

2.2. INTERNAL ENERGY

When a fluid flows from one location to another, energy will, in general, be converted from one form to another. The energy which is attributable to the physical state of the fluid is known as internal energy; it is arbitrarily taken as zero at some reference state, such as the absolute zero of temperature or the melting point of ice at atmospheric pressure. A change in the physical state of a fluid will, in general, cause an alteration in the internal energy. An elementary reversible change results from an infinitesimal change in

one of the intensive factors acting on the system; the change proceeds at an infinitesimal rate and a small change in the intensive factor in the opposite direction would have caused the process to take place in the reverse direction. Truly reversible changes never occur in practice but they provide a useful standard with which actual processes can be compared. In an irreversible process, changes are caused by a finite difference in the intensive factor and take place at a finite rate. In general the process will be accompanied by the conversion of electrical or mechanical energy into heat, or by the reduction of the temperature difference between different parts of the system.

For a stationary material the change in the internal energy is equal to the difference between the net amount of heat added to the system and the net amount of work done by the system on its surroundings. For an infinitesimal change:

$$dU = \delta q - \delta W \quad (2.1)$$

where dU is the small change in the internal energy, δq the small amount of heat added, and δW the net amount of work done on the surroundings.

In this expression consistent units must be used. In the SI system each of the terms in equation 2.1 is expressed in Joules per kilogram (J/kg). In other systems either heat units (e.g. cal/g) or mechanical energy units (e.g. erg/g) may be used. dU is a small change in the internal energy which is a property of the system; it is therefore a perfect differential. On the other hand, δq and δW are small quantities of heat and work; they are not properties of the system and their values depend on the manner in which the change is effected; they are, therefore, not perfect differentials. For a reversible process, however, both δq and δW can be expressed in terms of properties of the system. For convenience, reference will be made to systems of unit mass and the effects on the surroundings will be disregarded.

A property called entropy is defined by the relation:

$$dS = \frac{\delta q}{T} \quad (2.2)$$

where dS is the small change in entropy resulting from the addition of a small quantity of heat δq , at a temperature T , under reversible conditions. From the definition of the thermodynamic scale of temperature, $\oint \delta q/T = 0$ for a reversible cyclic process, and the net change in the entropy is also zero. Thus, for a particular condition of the system, the entropy has a definite value and must be a property of the system; dS is, therefore, a perfect differential.

For an irreversible process:

$$\frac{\delta q}{T} < dS = \frac{\delta q}{T} + \frac{\delta F}{T} \quad (\text{say}) \quad (2.3)$$

δF is then a measure of the degree of irreversibility of the process. It represents the amount of mechanical energy converted into heat or the conversion of heat energy at one temperature to heat energy at another temperature. For a finite process:

$$\int_{S_1}^{S_2} T dS = \Sigma \delta q + \Sigma \delta F = q + F \quad (\text{say}) \quad (2.4)$$

When a process is isentropic, $q = -F$; a reversible process is isentropic when $q = 0$, that is a reversible adiabatic process is isentropic.

The increase in the entropy of an irreversible process may be illustrated in the following manner. Considering the spontaneous transfer of a quantity of heat δq from one part of a system at a temperature T_1 to another part at a temperature T_2 , then the net change in the entropy of the system as a whole is then:

$$dS = \frac{\delta q}{T_2} - \frac{\delta q}{T_1}$$

T_1 must be greater than T_2 and dS is therefore positive. If the process had been carried out reversibly, there would have been an infinitesimal difference between T_1 and T_2 and the change in entropy would have been zero.

The change in the internal energy may be expressed in terms of properties of the system itself. For a reversible process:

$$\delta q = T dS \quad (\text{from equation 2.2}) \quad \text{and} \quad \delta W = P dv$$

if the only work done is that resulting from a change in volume, dv .

Thus, from equation 2.1:

$$dU = T dS - P dv \quad (2.5)$$

Since this relation is in terms of properties of the system, it must also apply to a system in motion and to irreversible changes where the only work done is the result of change of volume.

Thus, in an irreversible process, for a stationary system:

from equations 2.1 and 2.2: $dU = \delta q - \delta W = T dS - P dv$

and from equation 2.3:

$$\delta q + \delta F = T dS$$

\therefore

$$\delta W = P dv - \delta F \quad (2.6)$$

that is, the useful work performed by the system is less than $P dv$ by an amount δF , which represents the amount of mechanical energy converted into heat energy.

