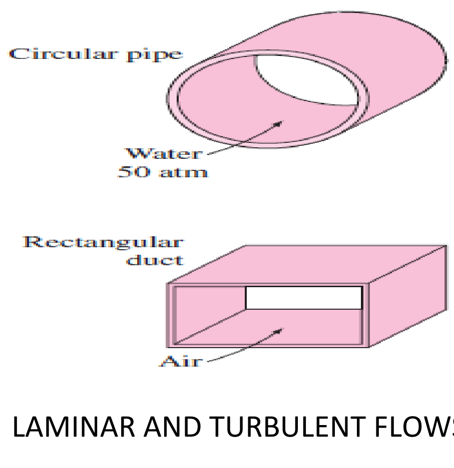
F L OW IN P I P E S

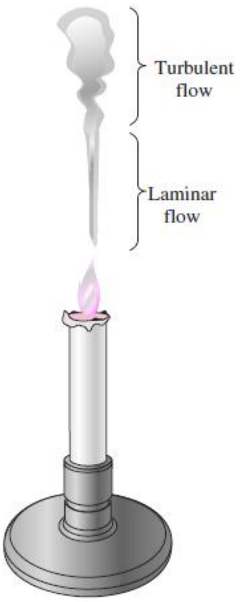
Fluid flow in circular and noncircular pipes is commonly encountered in practice. The hot and cold water that we use in our homes is pumped through pipes. Water in a city is distributed by extensive piping networks. Oil and natural gas are transported hundreds of miles by large pipelines. Blood is carried throughout our bodies by arteries and veins. The cooling water in an engine is transported by hoses to the pipes in the radiator where it is cooled as it flows. Thermal energy in a hydronic space heating system is transferred to the circulating water in the boiler, and then it is transported to the desired locations through pipes. Fluid flow is classified as *external* and *internal,* depending on whether the fluid is forced to flow over a surface or in a conduit. Internal and external flows exhibit very different characteristics. In this chapter we consider *internal flow* where the conduit is completely filled with the fluid, and flow is driven primarily by a pressure difference. This should not be confused with *open-channel flow* where the conduit is partially filled by the fluid and thus the flow is partially bounded by solid surfaces, as in an irrigation ditch, and flow is driven by gravity alone.

Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications and fluid distribution networks. The fluid in such applications is usually forced to flow by a fan or pump through a flow section. We pay particular attention to *friction,* which is directly related to the *pressure drop* and *head loss* during flow through pipes and ducts. The pressure drop is then used to determine the pumping power requirement. A typical piping system involves pipes of different diameters connected to each other by various fittings or elbows to route the fluid, valves to control the flow rate, and pumps to pressurize the fluid. The terms *pipe, duct,* and *conduit* are usually used interchangeably for flow sections. In general, flow sections of circular cross section are referred to as *pipes* (especially when the fluid is a liquid), and flow sections of noncircular cross section as *ducts* (especially when the fluid is a gas). Small diameter pipes are usually referred to as *tubes.*

