Liquid Filtration

The separation of solids from a suspension in a liquid by means of a porous medium or screen which retains the solids and allows the liquid to pass is termed filtration.

In general, the pores of the medium are larger than the particles which are to be removed, and the filter works efficiently only after an initial deposit has been trapped in the medium. In the laboratory, filtration is often carried out using a form of Buchner funnel, and the liquid is sucked through the thin layer of particles using a source of vacuum. In even simpler cases the suspension is poured into a conical funnel fitted with a filter paper. In the industrial equivalent, difficulties are encountered in the mechanical handling of much larger quantities of suspension and solids. A thicker layer of solids has to form and, in order to achieve a high rate of passage of liquid through the solids, higher pressures are needed, and a far greater area has to be provided. A typical filtration operation is illustrated in Figure 7.1, which shows the filter medium, in this case a cloth, its support and the layer of solids, or filter cake, which has already formed.

Volumes of the suspensions to be handled vary from the extremely large quantities involved in water purification and ore handling in the mining industry to relatively small quantities, as in the fine chemical industry where the variety of solids is considerable. In most industrial applications it is the solids that are required and their physical size and properties are of paramount importance. Thus, the main factors to be considered when selecting equipment and operating conditions are:

(a) The properties of the fluid, particularly its viscosity, density and corrosive properties.
(b) The nature of the solid — its particle size and shape, size distribution, and packing characteristics.
(c) The concentration of solids in suspension.
(d) The quantity of material to be handled, and its value.
(e) Whether the valuable product is the solid, the fluid, or both.
(f) Whether it is necessary to wash the filtered solids.
(g) Whether very slight contamination caused by contact of the suspension or filtrate with the various components of the equipment is detrimental to the product.
(h) Whether the feed liquor may be heated.
(i) Whether any form of pretreatment might be helpful.

Filtration is essentially a mechanical operation and is less demanding in energy than evaporation or drying where the high latent heat of the liquid, which is usually water, has to be provided. In the typical operation shown in Figure 7.1, the cake gradually builds up