The relation between the internal energy and the temperature of a fluid will now be considered. In a system consisting of unit mass of material and where the only work done is that resulting from volume change, the change in internal energy after a reversible change is given by:

$$dU = \delta q - P dv \quad (\text{from equation 2.1})$$

If there is no volume change:

$$dU = \delta q = C_v dT \quad (2.7)$$

where C_v is the specific heat at constant volume.

As this relation is in terms of properties of the system, it must be applicable to all changes at constant volume.

In an irreversible process:

$$\begin{aligned} dU &= \delta q - (P dv - \delta F) \quad (\text{from equations 2.1 and 2.6}) \quad (2.8) \\ &= \delta q + \delta F \quad (\text{under conditions of constant volume}) \end{aligned}$$

This quantity δF thus represents the mechanical energy which has been converted into heat and which is therefore available for increasing the temperature.

Thus:
$$\delta q + \delta F = C_v dT = dU. \quad (2.9)$$

For changes that take place under conditions of constant pressure, it is more satisfactory to consider variations in the enthalpy H . The enthalpy is defined by the relation:

$$H = U + Pv. \quad (2.10)$$

Thus:
$$\begin{aligned} dH &= dU + P dv + v dP \\ &= \delta q - P dv + \delta F + P dv + v dP \quad (\text{from equation 2.8}) \end{aligned}$$

for an irreversible process: (For a reversible process $\delta F = 0$)

$$\begin{aligned} \therefore dH &= \delta q + \delta F + v dP \\ &= \delta q + \delta F \quad (\text{at constant pressure}) \end{aligned} \quad (2.11)$$

$$= C_p dT \quad (2.12)$$

where C_p is the specific heat at constant pressure.

No assumptions have been made concerning the properties of the system and, therefore, the following relations apply to all fluids.

From equation 2.7:
$$\left(\frac{\partial U}{\partial T}\right)_v = C_v \quad (2.13)$$

From equation 2.12:
$$\left(\frac{\partial H}{\partial T}\right)_p = C_p \quad (2.14)$$

2.3. TYPES OF FLUID

Fluids may be classified in two different ways; either according to their behaviour under the action of externally applied pressure, or according to the effects produced by the action of a shear stress.

If the volume of an element of fluid is independent of its pressure and temperature, the fluid is said to be incompressible; if its volume changes it is said to be compressible. No real fluid is completely incompressible though liquids may generally be regarded as such when their flow is considered. Gases have a very much higher compressibility than liquids, and appreciable changes in volume may occur if the pressure or temperature is altered. However, if the percentage change in the pressure or in the absolute temperature is small, for practical purposes a gas may also be regarded as incompressible. Thus, in practice, volume changes are likely to be important only when the pressure or temperature of a gas changes by a large proportion. The relation between pressure, temperature, and volume of a real gas is generally complex though, except at very high pressures the behaviour of gases approximates to that of the ideal gas for which the volume of a given mass is inversely proportional to the pressure and directly proportional to the absolute temperature. At high pressures and when pressure changes are large, however, there may be appreciable deviations from this law and an approximate equation of state must then be used.

The behaviour of a fluid under the action of a shear stress is important in that it determines the way in which it will flow. The most important physical property affecting the stress distribution within the fluid is its viscosity. For a gas, the viscosity is low and even at high rates of shear, the viscous stresses are small. Under such conditions the gas approximates in its behaviour to an *inviscid* fluid. In many problems involving the flow of a gas or a liquid, the viscous stresses are important and give rise to appreciable velocity gradients within the fluid, and dissipation of energy occurs as a result of the frictional forces set up. In gases and in most pure liquids the ratio of the shear stress to the rate of shear is constant and equal to the viscosity of the fluid. These fluids are said to be *Newtonian* in their behaviour. However, in some liquids, particularly those containing a second phase in suspension, the ratio is not constant and the apparent viscosity of the fluid is a function of the rate of shear. The fluid is then said to be *non-Newtonian* and to exhibit rheological properties. The importance of the viscosity of the fluid in determining

2.3.1. The incompressible fluid (liquid)

By definition, v is independent of P , so that $(\partial v/\partial P)_T = 0$. The internal energy will be a function of temperature but not a function of pressure.