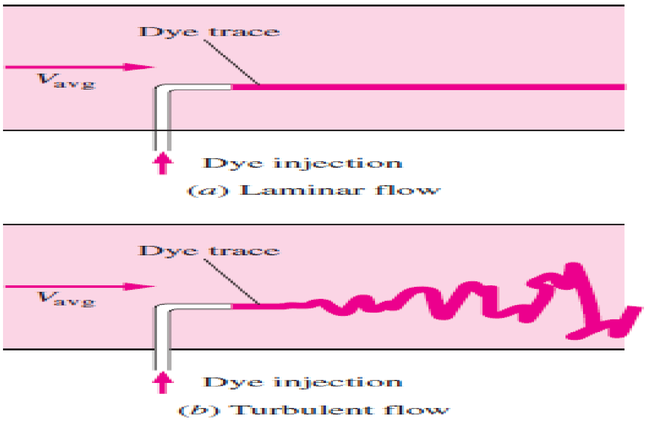


LAMINAR AND TURBULENT FLOWS

If you have been around smokers, you probably noticed that the cigarette smoke rises in a smooth plume for the first few centimeters and then starts fluctuating randomly in all directions as it continues its rise. Other plumes behave similarly . Likewise, a careful inspection of flow in a pipe reveals that the fluid flow is streamlined at low velocities but turns chaotic as the velocity is increased above a critical value, as shown below The flow regime in the first case is said to be laminar, characterized by *smooth streamlines* and *highly ordered motion,* and turbulent in the second case, where it is characterized by *velocity fluctuations* and *highly disordered motion.* The transition from laminar to turbulent flow does not occur suddenly; rather, it occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent. Most flows encountered in practice are turbulent. Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.



Laminar and turbulent flow regimes of candle smoke



The behavior of colored fluid injected into the flow in laminar and turbulent flows in a pipe

Reynolds Number

The transition from laminar to turbulent flow depends on the *geometry, surface roughness, flow velocity, surface temperature,* and *type of fluid,* among other things. After exhaustive experiments in the 1880s, Osborne Reynolds discovered that the flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* in the fluid. This ratio is called the Reynolds number and is expressed for internal flow in a circular pipe as:

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{avg}D}{\nu} = \frac{\rho V_{avg}D}{\mu}$$

where V_{avg} = average flow velocity (m/s), D = characteristic length of the geometry (diameter in this case, in m), and $\nu = \mu/\rho$ =kinematic viscosity of the fluid (m^2/s). Note that the Reynolds number is a *dimensionless* quantity.

At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid. At *small* or *moderate* Reynolds numbers, however, the viscous forces are large enough to suppress these fluctuations and to keep the fluid "in line." Thus the flow is *turbulent* in the first case and *laminar* in the second.

The Reynolds number at which the flow becomes turbulent is called the **critical Reynolds number,** Re_{cr}. The value of the critical Reynolds number is different for different geometries and flow conditions. For internal flow in a circular pipe, the generally accepted value of the critical Reynolds number is $\mathrm{Re}_{cr} = 2300$.

For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter** D_h defined as (Fig. 8–6)

Hydraulic diameter: $$D_h = \frac{4A_c}{p} \qquad \textbf{(8–4)}$$

where A_c is the cross-sectional area of the pipe and p is its wetted perimeter. The hydraulic diameter is defined such that it reduces to ordinary diameter D for circular pipes,

Circular pipes: $$D_h = \frac{4A_c}{p} = \frac{4(\pi D^2/4)}{\pi D} = D$$

It certainly is desirable to have precise values of Reynolds numbers for laminar, transitional, and turbulent flows, but this is not the case in practice. It turns out that the transition from laminar to turbulent flow also depends on the degree of disturbance of the flow by *surface roughness, pipe vibrations,* and *fluctuations in the flow.* Under most practical conditions, the flow in a circular pipe is laminar for $\mathrm{Re} \lesssim 2300$, turbulent for $\mathrm{Re} \gtrsim 4000$, and transitional in between. That is,

$$\begin{aligned} \mathrm{Re} &\lesssim 2300 && \text{laminar flow} \\ 2300 \lesssim \mathrm{Re} &\lesssim 4000 && \text{transitional flow} \\ \mathrm{Re} &\gtrsim 4000 && \text{turbulent flow} \end{aligned}$$

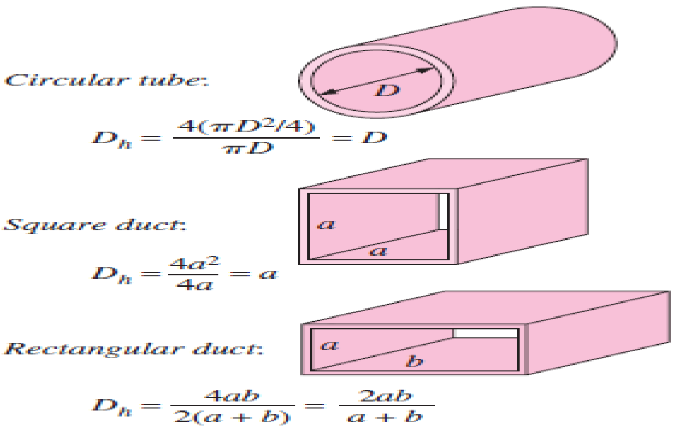


FIGURE 8–6
The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes.

