The most important parameters measured to provide information on the operating conditions in a plant are flowrates, pressures, and temperatures. The instruments used may give either an instantaneous reading or, in the case of flow, may be arranged to give a cumulative flow over any given period. In either case, the instrument may be required to give a signal to some control unit which will then govern one or more parameters on the plant. It should be noted that on industrial plants it is usually more important to have information on the change in the value of a given parameter than to use meters that give particular absolute accuracy. To maintain the value of a parameter at a desired value a control loop is used.

A simple control system, or *loop*, is illustrated in Figure 6.1. The temperature  $T_0$  of the water at Y is measured by means of a thermocouple, the output of which is fed to a controller mechanism. The latter can be divided into two sections (normally housed in the same unit). In the first (the *comparator*), the *measured value*  $(T_0)$  is compared with the desired value  $(T_d)$  to produce an *error* (e), where:

$$e = T_d - T_0 \tag{6.1}$$

The second section of the mechanism (the *controller*) produces an output which is a function of the magnitude of e. This is fed to a control valve in the steam line, so that the valve closes when  $T_0$  increases and vice versa. The system as shown may be used to counteract fluctuations in temperature due to extraneous causes such as variations in water flowrate or upstream temperature — termed *load* changes. It may also be employed to change the water temperature at Y to a new value by adjustment of the desired value.

It is very important to note that in this loop system the parameter  $T_0$ , which must be kept constant, is measured, though all subsequent action is concerned with the magnitude of the error and not with the actual value of  $T_0$ . This simple loop will frequently be complicated by there being several parameters to control, which may necessitate considerable instrumental analysis and the control action will involve operation of several control valves.

This represents a simple form of control for a single variable, though in a modern plant many parameters are controlled at the same time from various measuring instruments, and the variables on a plant such as a distillation unit are frequently linked together, thus increasing the complexity of control that is required.

On industrial plants, the instruments are therefore required not only to act as indicators but also to provide some link which can be used to help in the control of the plant. In this chapter, pressure measurement is briefly described and methods of measurement of flowrate are largely confined to those which depend on the application of the energy

200

#### FLOW AND PRESSURE MEASUREMENT

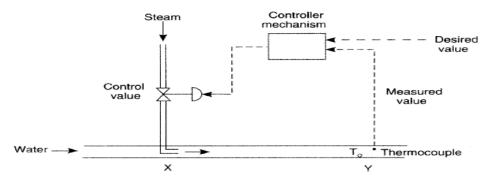


Figure 6.1. Simple feedback control system

233

## 6.2. FLUID PRESSURE

In a stationary fluid the pressure is exerted equally in all directions and is referred to as the *static pressure*. In a moving fluid, the static pressure is exerted on any plane parallel to the direction of motion. The pressure exerted on a plane at right angles to the direction of flow is greater than the static pressure because the surface has, in addition, to exert sufficient force to bring the fluid to rest. This additional pressure is proportional to the kinetic energy of the fluid; it cannot be measured independently of the static pressure.

# 6.2.1. Static pressure

The energy balance equation can be applied between any two sections in a continuous fluid. If the fluid is not moving, the kinetic energy and the frictional loss are both zero, and therefore:

$$v dP + g dz = 0$$
 (from equation 2.57)

For an incompressible fluid:

or:

$$v(P_2 - P_1) + g(z_2 - z_1) = 0$$
  

$$(P_2 - P_1) = -\rho g(z_2 - z_1)$$
(6.2)

Thus the pressure difference can be expressed in terms of the height of a vertical column of fluid.

If the fluid is compressible and behaves as an ideal gas, for isothermal conditions:

$$P_1 v_1 \ln \frac{P_2}{P_1} + g(z_2 - z_1) = 0 \quad \text{(from equation 2.69)}$$

$$\frac{P_2}{P_1} = \exp \frac{-gM}{RT} (z_2 - z_1) \quad \text{(from equation 2.16)}$$
(6.3)

This expression enables the pressure distribution within an ideal gas to be calculated for isothermal conditions.