2.3.2. The ideal gas

An *ideal gas* is defined as a gas whose properties obey the law:

$$PV = nRT \quad (2.15)$$

where V is the volume occupied by n molar units of the gas, R the universal gas constant, and T the absolute temperature. Here n is expressed in kmol when using the SI system.

This law is closely obeyed by real gases under conditions where the actual volume of the molecules is small compared with the total volume, and where the molecules exert only a very small attractive force on one another. These conditions are met at very low pressures when the distance apart of the individual molecules is large. The value of R is then the same for all gases and in SI units has the value of 8314 J/kmol K.

When the only external force on a gas is the fluid pressure, the equation of state is:

$$f(P, V, T, n) = 0$$

Any property may be expressed in terms of any three other properties. Considering the dependence of the internal energy on temperature and volume, then:

$$U = f(T, V, n)$$

For unit mass of gas:

$$U = f(T, v)$$

and:

$$Pv = \frac{\mathbf{RT}}{M} \quad (2.16)$$

where M is the molecular weight of the gas and v is the volume per unit mass.

Thus:
$$dU = \left(\frac{\partial U}{\partial T}\right)_v dT + \left(\frac{\partial U}{\partial v}\right)_T dv \quad (2.17)$$

and:
$$T dS = dU + P dv \quad (\text{from equation 2.5})$$

∴
$$T dS = \left(\frac{\partial U}{\partial T}\right)_v dT + \left[P + \left(\frac{\partial U}{\partial v}\right)_T\right] dv$$

and:
$$dS = \left(\frac{\partial U}{\partial T}\right)_v \frac{dT}{T} + \frac{1}{T} \left[P + \left(\frac{\partial U}{\partial v}\right)_T\right] dv \quad (2.18)$$

Thus:
$$\left(\frac{\partial S}{\partial T}\right)_v = \frac{1}{T} \left(\frac{\partial U}{\partial T}\right)_v \quad (2.19)$$

and:
$$\left(\frac{\partial S}{\partial v}\right)_T = \frac{1}{T} \left[P + \left(\frac{\partial U}{\partial v}\right)_T\right] \quad (2.20)$$

Then differentiating equation 2.19 by v and equation 2.20 by T and equating:

$$\frac{1}{T} \frac{\partial^2 U}{\partial T \partial v} = \frac{1}{T} \left[\left(\frac{\partial P}{\partial T}\right)_v + \frac{\partial^2 U}{\partial v \partial T} \right] - \frac{1}{T^2} \left[P + \left(\frac{\partial U}{\partial v}\right)_T \right]$$

or
$$\left(\frac{\partial U}{\partial v}\right)_T = T \left(\frac{\partial P}{\partial T}\right)_v - P \quad (2.21)$$

This relation applies to any fluid. For the particular case of an ideal gas, since $Pv = \mathbf{RT}/M$ (equation 2.16):

$$T \left(\frac{\partial P}{\partial T}\right)_v = T \frac{\mathbf{R}}{Mv} = P$$

so that:
$$\left(\frac{\partial U}{\partial v}\right)_T = 0 \quad (2.22)$$

and:
$$\left(\frac{\partial U}{\partial P}\right)_T = \left(\frac{\partial U}{\partial v}\right)_T \left(\frac{\partial v}{\partial P}\right)_T = 0 \quad (2.23)$$

Thus the internal energy of an ideal gas is a function of temperature only. The variation of internal energy and enthalpy with temperature will now be calculated.

$$dU = \left(\frac{\partial U}{\partial T}\right)_v dT + \left(\frac{\partial U}{\partial v}\right)_T dv \quad (\text{equation 2.17})$$

$$= C_v dT \quad (\text{from equations 2.13 and 2.22}) \quad (2.24)$$

Thus for an ideal gas under all conditions:

$$\frac{dU}{dT} = C_v \quad (2.25)$$

In general, this relation applies only to changes at constant volume. For the particular case of the ideal gas, however, it applies under all circumstances.

Again, since $H = f(T, P)$:

$$\begin{aligned} dH &= \left(\frac{\partial H}{\partial T}\right)_P dT + \left(\frac{\partial H}{\partial P}\right)_T dP \\ &= C_p dT + \left(\frac{\partial U}{\partial P}\right)_T dP + \left(\frac{\partial(Pv)}{\partial P}\right)_T dP \quad (\text{from equations 2.12 and 2.10}) \\ &= C_p dT \end{aligned}$$

since $(\partial U/\partial P)_T = 0$ and $[\partial(Pv)/\partial P]_T = 0$ for an ideal gas.