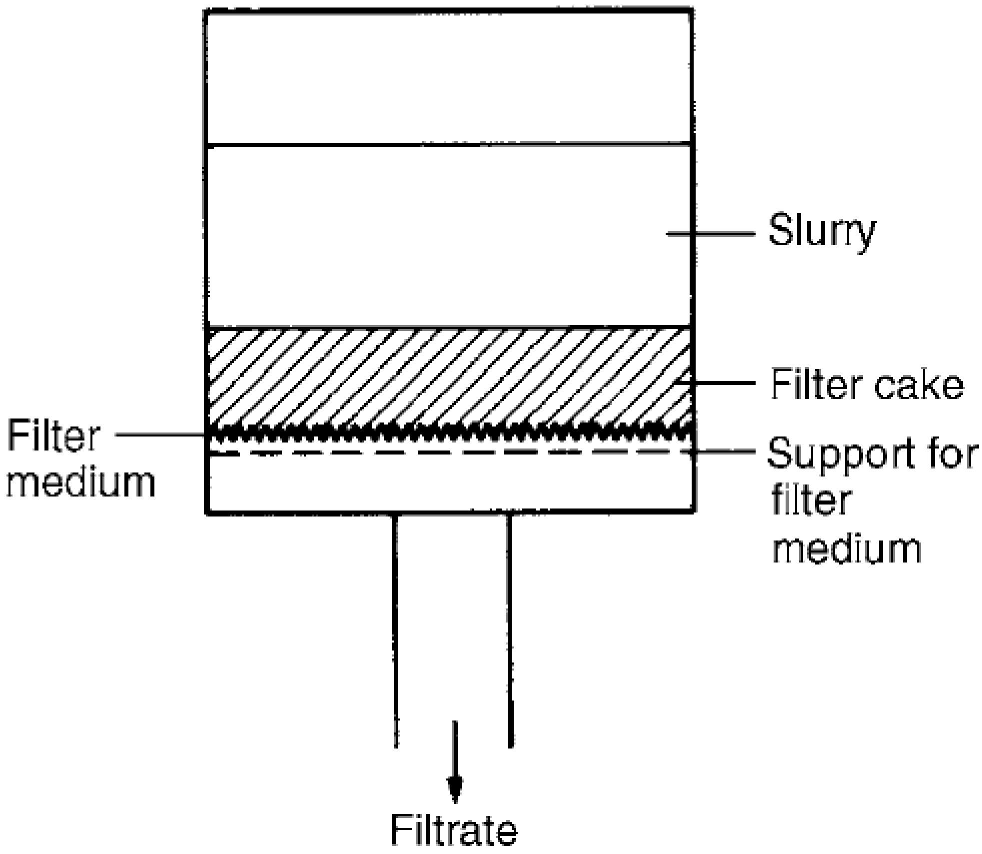

Figure 7.1. Principle of filtration

on the medium and the resistance to flow progressively increases. During the initial period of flow, particles are deposited in the surface layers of the cloth to form the true filtering medium. This initial deposit may be formed from a special initial flow of precoat material which is discussed later. The most important factors on which the rate of filtration then depends will be:

(a) The drop in pressure from the feed to the far side of the filter medium.
(b) The area of the filtering surface.
(c) The viscosity of the filtrate.
(d) The resistance of the filter cake.
(e) The resistance of the filter medium and initial layers of cake.

Two basic types of filtration processes may be identified, although there are cases where the two types appear to merge. In the first, frequently referred to as *cake filtration*, the particles from the suspension, which usually has a high proportion of solids, are deposited on the surface of a porous septum which should ideally offer only a small resistance to flow. As the solids build up on the septum, the initial layers form the effective filter medium, preventing the particles from embedding themselves in the filter cloth, and ensuring that a particle-free filtrate is obtained.

In the second type of filtration, *depth* or *deep-bed filtration*, the particles penetrate into the pores of the filter medium, where impacts between the particles and the surface of the medium are largely responsible for their removal and retention. This configuration is commonly used for the removal of fine particles from very dilute suspensions, where the recovery of the particles is not of primary importance. Typical examples here include air and water filtration. The filter bed gradually becomes clogged with particles, and its resistance to flow eventually reaches an unacceptably high level. For continued operation, it is therefore necessary to remove the accumulated solids, and it is important that this can be readily achieved. For this reason, the filter commonly consists of a bed of particulate solids, such as sand, which can be cleaned by back-flushing, often accompanied by fluidisation. In this chapter, the emphasis is on cake filtration although deep-bed filtration, which has been discussed in detail by Ives[1,2] is considered in the section on bed filters.

There are two principal modes under which deep bed filtration may be carried out. In the first, *dead-end filtration* which is illustrated in Figure 7.1, the slurry is filtered in such a way that it is fed perpendicularly to the filter medium and there is little flow parallel to the surface of the medium. In the second, termed *cross-flow filtration* which is discussed in Section 7.3.5. and which is used particularly for very dilute suspensions, the slurry is continuously recirculated so that it flows essentially across the surface of the filter medium at a rate considerably in excess of the flowrate through the filter cake.

FILTRATION THEORY

Chapter 4 is useful in describing the flow within the filter cake. The general theory of filtration and its importance in design has been considered by SUTTLE[3]. It may be noted that there are two quite different methods of operating a batch filter. If the pressure is kept constant then the rate of flow progressively diminishes, whereas if the flowrate is kept constant then the pressure must be gradually increased. Because the particles forming the cake are small and the flow through the bed is slow, streamline conditions are almost invariably obtained, and, at any instant, the flowrate of the filtrate may be represented by the following form of equation 4.9:

$$u_c = \frac{1}{A}\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{5}\frac{e^3}{(1-e)^2}\frac{-\Delta P}{S^2 \mu l} \tag{7.1}$$

where V is the volume of filtrate which has passed in time t, A is the total cross-sectional area of the filter cake, u_c is the superficial velocity of the filtrate, l is the cake thickness, S is the specific surface of the particles, e is the voidage, μ is the viscosity of the filtrate, and ΔP is the applied pressure difference.

In deriving this equation it is assumed that the cake is uniform and that the voidage is constant throughout. In the deposition of a filter cake this is unlikely to be the case and the voidage, e will depend on the nature of the support, including its geometry and surface structure, and on the rate of deposition. The initial stages in the formation of the cake are therefore of special importance for the following reasons:

(a) For any filtration pressure, the rate of flow is greatest at the beginning of the process since the resistance is then a minimum.
(b) High initial rates of filtration may result in plugging of the pores of the filter cloth and cause a very high resistance to flow.
(c) The orientation of the particle in the initial layers may appreciably influence the structure of the whole filter cake.