Pressure Drop and Head Loss

A quantity of interest in the analysis of pipe flow is the *pressure drop* ΔP since it is directly related to the power requirements of the fan or pump to maintain flow. We note that dP/dx = constant, and integrating from $x = x_1$ where the pressure is P_1 to $x = x_1 + L$ where the pressure is P_2 gives

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} \tag{8–19}$$

Substituting Eq. 8–19 into the V_{avg} expression in Eq. 8–16, the pressure drop can be expressed as

Laminar flow:
$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2} \tag{8–20}$$

The symbol Δ is typically used to indicate the difference between the final and initial values, like $\Delta y = y_2 - y_1$. But in fluid flow, ΔP is used to designate pressure drop, and thus it is $P_1 - P_2$. A pressure drop due to viscous effects represents an irreversible pressure loss, and it is called **pressure loss** ΔP_L to emphasize that it is a *loss* (just like the head loss h_L, which is proportional to it).

Note from Eq. 8–20 that the pressure drop is proportional to the viscosity μ of the fluid, and ΔP would be zero if there were no friction. Therefore, the drop of pressure from P_1 to P_2 in this case is due entirely to viscous effects, and Eq. 8–20 represents the pressure loss ΔP_L when a fluid of viscosity μ flows through a pipe of constant diameter D and length L at average velocity V_{avg}.

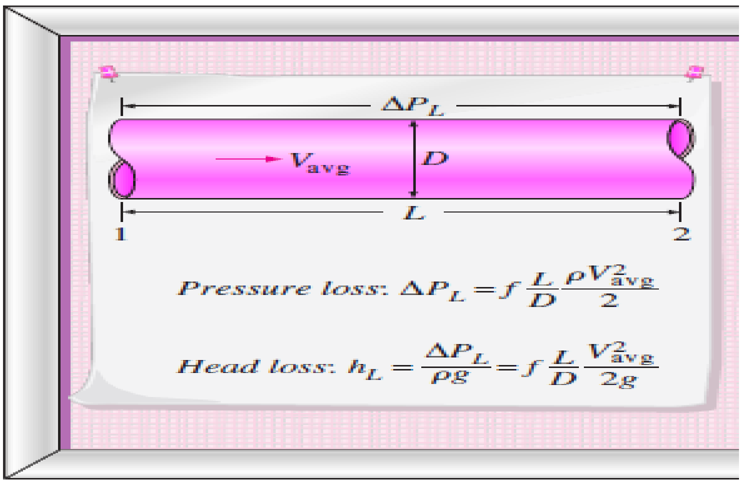
In practice, it is found convenient to express the pressure loss for all types of fully developed internal flows (laminar or turbulent flows, circular or noncircular pipes, smooth or rough surfaces, horizontal or inclined pipes) as (Fig. 8–13)

Pressure loss:
$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2} \tag{8–21}$$

where $\rho V_{avg}^2/2$ is the *dynamic pressure* and f is the **Darcy friction factor,**

$$f = \frac{8\tau_w}{\rho V_{avg}^2} \tag{8–22}$$

It is also called the **Darcy–Weisbach friction factor,** named after the Frenchman Henry Darcy (1803–1858) and the German Julius Weisbach (1806–1871), the two engineers who provided the greatest contribution in its development. It should not be confused with the *friction coefficient* C_f



[also called the *Fanning friction factor*, named after the American engineer John Fanning (1837–1911)], which is defined as $C_f = 2\tau_w/(\rho V_{avg}^2) = f/4$.