Thus under all conditions for an ideal gas:

$$\frac{dH}{dT} = C_p \quad (2.26)$$

$$C_p - C_v = \frac{dH}{dT} - \frac{dU}{dT} = \frac{d(Pv)}{dT} = \frac{\mathbf{R}}{M} \quad (2.27)$$

2.3.3. The non-ideal gas

For a non-ideal gas, equation 2.15 is modified by including a compressibility factor Z which is a function of both temperature and pressure:

$$PV = Zn\mathbf{R}T \quad (2.31)$$

At very low pressures, deviations from the ideal gas law are caused mainly by the attractive forces between the molecules and the compressibility factor has a value less than unity. At higher pressures, deviations are caused mainly by the fact that the volume of the molecules themselves, which can be regarded as incompressible, becomes significant compared with the total volume of the gas.

Many equations have been given to denote the approximate relation between the properties of a non-ideal gas. Of these the simplest, and probably the most commonly used,

is van der Waals' equation:

$$\left(P + a\frac{n^2}{V^2}\right)(V - nb) = nRT \quad (2.32)$$

where b is a quantity which is a function of the incompressible volume of the molecules themselves, and a/V^2 is a function of the attractive forces between the molecules. Values of a and b can be expressed in terms of the critical pressure P_c and the critical temperature T_c as $a = \frac{27R^2T_c^2}{64P_c}$ and $b = \frac{RT_c}{8P_c}$. It is seen that as P approaches zero and V approaches infinity, this equation reduces to the equation of state for the ideal gas.

A chart which correlates experimental $P - V - T$ data for all gases is included as Figure 2.1 and this is known as the generalised compressibility-factor chart.⁽¹⁾ Use is made of *reduced* coordinates where the *reduced temperature* T_R , the *reduced pressure* P_R , and the *reduced volume* V_R are defined as the ratio of the actual temperature, pressure, and volume of the gas to the corresponding values of these properties at the critical state. It is found that, at a given value of T_R and P_R , nearly all gases have the same molar volume, compressibility factor, and other thermodynamic properties. This empirical relationship applies to within about 2 per cent for most gases; the most important exception to the rule is ammonia.

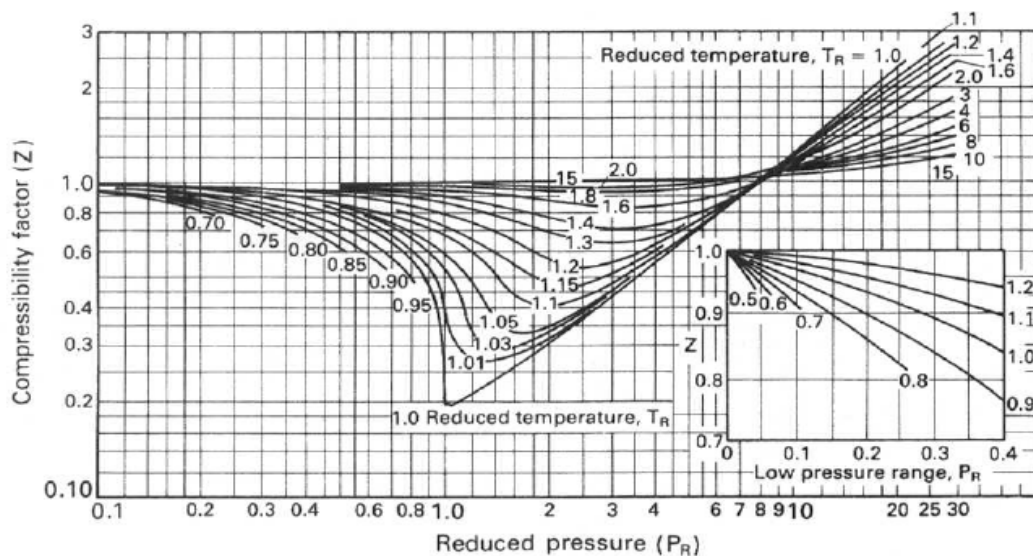


Figure 2.1. Compressibility factors of gases and vapours

Example 2.1

It is required to store 1 kmol of methane at 320 K and 60 MN/m^2 . Using the following methods, estimate the volume of the vessel which must be provided:

- (a) ideal gas law;
- (b) van der Waals' equation;
- (c) generalised compressibility-factor chart;

Solution

For 1 kmol of methane,

(a) $PV = 1 \times RT$, where $R = 8314 \text{ J/kmol K}$.