Filter cakes may be divided into two classes — incompressible cakes and compressible cakes. In the case of an incompressible cake, the resistance to flow of a given volume of cake is not appreciably affected either by the pressure difference across the cake or by the rate of deposition of material. On the other hand, with a compressible cake, increase of the pressure difference or of the rate of flow causes the formation of a denser cake with a higher resistance. For incompressible cakes e in equation 7.1 may be taken as constant and the quantity $e^3/[5(1-e)^2S^2]$ is then a property of the particles forming the cake and should be constant for a given material.

Thus:
$$\frac{1}{A}\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{-\Delta P}{\mathbf{r}\mu l} \tag{7.2}$$

where:
$$\mathbf{r} = \frac{5(1-e)^2S^2}{e^3} \tag{7.3}$$

It may be noted that, when there is a hydrostatic pressure component such as with a horizontal filter surface, this should be included in the calculation of $-\Delta P$.

Equation 7.2 is the basic filtration equation and **r** is termed the specific resistance which is seen to depend on e and S. For incompressible cakes, **r** is taken as constant, although it depends on rate of deposition, the nature of the particles, and on the forces between the particles. **r** has the dimensions of $\mathbf{L}^{-2}$ and the units m^{-2} in the SI system.

7.2.2. Relation between thickness of cake and volume of filtrate

In equation 7.2, the variables l and V are connected, and the relation between them may be obtained by making a material balance between the solids in both the slurry and the cake as follows.

Mass of solids in filter cake $= (1-e)Al\rho_s$, where ρ_s is the density of the solids

Mass of liquid retained in the filter cake $= eAl\rho$, where ρ is the density of the filtrate.

If J is the mass fraction of solids in the original suspension then:

$$(1-e)lA\rho_s = \frac{(V+eAl)\rho J}{1-J}$$

or:
$$(1-J)(1-e)Al\rho_s = JV\rho + AeJl\rho$$

so that:
$$l = \frac{JV\rho}{A\{(1-J)(1-e)\rho_s - Je\rho\}} \tag{7.4}$$

and: $$V = \frac{\{\rho_s(1-e)(1-J) - e\rho J\}Al}{\rho J} \tag{7.5}$$

If v is the volume of cake deposited by unit volume of filtrate then:

$$v = \frac{lA}{V} \quad \text{or} \quad l = \frac{vV}{A} \tag{7.6}$$

and from equation 7.5:

$$v = \frac{J\rho}{(1-J)(1-e)\rho_s - Je\rho} \tag{7.7}$$

Substituting for l in equation 7.2:

$$\frac{1}{A}\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{(-\Delta P)}{\mathbf{r}\mu}\frac{A}{vV}$$

or: $$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{A^2(-\Delta P)}{\mathbf{r}\mu vV} \tag{7.8}$$

Equation 7.8 may be regarded as the basic relation between $-\Delta P$, V, and t. Two important types of operation are: (i) where the pressure difference is maintained constant and (ii) where the rate of filtration is maintained constant.

For a filtration at constant rate

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{V}{t} = \text{constant}$$

so that: $$\frac{V}{t} = \frac{A^2(-\Delta P)}{\mathbf{r}\mu Vv} \tag{7.9}$$

or: $$\frac{t}{V} = \frac{\mathbf{r}\mu v}{A^2(-\Delta P)}V \tag{7.10}$$

and $-\Delta P$ is directly proportional to V.

For a filtration at constant pressure difference

$$\frac{V^2}{2} = \frac{A^2(-\Delta P)t}{\mathbf{r}\mu v} \tag{7.11}$$

or: $$\frac{t}{V} = \frac{\mathbf{r}\mu v}{2A^2(-\Delta P)}V \tag{7.12}$$

Thus for a constant pressure filtration, there is a linear relation between V^2 and t or between t/V and V.

Filtration at constant pressure is more frequently adopted in practice, although the pressure difference is normally gradually built up to its ultimate value.