When the static pressure in a moving fluid is to be determined, the measuring surface must be parallel to the direction of flow so that no kinetic energy is converted into pressure energy at the surface. If the fluid is flowing in a circular pipe the measuring surface must be perpendicular to the radial direction at any point. The pressure connection, which is known as a *piezometer tube*, should terminate flush with the wall of the pipe so that the flow is not disturbed: the pressure is then measured near the walls where the velocity is a minimum and the reading would be subject only to a small error if the surface were not quite parallel to the direction of flow. A piezometer tube of narrow diameter is used for accurate measurements.

The static pressure should always be measured at a distance of not less than 50 diameters from bends or other obstructions, so that the flow lines are almost parallel to the walls of the tube. If there are likely to be large cross-currents or eddies, a piezometer ring should be used. This consists of four pressure tappings equally spaced at 90° intervals round the circumference of the tube; they are joined by a circular tube which is connected to the pressure measuring device. By this means, false readings due to irregular flow are avoided. If the pressure on one side of the tube is relatively high, the pressure on the opposite side is generally correspondingly low; with the piezometer ring a mean value is obtained. The cross-section of the piezometer tubes and ring should be small to prevent any appreciable circulation of the fluid.

## 6.2.2. Pressure measuring devices

(a) The simple manometer, shown in Figure 6.2a, consists of a transparent U-tube containing the fluid **A** of density  $\rho$  whose pressure is to be measured and an immiscible fluid **B** of higher density  $\rho_m$ . The limbs are connected to the two points between which the pressure difference  $(P_2 - P_1)$  is required; the connecting leads should be completely full of fluid **A**. If  $P_2$  is greater than  $P_1$ , the interface between the two liquids in limb 2 will be depressed a distance  $h_m$  (say) below that in limb 1. The pressure at the level a - a must be the same in each of the limbs and, therefore:

$$P_{2} + z_{m}\rho g = P_{1} + (z_{m} - h_{m})\rho g + h_{m}\rho_{m}g$$

$$\Delta P = P_{2} - P_{1} = h_{m}(\rho_{m} - \rho)g$$
(6.4)

and:

If fluid **A** is a gas, the density  $\rho$  will normally be small compared with the density of the manometer fluid  $\rho_m$  so that:

$$\Delta P = h_m \rho_m g \tag{6.5}$$

(b) In order to avoid the inconvenience of having to read two limbs, the *well-type* manometer shown in Figure 6.2b can be used. If  $A_w$  and  $A_c$  are the cross-sectional areas of the well and the column and  $h_m$  is the increase in the level of the column and  $h_w$  the decrease in the level of the well, then:

$$P_2 = P_1 + \rho g(h_m + h_w)$$
  
$$P_2 - P_1 = \rho g(h_m + h_w)$$

or:

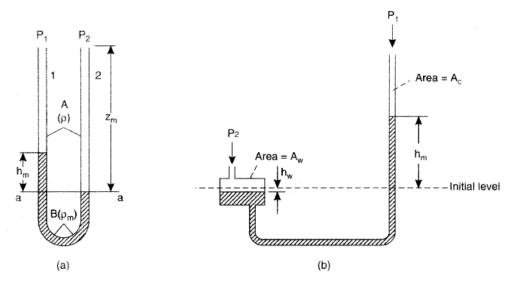


Figure 6.2. (a) The simple manometer (b) The well-type manometer

The quantity of liquid expelled from the well is equal to the quantity pushed into the column so that:

$$A_w h_w = A_c h_m$$
$$h_w = \frac{A_c}{A_w} h_m$$

and:

Substituting:

$$P_2 - P_1 = \rho g h_m \left( 1 + \frac{A_c}{A_w} \right) \tag{6.6}$$

If the well is large in comparison to the column then:

$$P_2 - P_1 = \rho g h_m \tag{6.7}$$

(c) The *inclined manometer* shown in Figure 6.3 enables the sensitivity of the manometers described previously to be increased by measuring the length of the column of liquid.

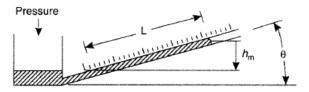


Figure 6.3. An inclined manometer

If  $\theta$  is the angle of inclination of the manometer (typically about  $10-20^{\circ}$ ) and L is the movement of the column of liquid along the limb, then:

$$L = \frac{h_m}{\sin \theta} \tag{6.8}$$

(d) The *inverted manometer* (Figure 6.4) is used for measuring pressure differences in liquids. The space above the liquid in the manometer is filled with air which can be admitted or expelled through the tap A in order to adjust the level of the liquid in the manometer.