In this case:

$$P = 60 \times 10^6 \text{ N/m}^2; T = 320 \text{ K}$$

$$\therefore V = 8314 \times \frac{320}{(60 \times 10^6)} = \underline{\underline{0.0443 \text{ m}^3}}$$

(b) In van der Waals' equation (2.32), the constants may be taken as:

$$a = \frac{27R^2T_c^2}{64P_c}; \quad b = \frac{RT_c}{8P_c}$$

where the critical temperature $T_c = 191 \text{ K}$ and the critical pressure $P_c = 4.64 \times 10^6 \text{ N/m}^2$ for methane as shown in the Appendix tables.

$$\therefore a = \frac{27 \times 8314^2 \times 191^2}{(64 \times 4.64 \times 10^6)} = 229,300 \text{ (N/m}^2\text{)(m}^3\text{)}^2\text{/kmol}^2$$

and:
$$b = 8314 \times \frac{191}{(8 \times 4.64 \times 10^6)} = 0.0427 \text{ m}^3\text{/kmol}$$

Thus in equation 2.32:

$$\left(60 \times 10^6 + 229,300 \times \frac{1}{V^2}\right) \left(V - (1 \times 0.0427)\right) = 1 \times 8314 \times 320$$

or:
$$V^3 - 0.0427V^2 + 0.000382 = 0.0445$$

Solving by trial and error:

$$V = \underline{\underline{0.066 \text{ m}^3}}$$

(c)
$$T_r = \frac{T}{T_c} = \frac{320}{191} = 1.68$$

$$P_r = \frac{P}{P_c} = \frac{60 \times 10^3}{4640} = 12.93$$

and:

$$V = \frac{ZnRT}{P} \quad (\text{from equation 2.31})$$

$$= \frac{1.33 \times 1.0 \times 8314 \times 320}{(60 \times 10^6)} = \underline{\underline{0.0589 \text{ m}^3}}$$

2.4. THE FLUID IN MOTION

When a fluid flows through a duct or over a surface, the velocity over a plane at right angles to the stream is not normally uniform. The variation of velocity can be shown by the use of streamlines which are lines so drawn that the velocity vector is always tangential to them. The flowrate between any two streamlines is always the same. Constant velocity over a cross-section is shown by equidistant streamlines and an increase in velocity by closer spacing of the streamlines. There are two principal types of flow which are discussed in detail later, namely streamline and turbulent flow. In streamline flow, movement across streamlines occurs solely as the result of diffusion on a molecular scale and the flowrate is steady. In turbulent flow the presence of circulating current results in transference of fluid on a larger scale, and cyclic fluctuations occur in the flowrate, though the time-average rate remains constant.

A group of streamlines can be taken together to form a streamtube, and thus the whole area for flow can be regarded as being composed of bundles of streamtubes.

Figures 2.3, 2.4, and 2.5 show the flow patterns in a straight tube, through a constriction and past an immersed object. In the first case, the streamlines are all parallel to one another, whereas in the other two cases the streamlines approach one another as the passage becomes constricted, indicating that the velocity is increasing.

2.4.1. Continuity

Considering the flow of a fluid through a streamtube, as shown in Figure 2.6, then equating the mass rates of flow at sections 1 and 2:

$$G = \rho_1 \dot{u}_1 dA_1 = \rho_2 \dot{u}_2 dA_2 \quad (2.35)$$

where ρ_1, ρ_2 are the densities, \dot{u}_1, \dot{u}_2 the velocities in the streamtube; and dA_1, dA_2 the flow areas at sections 1 and 2 respectively.

