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2017/2018 Session: EEE 316 Electromagnetic Waves: Practice Exercises

Take $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m

Q1 (a) Write Maxwell's equations in their integral and differential forms. Explain the terms and write the units of the quantities of the equations. State which of the equations are derived from Gauss', Ampere's and Faraday's laws.

(b) Explain the modification introduced by Maxwell to explain the difference between Ampere's law for static and time-varying magnetic fields.

Q2 Assume the E-component of the em wave is polarized in the y-direction while the H-component is polarized in the z-direction. Maxwell's equations show the relationship between the space and time variations of these field components in vacuum to be expressed as follows:

$$\frac{\partial H_z}{\partial x} = -\varepsilon_0 \frac{\partial E_y}{\partial t} \quad (1)$$

$$\frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \quad (2)$$

(a) Show how this pair of equations could be decoupled to give the wave equations in E_y and H_z respectively.

(b) Solve the wave equation in E_y and H_z respectively, and derive the expression for the wave velocity and the characteristic impedance of the medium.

(c) Calculate the value of this velocity and the characteristic impedance.

Q 3. $\frac{\partial^2 E_y}{\partial x^2} = (j\omega\mu\sigma - \omega^2 \mu\varepsilon)\, E_y$ is a familiar equation in em wave propagation in a material medium. Answer the following questions.

(a) Define E_y, ω, μ, σ and ε, stating their units.

(b) If the medium is lossless, state the value of σ and write the expression for the phase velocity, v_p, in terms of μ and ε.

(c) If the medium is lossless and has $\mu_r = 1$ and $\varepsilon_r = 1$, determine the value of v_p and the characteristic impedance Z_0

(d) If the wave travels in the positive x-direction and the electric field is lined up in the y-direction, in what direction is the magnetic field lined up, and why?

Q 4. If the medium through which the em wave propagates is a good conductor,

(a) Analyse the equation in Q3 to show that the wave amplitude decreases exponentially as it penetrates the medium.
(b) Define the depth of penetration known as the skin depth and derive its value in terms of the parameters of the medium and the frequency of the signal.
(c) Calculate the depth of penetration of the wave in a sheet of copper at a frequency of 10 MHz at which the wave amplitude decreases to 1% of its value upon entering the sheet.

Take $\sigma = 5.8 \times 10^7$ S/m, $\mu_r = 1$ for copper.

Q5 (a) Define the skin depth or the depth of penetration, δ, when an em wave enters a conducting medium.

(b) Given $\delta = \frac{1}{\sqrt{(\pi f \mu_0 \sigma)}}$ calculate δ in copper at a frequency of 1 MHz, where $\sigma = 5.8 \times 10^7$ S/m

(c) Calculate δ at 100 MHz, considering the frequency-dependence relation only.

(d) If the magnitude of the electric field at the surface of the conductor is 100 mV /m, calculate the value of the field at (i) skin depth, and (ii) twice the skin depth.

Q6. (a) Draw the network representation of a transmission line in terms of its primary constants.

(b) Derive the wave equations for the voltage, V and current, I, of a two-wire transmission line.

(c) Derive the expression for the characteristic impedance, Z_0, of the line in terms of its primary constants, and deduce its value for a lossless line.

Q7 (a) An air-filled coaxial transmission line has an outer conductor inside diameter, b = 10 mm and an inner conductor outside diameter, a = 3 mm. Calculate the,

(a) Capacitance per meter, C
(b) Inductance per meter, L
(c) Characteristic impedance, Z_0
(d) Phase velocity, v_p of an em wave propagate through it.

Hint: $C = \frac{2\pi\varepsilon_0}{\log_e \frac{b}{a}}$ $\quad L = \frac{\mu_0}{2\pi} \log_e \frac{b}{a}$

Q8 (a) Define the terms, voltage reflection coefficient, Γ, and the voltage standing wave ratio, S of an em wave on a transmission line.

(b) Derive the expression for Γ *in terms of the load*, Z_L and the characteristic impedance, Z_0 of the line.

(c) Derive the relation between Γ and S.

(d) A lossless 50 Ω line is terminated by a load of 25 Ω. Calculate Γ and S.

(e) If the maximum voltage measured at a point on the line is 10 mV, determine the minimum voltage on the line.

Q9, The impedance Z_x at a point x from a load of impedance Z_L of a lossless transmission line with characteristic impedance Z_0 is given by,

$$Z_x = Z_0\left[\frac{Z_L + jZ_0 \tan\beta x}{Z_0 + jZ_L \tan\beta x}\right]$$

(a) Calculate Z_x for a 50 Ω line at a point x = 0.25λ, when

(i) Z_L = 25 Ω
(ii) Z_L = 50 Ω
(iii) The line is short-circuited at the end
(iv) The line is open-circuited at the end

(b) Show that $Z_0 = \sqrt{Z_x Z_L}$ in cases (i) and (ii) above

(c) Comment on the results of (iii) and (iv).

Q10 (a) Describe the following methods of matching a transmission line to a load.

(i) Using a $\frac{\lambda}{4}$ – line transformer
(ii) Single-stub matching

(b) Describe the Smith chart technique of solving transmission line problems

(c) A 100 Ω transmission line is terminated in a load of 50 + j40 Ω. Using the Smith chart, determine the,

(i) Voltage standing wave ratio on the line
(ii) Impedance, Z_x at a point, x = 0.2λ from the load
(iii) Admittance of the load

(d) Verify the admittance of the load by calculation.