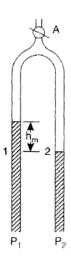


Figure 6.4. Inverted manometer

(e) The two-liquid manometer. Small differences in pressure in gases are often measured with a manometer of the form shown in Figure 6.5. The reservoir at the top of each limb is of a sufficiently large cross-section for the liquid level to remain approximately the

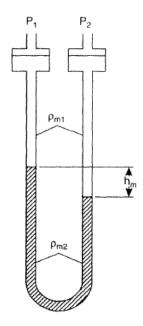


Figure 6.5. Two-liquid manometer

same on each side of the manometer. The difference in pressure is then given by:

$$\Delta P = (P_2 - P_1) = h_m(\rho_{m1} - \rho_{m2})g \tag{6.9}$$

where  $\rho_{m1}$  and  $\rho_{m2}$  are the densities of the two manometer liquids. The sensitivity of the instrument is very high if the densities of the two liquids are nearly the same. To obtain accurate readings it is necessary to choose liquids which give sharp interfaces: paraffin oil and industrial alcohol are commonly used. According to OWER and PANKHURST<sup>(2)</sup>, benzyl alcohol (specific gravity 1.048) and calcium chloride solutions give the most satisfactory results. The difference in density can be varied by altering the concentration of the calcium chloride solution.

(f) The Bourdon gauge (Figure 6.6). The pressure to be measured is applied to a curved tube, oval in cross-section, and the deflection of the end of the tube is communicated through a system of levers to a recording needle. This gauge is widely used for steam and compressed gases, and frequently forms the indicating element on flow controllers. The simple form of the gauge is illustrated in Figures 6.6a and 6.6b. Figure 6.6c shows a Bourdon type gauge with the sensing element in the form of a helix; this instrument has a very much greater sensitivity and is suitable for very high pressures.

It may be noted that the pressure measuring devices (a) to (e) all measure a pressure difference  $\Delta P (= P_2 - P_1)$ . In the case of the Bourdon gauge (f), the pressure indicated is the difference between that communicated by the system to the tube and the external (ambient) pressure, and this is usually referred to as the gauge pressure. It is then necessary to add on the ambient pressure in order to obtain the (absolute) pressure. Even the mercury barometer measures, not atmospheric pressure, but the difference between atmospheric pressure and the vapour pressure of mercury which, of course, is negligible. Gauge pressures are not, however, used in the SI System of units.

### 6.3. MEASUREMENT OF FLUID FLOW

The most important class of flowmeter is that in which the fluid is either accelerated or retarded at the measuring section by reducing the flow area, and the change in the kinetic energy is measured by recording the pressure difference produced.

This class includes:

The pitot tube, in which a small element of fluid is brought to rest at an orifice situated at right angles to the direction of flow. The flowrate is then obtained from the difference

between the impact and the static pressure. With this instrument the velocity measured is that of a small filament of fluid.

The orifice meter, in which the fluid is accelerated at a sudden constriction (the orifice) and the pressure developed is then measured. This is a relatively cheap and reliable instrument though the overall pressure drop is high because most of the kinetic energy of the fluid at the orifice is wasted.

The venturi meter, in which the fluid is gradually accelerated to a throat and gradually retarded as the flow channel is expanded to the pipe size. A high proportion of the kinetic energy is thus recovered but the instrument is expensive and bulky.

The nozzle, in which the fluid is gradually accelerated up to the throat of the instrument but expansion to pipe diameter is sudden as with an orifice. This instrument is again expensive because of the accuracy required over the inlet section.

The notch or weir, in which the fluid flows over the weir so that its kinetic energy is measured by determining the head of the fluid flowing above the weir. This instrument is used in open-channel flow and extensively in tray towers<sup>(3)</sup> where the height of the weir is adjusted to provide the necessary liquid depth for a given flow.

Each of these devices will now be considered in more detail together with some less common and special purpose meters.