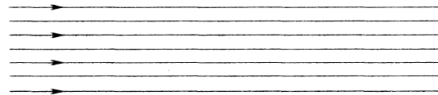


Figure 2.3. Streamlines in a straight tube

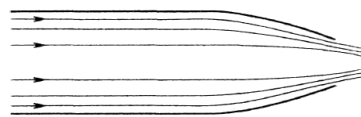


Figure 2.4. Streamlines in a constriction

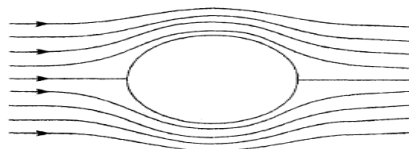


Figure 2.5. Streamlines for flow past an immersed object

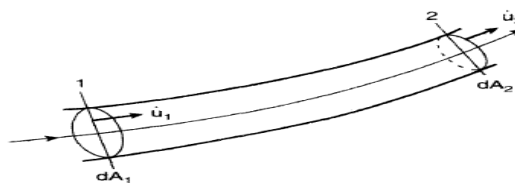


Figure 2.6. Flow through a streamtube

On integration:

$$G = \int \rho_1 \dot{u}_1 dA_1 = \int \rho_2 \dot{u}_2 dA_2 = \rho_1 u_1 A_1 = \rho_2 u_2 A_2 \quad (2.36)$$

where u_1, u_2 are the average velocities (defined by the previous equations) at the two sections. In many problems, the mass flowrate per unit area G' is the important quantity.

$$G' = \frac{G}{A} = \rho u \quad (2.37)$$

For an incompressible fluid, such as a liquid or a gas where the pressure changes are small:

$$u_1 A_1 = u_2 A_2 \quad (2.38)$$

2.4.2. Momentum changes in a fluid

As a fluid flows through a duct its momentum and pressure may change. The magnitude of the changes can be considered by applying the momentum equation (force equals rate of change of momentum) to the fluid in a streamtube and then integrating over the cross-section of the duct. The effect of frictional forces will be neglected at first and the relations thus obtained will strictly apply only to an inviscid (frictionless) fluid. Considering an element of length dl of a streamtube of cross-sectional area dA , increasing to $dA + (d(dA)/dl)dl$, as shown in Figure 2.7, then the upstream pressure = P and force attributable to upstream pressure = $P dA$.

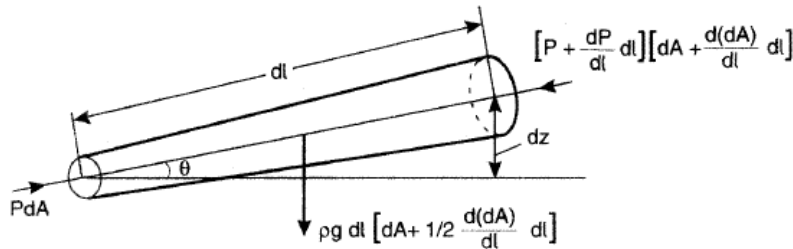


Figure 2.7. Forces on fluid in a streamtube

The downstream pressure = $P + \left(\frac{dP}{dl}\right) dl$

This pressure acts over an area $dA + (d(dA)/dl)dl$ and gives rise to a total force of $-[P + (dP/dl)dl]\{dA + [d(dA)/dl]dl\}$.

In addition, the mean pressure of $P + \frac{1}{2}(dP/dl)dl$ acting on the sides of the streamtube will give rise to a force having a component $[P + \frac{1}{2}(dP/dl)dl][d(dA)/dl]dl$ along the

streamtube. Thus, the net force along the streamtube due to the pressure gradient is:

$$P dA - \left(P + \frac{dP}{dl} dl \right) \left(dA + \frac{d(dA)}{dl} dl \right) + \left(P + \frac{1}{2} \frac{dP}{dl} dl \right) \frac{d(dA)}{dl} dl \approx -\frac{dP}{dl} dl dA$$

The other force acting is the weight of the fluid

$$= \rho g dl \left(dA + \frac{1}{2} \frac{d(dA)}{dl} dl \right)$$

The component of this force along the streamtube is

$$-\rho g dl \left(dA + \frac{1}{2} \frac{d(dA)}{dl} dl \right) \sin \theta$$

Neglecting second order terms and noting that $\sin \theta = dz/dl$:

$$\text{Total force on fluid} = -\frac{dP}{dl} dl dA - \frac{dz}{dl} \rho g dl dA \quad (2.39)$$

The rate of change of momentum of the fluid along the streamtube

$$\begin{aligned} &= (\rho \dot{u} dA) \left[\left(\dot{u} + \frac{d\dot{u}}{dl} dl \right) - \dot{u} \right] \\ &= \rho \dot{u} \frac{d\dot{u}}{dl} dl dA \end{aligned} \quad (2.40)$$