Setting Eqs. 8–20 and 8–21 equal to each other and solving for f gives the friction factor for fully developed laminar flow in a circular pipe,

Circular pipe, laminar:
$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{\text{Re}} \tag{8–23}$$

This equation shows that *in laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface.*

In the analysis of piping systems, pressure losses are commonly expressed in terms of the *equivalent fluid column height*, called the **head loss** h_L. Noting from fluid statics that $\Delta P = \rho g h$ and thus a pressure difference of ΔP corresponds to a fluid height of $h = \Delta P/\rho g$, the *pipe head loss* is obtained by dividing ΔP_L by ρg to give

Head loss:
$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{avg}^2}{2g} \tag{8–24}$$

The head loss h_L represents *the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe*. The head loss is caused by viscosity, and it is directly related to the wall shear stress. Equations 8–21 and 8–24 are valid for both laminar and turbulent flows in both circular and noncircular pipes, but Eq. 8–23 is valid only for fully developed laminar flow in circular pipes.

Once the pressure loss (or head loss) is known, the required pumping power *to overcome the pressure loss* is determined from

$$\dot{W}_{\text{pump}, L} = \dot{V}\,\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L \tag{8–25}$$

where $\dot{V}$ is the volume flow rate and $\dot{m}$ is the mass flow rate.

The average velocity for laminar flow in a horizontal pipe is, from Eq. 8–20,

Horizontal pipe:
$$V_{avg} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P\, D^2}{32\mu L} \tag{8–26}$$

Then the volume flow rate for laminar flow through a horizontal pipe of diameter D and length L becomes

$$\dot{V} = V_{avg} A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P\, \pi D^4}{128\mu L} \tag{8–27}$$

This equation is known as **Poiseuille's law,** and this flow is called *Hagen–Poiseuille flow* in honor of the works of G. Hagen (1797–1884) and J. Poiseuille (1799–1869) on the subject. Note from Eq. 8–27 that *for a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the radius (or diameter) of the pipe*. Therefore, the pumping power requirement for a piping system can be reduced by a factor of 16 by doubling the pipe diameter (Fig. 8–14). Of course the benefits of the reduction in the energy costs must be weighed against the increased cost of construction due to using a larger-diameter pipe.

The pressure drop ΔP equals the pressure loss ΔP_L in the case of a horizontal pipe, but this is not the case for inclined pipes or pipes with variable cross-sectional area. This can be demonstrated by writing the energy equation for steady, incompressible one-dimensional flow in terms of heads as (see Chap. 5)

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine}, e} + h_L \qquad \textbf{(8–28)}$$

where $h_{\text{pump}, u}$ is the useful pump head delivered to the fluid, $h_{\text{turbine}, e}$ is the turbine head extracted from the fluid, h_L is the irreversible head loss between sections 1 and 2, V_1 and V_2 are the average velocities at sections 1 and 2, respectively, and α_1 and α_2 are the *kinetic energy correction factors* at sections 1 and 2 (it can be shown that $\alpha = 2$ for fully developed laminar flow and about 1.05 for fully developed turbulent flow). Equation 8–28 can be rearranged as

$$P_1 - P_2 = \rho(\alpha_2 V_2^2 - \alpha_1 V_1^2)/2 + \rho g[(z_2 - z_1) + h_{\text{turbine}, e} - h_{\text{pump}, u} + h_L] \qquad \textbf{(8–29)}$$

Therefore, the pressure drop $\Delta P = P_1 - P_2$ and pressure loss $\Delta P_L = \rho g h_L$ for a given flow section are equivalent if (1) the flow section is horizontal so that there are no hydrostatic or gravity effects ($z_1 = z_2$), (2) the flow section does not involve any work devices such as a pump or a turbine since they change the fluid pressure ($h_{\text{pump}, u} = h_{\text{turbine}, e} = 0$), (3) the cross-sectional area of the flow section is constant and thus the average flow velocity is constant ($V_1 = V_2$), and (4) the velocity profiles at sections 1 and 2 are the same shape ($\alpha_1 = \alpha_2$).