# 6.3.1. The pitot tube

The pitot tube is used to measure the difference between the impact and static pressures in a fluid. It normally consists of two concentric tubes arranged parallel to the direction of flow; the impact pressure is measured on the open end of the inner tube. The end of the outer concentric tube is sealed and a series of orifices on the curved surface give an accurate indication of the static pressure. The position of these orifices must be carefully chosen because there are two disturbances which may cause an incorrect reading of the static pressure. These are due to:

- (1) the head of the instrument;
- (2) the portion of the stem which is at right angles to the direction of flow of the fluid.

These two disturbances cause errors in opposite directions, and the static pressure should therefore be measured at the point where the effects are equal and opposite.

If the head and stem are situated at a distance of 14 diameters from each other as on the standard instrument, (4) the two disturbances are equal and opposite at a section 6 diameters from the head and 8 from the stem. This is, therefore, the position at which the static pressure orifices should be located. If the distance between the head and the stem is too great, the instrument will be unwieldy; if it is too short, the magnitude of each of the disturbances will be relatively great, and a small error in the location of the static pressure orifices will appreciably affect the reading.

# 6.3.2. Measurement by flow through a constriction

In measuring devices where the fluid is accelerated by causing it to flow through a constriction, the kinetic energy is thereby increased and the pressure energy therefore

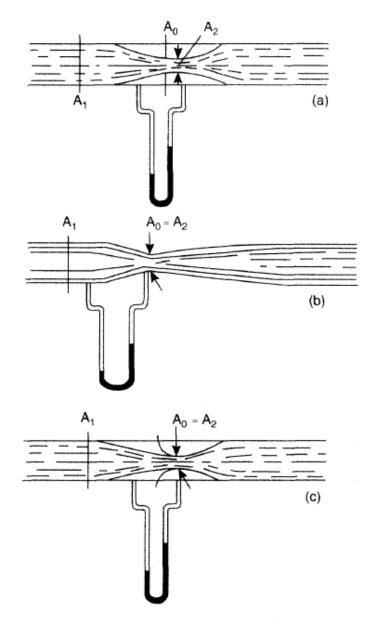


Figure 6.14. (a) Orifice meter (b) Venturi meter (c) Nozzle

decreases. The flowrate is obtained by measuring the pressure difference between the inlet of the meter and a point of reduced pressure, as shown in Figure 6.14 where the orifice meter, the nozzle and the venturi meter are illustrated. If the pressure is measured a short distance upstream where the flow is undisturbed (section 1) and at the position where the area of flow is a minimum (section 2), application of the energy and material balance equations gives:

$$\frac{u_2^2}{2\alpha_2} - \frac{u_1^2}{2\alpha_1} + g(z_2 - z_1) + \int_{P_1}^{P_2} v \, dP + W_s + F = 0 \quad \text{(from equation 2.55)}$$

and the mass flow,  $G = \frac{u_1 A_1}{v_1} = \frac{u_2 A_2}{v_2}$  (from equation 2.37)

If the frictional losses are neglected, and the fluid does no work on the surroundings, that is  $W_s$  and F are both zero, then:

$$\frac{u_2^2}{2\alpha_2} - \frac{u_1^2}{2\alpha_1} = g(z_1 - z_2) - \int_{P_1}^{P_2} v \, dP$$
 (6.13)

Inserting the value of  $u_1$  in terms of  $u_2$  in equation 6.13 enables  $u_2$  and G to be obtained. For an incompressible fluid:

$$\int_{P_1}^{P_2} v \, dP = v(P_2 - P_1)$$
 (equation 2.65)  
$$u_1 = u_2 \frac{A_2}{A_1}$$

and:

Substituting these values in equation 6.13:

$$\frac{u_2^2}{2\alpha_2} \left( 1 - \frac{\alpha_2}{\alpha_1} \frac{A_2^2}{A_1^2} \right) = g(z_1 - z_2) + v(P_1 - P_2)$$

$$u_2^2 = \frac{2\alpha_2 [g(z_1 - z_2) + v(P_1 - P_2)]}{1 - \frac{\alpha_2}{\alpha_1} \left( \frac{A_2}{A_1} \right)^2} \tag{6.14}$$