Equating equations 2.39 and 2.40:

$$\begin{aligned} \rho \dot{u} dl dA \frac{d\dot{u}}{dl} &= -dl dA \frac{dP}{dl} - \rho g dl dA \frac{dz}{dl} \\ \dot{u} d\dot{u} + \frac{dP}{\rho} + g dz &= 0 \end{aligned} \quad (2.41)$$

On integration:

$$\frac{\dot{u}^2}{2} + \int \frac{dP}{\rho} + gz = \text{constant} \quad (2.42)$$

For the simple case of the incompressible fluid, ρ is independent of pressure, and:

$$\frac{\dot{u}^2}{2} + \frac{P}{\rho} + gz = \text{constant} \quad (2.43)$$

Equation 2.43 is known as Bernoulli's equation, which relates the pressure at a point in the fluid to its position and velocity. Each term in equation 2.43 represents energy per unit mass of fluid. Thus, if all the fluid is moving with a velocity u , the total energy per unit mass ψ is given by:

$$\psi = \frac{u^2}{2} + \frac{P}{\rho} + gz \quad (2.44)$$

Dividing equation 2.44 by g :

$$\frac{u^2}{2g} + \frac{P}{\rho g} + z = \text{constant} \quad (2.45)$$

In equation 2.45 each term represents energy per unit weight of fluid and has the dimensions of length and can be regarded as representing a contribution to the total fluid head.

Thus: $\frac{u^2}{2g}$ is the velocity head

$\frac{P}{\rho g}$ is the pressure head

and: z is the potential head

Equation 2.42 can also be obtained from consideration of the energy changes in the fluid.

Example 2.3

Water leaves the 25 mm diameter nozzle of a fire hose at a velocity of 25 m/s. What will be the reaction force at the nozzle which the fireman will need to counterbalance?

Solution

$$\begin{aligned} \text{Mass rate of discharge of water, } G &= \rho u A \\ &= 1000 \times 25 \times \frac{\pi}{4} (0.025)^2 \\ &= 12.27 \text{ kg/s} \end{aligned}$$

$$\begin{aligned} \text{Momentum of fluid per second} &= Gu \\ &= 12.27 \times 25 \\ &= 307 \text{ N} \end{aligned}$$

$$\text{Reaction force} = \text{Rate of change of momentum} = \underline{\underline{307 \text{ N}}}$$

Example 2.4

Water is flowing at 5 m/s in a 50 mm diameter pipe which incorporates a 90° bend, as shown in Figure 2.8. What is the additional force to which a retaining bracket will be subjected, as a result of the momentum changes in the liquid, if it is arranged symmetrically in the pipe bend?

Solution

Momentum per second of approaching liquid in Y -direction

$$\begin{aligned} &= \rho u^2 A \\ &= 1000 \times 25 \times \frac{\pi}{4} (0.050)^2 \\ &= 49.1 \text{ N} \end{aligned}$$

The pipe bracket must therefore exert a reaction force of -49.1 N in the Y -direction, that is in the direction in which the fluid is accelerating. Similarly, the force in the X -direction = 49.1 N

$$\begin{aligned} \text{The resultant force in direction of arm of bracket} &= 49.1 \cos 45^\circ + 49.1 \sin 45^\circ \\ &= 49.1 \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \\ &= \underline{\underline{69.4 \text{ N}}} \end{aligned}$$

2.4.5. Constant flow per unit area

When the flow rate of the fluid per unit area G' is constant, equation 2.37 can be written:

$$\frac{G}{A} = G' = \frac{u_1}{v_1} = \frac{u_2}{v_2} = \frac{u}{v} \quad (2.62)$$

or:
$$G' = u_1 \rho_1 = u_2 \rho_2 = u \rho \quad (2.63)$$

Equation 2.58 is the momentum balance for horizontal turbulent flow:

$$u \, du + v \, dP = 0 \quad (\text{equation 2.58})$$

or:
$$u \frac{du}{v} + dP = 0$$

Because u/v is constant, on integration this gives:

$$\frac{u_1(u_2 - u_1)}{v_1} + P_2 - P_1 = 0$$

or:
$$\frac{u_1^2}{v_1} + P_1 = \frac{u_2^2}{v_2} + P_2 \quad (2.64)$$

2.5. PRESSURE–VOLUME RELATIONSHIPS

2.5.1. Incompressible fluids

For incompressible fluids v is independent of pressure so that

$$\int_{P_1}^{P_2} v \, dP = (P_2 - P_1)v \quad (2.65)$$

Therefore equation 2.55 becomes:

$$\frac{u_1^2}{2\alpha_1} + gz_1 + P_1v = \frac{u_2^2}{2\alpha_2} + gz_2 + P_2v + W_s + F \quad (2.66)$$

or:
$$\Delta \frac{u^2}{2\alpha} + g\Delta z + v\Delta P + W_s + F = 0 \quad (2.67)$$

