline{1121 \text{ K}}}$$

With an error of +2 per cent, the correct value is given by:

$$E_{\lambda,b} = (100 - 2)(1 \times 10^9)/100 = 9.8 \times 10^8 \text{ W/m}^3$$

In equation 9.108:

$$9.8 \times 10^8 = (3.742 \times 10^{-16})/[(1 \times 10^{-6})^5]$$

$$(\exp(1.439 \times 10^{-2}/(1.0 \times 10^{-6}T)) - 1)]$$

and:

$$\underline{\underline{T = 1120 \text{ K}}}$$

Thus, the error in the calculated temperature of the surface is only 1 K.

The wavelength at which maximum emission takes place is related to the absolute temperature by *Wein's Displacement Law*, which states that the wavelength for maximum emission varies inversely with the absolute temperature of the source, or:

$$\lambda_{\max} T = \text{constant}, C_3 (= 2.898 \times 10^{-3} \text{ mK in SI units}) \quad (9.109)$$

Thus, combining equations 9.108 and 9.109:

$$E_{\lambda_{\max},b} = C_1 / [(C_3/T)^5 [\exp(C_2/C_3) - 1]]$$

$$\text{or: } E_{\lambda_{\max},b} = C_4 T^5 \quad (9.110)$$

where, in SI units, the fourth radiation constant, $C_4 = 12.86 \times 10^{-6} \text{ W/m}^3 \text{ K}^5$. Values of the maximum emissive power are shown by the broken line in Figure 9.33.

The *total emissive power* E is defined as the rate at which radiation energy is emitted per unit time per unit area of surface over all wavelengths and in all directions. This may be determined by making a summation of all the radiation at all wavelengths, that is by determining the area corresponding to a particular temperature under the Planck distribution curve, Figure 9.33. In this way, from equation 9.108, the total emissive power is given by:

$$E_b = \int_0^{\infty} C_1 d\lambda / [\lambda^5 (\exp(C_2/\lambda T) - 1)] \quad (9.111)$$

which is known as the *Stefan-Boltzmann Law*. This may be integrated for any constant value of T to give:

$$E_b = \sigma T^4 \quad (9.112)$$

where, in SI units, the Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$.

Example 9.13

Electrically-heated carbide elements, 10 mm in diameter and 0.5 m long, radiating essentially as black bodies, are to be used in the construction of a heater in which thermal radiation from the surroundings is negligible. If the surface temperature of the carbide is limited to 1750 K, how many elements are required to provide a radiated thermal output of 500 kW?

Solution

From equation 9.112, the total emissive power is given by:

$$E_b = \sigma T^4 = (5.67 \times 10^{-8} \times 1750^4) = 5.32 \times 10^5 \text{ W/m}^2$$

$$\text{The area of one element} = \pi(10/1000)0.5 = 1.571 \times 10^{-2} \text{ m}^2$$

$$\text{and: Power dissipated by one element} = (5.32 \times 10^5 \times 1.571 \times 10^{-2}) = 8.367 \times 10^3 \text{ W}$$

$$\text{Thus: Number of elements required} = (500 \times 1000)/(8.357 \times 10^3) = 59.8 \text{ say } \underline{\underline{60}}$$

9.5.3. Radiation from real surfaces

The *emissivity* of a material is defined as the ratio of the radiation per unit area emitted from a "real" or from a grey surface (one for which the emissivity is independent of wavelength) to that emitted by a black body at the same temperature. Emissivities of "real" materials are always less than unity and they depend on the type, condition and roughness of the material, and possibly on the wavelength and direction of the emitted radiation as well. For diffuse surfaces where emissivities are independent of direction, the emissivity, which represents an average over all directions, is known as the *hemispherical emissivity*. For a particular wavelength λ this is given by:

$$e_\lambda = E_\lambda / E_b \quad (9.113)$$

and, similarly, the total hemispherical emissivity, an average over all wavelengths, is given by:

$$e = E/E_b \quad (9.114)$$

Equation 9.114 leads to *Kirchoff's Law* which states that the absorptivity, or fraction of incident radiation absorbed, and the emissivity of a surface are equal. If two bodies **A** and **B** of areas A_1 and A_2 are in a large enclosure from which no energy is lost, then the energy absorbed by **A** from the enclosure is $A_1 a_1 I$ where I is the rate at which energy is falling on unit area of **A** and a_1 is the absorptivity. The energy given out by **A** is $E_1 A_1$ and, at equilibrium, these two quantities will be equal or:

$$I A_1 a_1 = A_1 E_1$$

and, for **B**: $I A_2 a_2 = A_2 E_2$

Thus: $E_1/a_1 = E_2/a_2 = E/a$ for any other body.

Since $E/a = E_b/a_b$, then, from equation 9.114:

$$e = E/E_b = a/a_b$$

and, as by definition, $a_b = 1$, the emissivity of any body is equal to its absorptivity, or:

$$e = a \quad (9.115)$$