Thus:

For a horizontal meter  $z_1 = z_2$ , and:

$$u_{2} = \sqrt{\frac{2\alpha_{2}v(P_{1} - P_{2})}{1 - \frac{\alpha_{2}}{\alpha_{1}} \left(\frac{A_{2}}{A_{1}}\right)^{2}}}$$

$$G = \frac{u_{2}A_{2}}{v_{2}} = \frac{A_{2}}{v} \sqrt{\frac{2\alpha_{2}v(P_{1} - P_{2})}{1 - \frac{\alpha_{2}}{\alpha_{1}} \left(\frac{A_{2}}{A_{1}}\right)^{2}}}$$
(6.15)

and:

For an ideal gas in isothermal flow:

$$\int_{P_1}^{P_2} v \, dP = P_1 v_1 \ln \frac{P_2}{P_1}.$$
 (equation 2.69)  
$$u_1 = u_2 \frac{A_2 v_1}{A_1 v_2}$$

and:

And again neglecting terms in z, from equation 6.13:

$$\frac{u_2^2}{2\alpha_2} \left[ 1 - \frac{\alpha_2}{\alpha_1} \left( \frac{v_1 A_2}{v_2 A_1} \right)^2 \right] = P_1 v_1 \ln \frac{P_1}{P_2}$$

$$u_2^2 = \frac{2\alpha_2 P_1 v_1 \ln \frac{P_1}{P_2}}{1 - \frac{\alpha_2}{\alpha_1} \left( \frac{v_1 A_2}{v_2 A_1} \right)^2}$$
(6.16)

and:

and the mass flow G is again  $u_2A_2/v_2$ .

For an ideal gas in non-isothermal flow. If the pressure and volume are related by  $Pv^k = \text{constant}$ , then a similar analysis gives:

$$\int_{P_1}^{P_2} v \, dP = P_1 v_1 \frac{k}{k-1} \left[ \left( \frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right] \qquad \text{(equation 2.73)}$$
and, hence: 
$$\frac{u_2^2}{2\alpha_2} \left[ 1 - \frac{\alpha_2}{\alpha_1} \left( \frac{v_1 A_2}{v_2 A_1} \right)^2 \right] = -P_1 v_1 \frac{k}{k-1} \left[ \left( \frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right]$$
or: 
$$u_2^2 = \frac{2\alpha_2 P_1 v_1 \frac{k}{k-1} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{(k-1)/k} \right]}{1 - \frac{\alpha_2}{\alpha_1} \left( \frac{v_1 A_2}{v_2 A_1} \right)^2} \tag{6.17}$$

and the mass flow G is again  $u_2A_2/v_2$ .

### 6.3.3. The orifice meter

The most important factors influencing the reading of an orifice meter (Figure 6.15) are the size of the orifice and the diameter of the pipe in which it is fitted, though a number of other factors do affect the reading to some extent. Thus the exact position and the method of fixing the pressure tappings are important because the area of flow, and hence the velocity, gradually changes in the region of the orifice. The meter should be located not less than 50 pipe diameters from any pipe fittings. Details of the exact shape of the orifice, the orifice thickness, and other details are given in BS1042<sup>(4)</sup> and the details must be followed if a standard orifice is to be used without calibration—otherwise the meter must be calibrated. It should be noted that the standard applies only for pipes of at least 150 mm diameter.

The most serious disadvantage of the meter is that most of the pressure drop is not recoverable, that is it is inefficient. The velocity of the fluid is increased at the throat without much loss of energy. The fluid is subsequently retarded as it mixes with the relatively slow-moving fluid downstream from the orifice. A high degree of turbulence is set up and most of the excess kinetic energy is dissipated as heat. Usually only about 5 or 10 per cent of the excess kinetic energy can be recovered as pressure energy. The pressure drop over the orifice meter is therefore high, and this may preclude it from being used in a particular instance.