If this takes a time t_1 during which a volume V_1 of filtrate passes, then integration of equation 7.12 gives:

$$\frac{1}{2}(V^2 - V_1^2) = \frac{A^2(-\Delta P)}{\mathbf{r}\mu v}(t - t_1) \tag{7.13}$$

or:
$$\frac{t-t_1}{V-V_1}=\frac{\mathbf{r}\mu v}{2A^2(-\Delta P)}(V-V_1)+\frac{\mathbf{r}\mu v V_1}{A^2(-\Delta P)} \tag{7.14}$$

Thus, there where is a linear relation between V^2 and t and between $(t-t_1)/(V-V_1)$ and $(V-V_1)$, where $(t-t_1)$ represents the time of the constant pressure filtration and $(V-V_1)$ the corresponding volume of filtrate obtained.

7.2.4. Flow of filtrate through the cloth and cake combined

If the filter cloth and the initial layers of cake are together equivalent to a thickness L of cake as deposited at a later stage in the process, and if $-\Delta P$ is the pressure drop across the cake and cloth combined, then:

$$\frac{1}{A}\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{(-\Delta P)}{\mathbf{r}\mu(l+L)} \tag{7.15}$$

which may be compared with equation 7.2.

Thus:
$$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{A(-\Delta P)}{\mathbf{r}\mu\left(\dfrac{Vv}{A}+L\right)}=\frac{A^2(-\Delta P)}{\mathbf{r}\mu v\left(V+\dfrac{LA}{v}\right)} \tag{7.16}$$

This equation may be integrated between the limits $t=0$, $V=0$ and $t=t_1$, $V=V_1$ for constant rate filtration, and $t=t_1$, $V=V_1$ and $t=t$, $V=V$ for a subsequent constant pressure filtration.

For the period of *constant rate filtration*:

$$\frac{V_1}{t_1}=\frac{A^2(-\Delta P)}{\mathbf{r}\mu v\left(V_1+\dfrac{LA}{v}\right)}$$

or:
$$\frac{t_1}{V_1}=\frac{\mathbf{r}\mu v}{A^2(-\Delta P)}V_1+\frac{\mathbf{r}\mu L}{A(-\Delta P)}$$

or:
$$V_1^2+\frac{LA}{v}V_1=\frac{A^2(-\Delta P)}{\mathbf{r}\mu v}t_1 \tag{7.17}$$

For a subsequent *constant pressure filtration*:

$$\frac{1}{2}(V^2-V_1^2)+\frac{LA}{v}(V-V_1)=\frac{A^2(-\Delta P)}{\mathbf{r}\mu v}(t-t_1) \tag{7.18}$$

or:
$$(V-V_1+2V_1)(V-V_1)+\frac{2LA}{v}(V-V_1)=\frac{2A^2(-\Delta P)}{\mathbf{r}\mu v}(t-t_1)$$

or:
$$\frac{t-t_1}{V-V_1}=\frac{\mathbf{r}\mu v}{2A^2(-\Delta P)}(V-V_1)+\frac{\mathbf{r}\mu v V_1}{A^2(-\Delta P)}+\frac{\mathbf{r}\mu L}{A(-\Delta P)} \tag{7.19}$$

7.4. FILTRATION EQUIPMENT

7.4.1. Filter selection

The most suitable filter for any given operation is the one which will fulfil the requirements at minimum overall cost. Since the cost of the equipment is closely related to the filtering area, it is normally desirable to obtain a high overall rate of filtration. This involves the use of relatively high pressures although the maximum pressures are often limited by mechanical design considerations. Although a higher throughput from a given filtering surface is obtained from a continuous filter than from a batch operated filter, it may sometimes be necessary to use a batch filter, particularly if the filter cake has a high resistance, since most continuous filters operate under reduced pressure and the maximum filtration pressure is therefore limited. Other features which are desirable in a filter include ease of discharge of the filter cake in a convenient physical form, and a method of observing the quality of the filtrate obtained from each section of the plant. These factors are important in considering the types of equipment available. The most common types are filter presses, leaf filters, and continuous rotary filters. In addition, there are filters for special purposes, such as bag filters, and the disc type of filter which is used for the removal of small quantities of solids from a fluid.