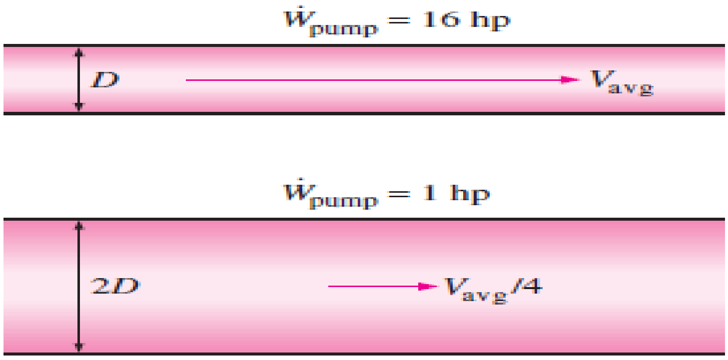


FIGURE 8–14
The pumping power requirement for a laminar flow piping system can be reduced by a factor of 16 by doubling the pipe diameter.

EXAMPLE 8–1 **Flow Rates in Horizontal and Inclined Pipes**

Oil at 20°C (ρ = 888 kg/m^3 and μ = 0.800 kg/m · s) is flowing steadily through a 5-cm-diameter 40-m-long pipe (Fig. 8–17). The pressure at the pipe inlet and outlet are measured to be 745 and 97 kPa, respectively. Determine the flow rate of oil through the pipe assuming the pipe is (*a*) horizontal, (*b*) inclined 15° upward, (*c*) inclined 15° downward. Also verify that the flow through the pipe is laminar.

SOLUTION The pressure readings at the inlet and outlet of a pipe are given. The flow rates are to be determined for three different orientations, and the flow is to be shown to be laminar.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as a pump or a turbine.

Properties The density and dynamic viscosity of oil are given to be ρ = 888 kg/m^3 and μ = 0.800 kg/m · s, respectively.

Analysis The pressure drop across the pipe and the pipe cross-sectional area are

$$\Delta P = P_1 - P_2 = 745 - 97 = 648\ \text{kPa}$$

$$A_c = \pi D^2/4 = \pi(0.05\ \text{m})^2/4 = 0.001963\ \text{m}^2$$

(*a*) The flow rate for all three cases can be determined from Eq. 8–34,

$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

where θ is the angle the pipe makes with the horizontal. For the horizontal case, θ = 0 and thus sin θ = 0. Therefore,

$$\dot{V}_{\text{horiz}} = \frac{\Delta P\, \pi D^4}{128\mu L} = \frac{(648\ \text{kPa})\pi(0.05\ \text{m})^4}{128(0.800\ \text{kg/m}\cdot\text{s})(40\ \text{m})}\left(\frac{1000\ \text{N/m}^2}{1\ \text{kPa}}\right)\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)$$

$$= \mathbf{0.00311\ m^3/s}$$

(*b*) For uphill flow with an inclination of 15°, we have θ = +15°, and

$$\dot{V}_{\text{uphill}} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

$$= \frac{[648{,}000\ \text{Pa} - (888\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(40\ \text{m})\sin 15°]\pi(0.05\ \text{m})^4}{128(0.800\ \text{kg/m}\cdot\text{s})(40\ \text{m})}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{Pa}\cdot\text{m}^2}\right)$$

$$= \mathbf{0.00267\ m^3/s}$$

(*c*) For downhill flow with an inclination of 15°, we have θ = −15°, and

$$\dot{V}_{\text{downhill}} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

$$= \frac{[648{,}000\ \text{Pa} - (888\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(40\ \text{m})\sin(-15°)]\pi(0.05\ \text{m})^4}{128(0.800\ \text{kg/m}\cdot\text{s})(40\ \text{m})}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{Pa}\cdot\text{m}^2}\right)$$

$$= \mathbf{0.00354\ m^3/s}$$

The flow rate is the highest for the downhill flow case, as expected. The average fluid velocity and the Reynolds number in this case are

$$V_{avg} = \frac{\dot{V}}{A_c} = \frac{0.00354\ m^3/s}{0.001963\ m^2} = 1.80\ m/s$$

$$Re = \frac{\rho V_{avg} D}{\mu} = \frac{(888\ kg/m^3)(1.80\ m/s)(0.05\ m)}{0.800\ kg/m \cdot s} = 100$$

which is much less than 2300. Therefore, the flow is *laminar* for all three cases and the analysis is valid.