The area of flow decreases from  $A_1$  at section 1 to  $A_0$  at the orifice and then to  $A_2$  at the vena contracta (Figure 6.14). The area at the vena contracta can be conveniently related to the area of the orifice by the coefficient of contraction  $C_c$ , defined by the relation:

$$C_c = \frac{A_2}{A_0}$$

Inserting the value  $A_2 = C_c A_0$  in equation 6.15, then for an *incompressible fluid* in a horizontal meter:

$$G = \frac{C_c A_0}{v} \sqrt{\frac{2\alpha_2 v (P_1 - P_2)}{1 - \frac{\alpha_2}{\alpha_1} \left(C_c \frac{A_0}{A_1}\right)^2}}$$
(6.18)

Using a coefficient of discharge  $C_D$  to take account of the frictional losses in the meter and of the parameters  $C_c$ ,  $\alpha_1$ , and  $\alpha_2$ :

$$G = \frac{C_D A_0}{v} \sqrt{\frac{2v(P_1 - P_2)}{1 - \left(\frac{A_0}{A_1}\right)^2}}$$
(6.19)

For a meter in which the area of the orifice is small compared with that of the pipe:

$$\sqrt{1 - \left(\frac{A_0}{A_1}\right)^2} \to 1$$

$$G = \frac{C_D A_0}{v} \sqrt{2v(P_1 - P_2)}$$

$$= C_D A_0 \sqrt{2\rho(P_1 - P_2)}$$
(6.20)

and:

$$=C_D A_0 \rho \sqrt{2gh_0} \tag{6.21}$$

where  $h_0$  is the difference in head across the orifice expressed in terms of the fluid in question.

This gives a simple working equation for evaluating G though the coefficient  $C_D$  is not a simple function and depends on the values of the Reynolds number in the orifice and the form of the pressure tappings. A value of 0.61 may be taken for the standard meter for Reynolds numbers in excess of  $10^4$ , though the value changes noticeably at lower values of Reynolds number as shown in Figure 6.16.

For the isothermal flow of an ideal gas, from equation 6.16 and using  $C_D$  as above:

$$G = \frac{C_D A_0}{v_2} \sqrt{\frac{2P_1 v_1 \ln\left(\frac{P_1}{P_2}\right)}{1 - \left(\frac{v_1 A_0}{v_2 A_1}\right)^2}}$$
(6.22)

For a meter in which the area of the orifice is small compared with that of the pipe:

$$G = \frac{C_D A_0}{v_2} \sqrt{2P_1 v_1 \ln\left(\frac{P_1}{P_2}\right)}$$

$$= C_D A_0 \sqrt{2\frac{P_2}{v_2} \ln\frac{P_1}{P_2}}$$
(6.23)

### Example 6.1

Water flows through an orifice of 25 mm diameter situated in a 75 mm diameter pipe, at a rate of 300 cm<sup>3</sup>/s. What will be the difference in level on a water manometer connected across the meter? The viscosity of water is 1 mN s/m<sup>2</sup>.

#### Solution

Area of orifice = 
$$\frac{\pi}{4} \times 25^2 = 491 \text{ mm}^2 \text{ or } 4.91 \times 10^{-4} \text{ m}^2$$

Flow of water = 
$$300 \text{ cm}^3/\text{s}$$
 or  $3.0 \times 10^{-4} \text{ m}^3/\text{s}$ 

. '. Velocity of water through the orifice = 
$$\frac{3.0 \times 10^{-4}}{4.91 \times 10^{-4}} = 0.61$$
 m/s

Re at the orifice = 
$$\frac{25 \times 10^{-3} \times 0.61 \times 1000}{1 \times 10^{-3}} = 15250$$

From Figure 6.16, the corresponding value of  $C_D = 0.61$  (diameter ratio = 0.33):

$$\left[1 - \left(\frac{A_0}{A_1}\right)^2\right]^{0.5} = \left[1 - \left(\frac{25^2}{75^2}\right)^2\right]^{0.5} = 0.994 \approx 1$$

Equation 6.21 may therefore be applied:

$$G = 3.0 \times 10^{-4} \times 10^{3} = 0.30 \text{ kg/s}.$$

$$0.30 = (0.61 \times 4.91 \times 10^{-4} \times 10^{3}) \sqrt{(2 \times 9.81 \times h_0)}$$

Hence:  $\sqrt{h_0} = 0.226$ 

and:  $h_0 = 0.051 \text{ m of water}$ 

= 51 mm of water