In a frictionless system in which the fluid does not work on the surroundings and α_1 and α_2 are taken as unity (turbulent flow), then:

$$\frac{u_1^2}{2} + gz_1 + P_1v = \frac{u_2^2}{2} + gz_2 + P_2v \quad (2.68)$$

Example 2.5

Water flows from a tap at a pressure of 250 kN/m² above atmospheric. What is the velocity of the jet if frictional effects are neglected?

Solution

From equation 2.68:

$$0.5(u_2^2 - u_1^2) = g(z_1 - z_2) + \frac{(P_1 - P_2)}{\rho}$$

Using suffix 1 to denote conditions in the pipe and suffix 2 to denote conditions in the jet and neglecting the velocity of approach in the pipe:

$$0.5(u_2^2 - 0) = 9.81 \times 0 + \frac{250 \times 10^3}{1000}$$

$$\underline{\underline{u_2 = 22.4 \text{ m/s}}}$$

2.5.2. Compressible fluids

For a gas, the mean value of the specific volume can be used unless the pressure or temperature change is very large, when the relation between pressure and volume must be taken into account.

The term $\int_{P_1}^{P_2} v dP$ will now be evaluated for the ideal gas under various conditions. In most cases the results so obtained may be applied to the non-ideal gas without introducing an error greater than is involved in estimating the other factors concerned in the process. The only common exception to this occurs in the flow of gases at very high pressures and for the flow of steam, when it is necessary to employ one of the approximate equations for the state of a non-ideal gas, in place of the equation for the ideal gas. Alternatively, equation 2.56 may be used and work expressed in terms of changes in enthalpy. For a gas, the potential energy term is usually small compared with the other energy terms.

The relation between the pressure and the volume of an ideal gas depends on the rate of transfer of heat to the surroundings and the degree of irreversibility of the process. The following conditions will be considered.

- (a) an isothermal process;
- (b) an isentropic process;
- (c) a reversible process which is neither isothermal nor adiabatic;
- (d) an irreversible process which is not isothermal.

Isothermal process

For an isothermal process, $Pv = \mathbf{RT}/M = P_1v_1$, where the subscript 1 denotes the initial values and M is the molecular weight.

$$\text{Thus} \quad \int_{P_1}^{P_2} v dP = P_1v_1 \int_{P_1}^{P_2} \frac{1}{P} dP = P_1v_1 \ln \frac{P_2}{P_1} \quad (2.69)$$

Isentropic process

From equation 2.30, for an isentropic process:

$$\begin{aligned} Pv^\gamma &= P_1v_1^\gamma = \text{constant} \\ \int_{P_1}^{P_2} v dP &= \int_{P_1}^{P_2} \left(\frac{P_1v_1^\gamma}{P} \right)^{1/\gamma} dP \\ &= P_1^{1/\gamma} v_1 \int_{P_1}^{P_2} P^{-1/\gamma} dP \\ &= P_1^{1/\gamma} v_1 \frac{1}{1 - (1/\gamma)} (P_2^{1-(1/\gamma)} - P_1^{1-(1/\gamma)}) \\ &= \frac{\gamma}{\gamma - 1} P_1 v_1 \left[\left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right] \end{aligned} \quad (2.70)$$

$$\begin{aligned} &= \frac{\gamma}{\gamma - 1} \left[P_1 \left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} \left(\frac{P_2}{P_1} \right)^{1/\gamma} v_2 - P_1 v_1 \right] \\ &= \frac{\gamma}{\gamma - 1} (P_2 v_2 - P_1 v_1) \end{aligned} \quad (2.71)$$

Further, from equations 2.29 and 2.26, taking C_p as constant:

$$\int_{P_1}^{P_2} v \, dP = \int_{H_1}^{H_2} dH = C_p \Delta T \quad (2.72)$$

The above relations apply for an ideal gas to a reversible adiabatic process which, as already shown, is isentropic.