Discussion Note that the flow is driven by the combined effect of pressure difference and gravity. As can be seen from the flow rates we calculated, gravity opposes uphill flow, but enhances downhill flow. Gravity has no effect on the flow rate in the horizontal case. Downhill flow can occur even in the absence of an applied pressure difference. For the case of $P_1 = P_2 = 97$ kPa (i.e., no applied pressure difference), the pressure throughout the entire pipe would remain constant at 97 Pa, and the fluid would flow through the pipe at a rate of 0.00043 m^3/s under the influence of gravity. The flow rate increases as the tilt angle of the pipe from the horizontal is increased in the negative direction and would reach its maximum value when the pipe is vertical.

EXAMPLE 8–2 Pressure Drop and Head Loss in a Pipe

Water at 40°F ($\rho = 62.42$ lbm/ft^3 and $\mu = 1.038 \times 10^{-3}$ lbm/ft · s) is flowing through a 0.12-in- (= 0.010 ft) diameter 30-ft-long horizontal pipe steadily at an average velocity of 3.0 ft/s (Fig. 8–18). Determine (*a*) the head loss, (*b*) the pressure drop, and (*c*) the pumping power requirement to overcome this pressure drop.

SOLUTION The average flow velocity in a pipe is given. The head loss, the pressure drop, and the pumping power are to be determined.

Assumptions 1 The flow is steady and incompressible. 2 The entrance effects are negligible, and thus the flow is fully developed. 3 The pipe involves no components such as bends, valves, and connectors.

Properties The density and dynamic viscosity of water are given to be $\rho = 62.42$ lbm/ft^3 and $\mu = 1.038 \times 10^{-3}$ lbm/ft · s, respectively.

Analysis (*a*) First we need to determine the flow regime. The Reynolds number is

$$Re = \frac{\rho V_{avg} D}{\mu} = \frac{(62.42\ lbm/ft^3)(3\ ft/s)(0.01\ ft)}{1.038 \times 10^{-3}\ lbm/ft \cdot s} = 1803$$

which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the head loss become

$$f = \frac{64}{Re} = \frac{64}{1803} = 0.0355$$

$$h_L = f \frac{L}{D} \frac{V_{avg}^2}{2g} = 0.0355 \frac{30\ ft}{0.01\ ft} \frac{(3\ ft/s)^2}{2(32.2\ ft/s^2)} = \mathbf{14.9\ ft}$$

(*b*) Noting that the pipe is horizontal and its diameter is constant, the pressure drop in the pipe is due entirely to the frictional losses and is equivalent to the pressure loss,

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 0.0355\frac{30 \text{ ft}}{0.01 \text{ ft}}\frac{(62.42 \text{ lbm/ft}^3)(3 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right)$$

$$= \mathbf{929 \text{ lbf/ft}^2 = 6.45 \text{ psi}}$$

(*c*) The volume flow rate and the pumping power requirements are

$$\dot{V} = V_{avg}A_c = V_{avg}(\pi D^2/4) = (3 \text{ ft/s})[\pi(0.01 \text{ ft})^2/4] = 0.000236 \text{ ft}^3\text{/s}$$

$$\dot{W}_{pump} = \dot{V}\,\Delta P = (0.000236 \text{ ft}^3\text{/s})(929 \text{ lbf/ft}^2)\left(\frac{1 \text{ W}}{0.737 \text{ lbf} \cdot \text{ft/s}}\right) = \mathbf{0.30 \text{ W}}$$

Therefore, power input in the amount of 0.30 W is needed to overcome the frictional losses in the flow due to viscosity.

Discussion The pressure rise provided by a pump is often listed by a pump manufacturer in units of head (Chap. 14). Thus, the pump in this flow needs to provide 14.9 ft of water head in order to overcome the irreversible head loss.