The most important factors in filter selection are the specific resistance of the filter cake, the quantity to be filtered, and the solids concentration. For free-filtering materials, a rotary vacuum filter is generally the most satisfactory since it has a very high capacity for its size and does not require any significant manual attention. If the cake has to be washed, the rotary drum is to be preferred to the rotary leaf. If a high degree of washing is required, however, it is usually desirable to repulp the filter cake and to filter a second time.

For large-scale filtration, there are three principal cases where a rotary vacuum filter will not be used. Firstly, if the specific resistance is high, a positive pressure filter will be required, and a filter press may well be suitable, particularly if the solid content is not so high that frequent dismantling of the press is necessary. Secondly, when efficient washing is required, a leaf filter is effective, because very thin cakes can be prepared and the risk of channelling during washing is reduced to a minimum. Finally, where only very small quantities of solids are present in the liquid, an edge filter may be employed.

Whilst it may be possible to predict qualitatively the effect of the physical properties of the fluid and the solid on the filtration characteristics of a suspension, it is necessary in all cases to carry out a test on a sample before the large-scale plant can be designed. A simple vacuum filter with a filter area of 0.0065 m^2 is used to obtain laboratory data, as illustrated in Figure 7.5. The information on filtration rates and specific resistance obtained in this way can be directly applied to industrial filters provided due account is taken of the compressibility of the filter cake. It cannot be stressed too

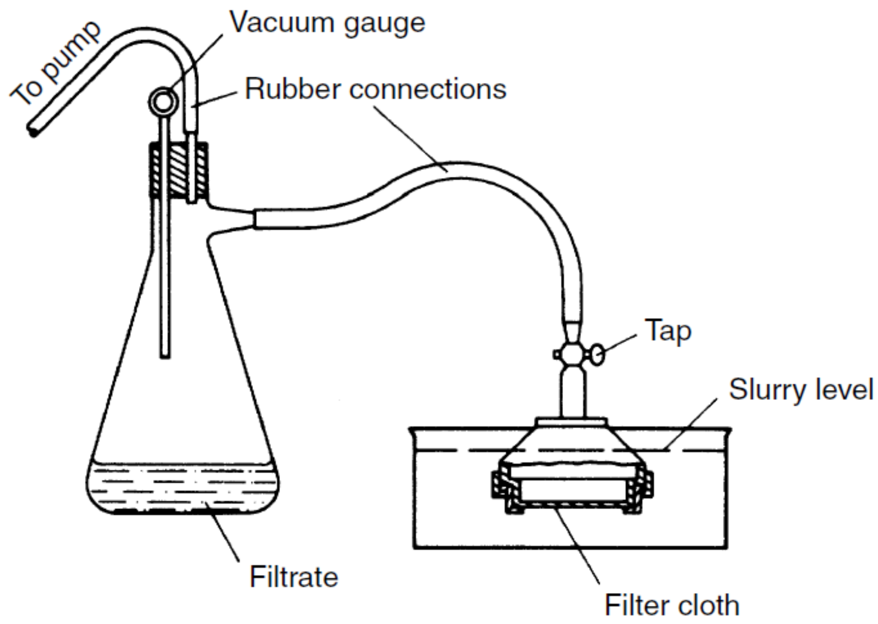

Figure 7.5. Laboratory test filter

7.4.2. Bed filters

Bed filters provide an example of the application of the principles of *deep bed filtration* in which the particles penetrate into the interstices of the filter bed where they are trapped following impingement on the surfaces of the material of the bed.

For the purification of water supplies and for waste water treatment where the solid content is about 10 g/m^3 or less, as noted by CLEASBY[23] granular bed filters have largely replaced the former very slow sand filters. The beds are formed from granular material of grain size 0.6–1.2 mm in beds 0.6–1.8 m deep. The very fine particles of solids are removed by mechanical action although the particles finally adhere as a result of surface electric forces or adsorption, as IVES[24] points out. This operation has been analysed by IWASAKI[25] who proposes the following equation:

$$-\frac{\partial C}{\partial l} = \lambda C \tag{7.36}$$

On integration:

$$C/C_0 = e^{-\lambda l} \tag{7.37}$$

where: C is the volume concentration of solids in suspension in the filter,
C_0 is the value of C at the surface of the filter,
l is the depth of the filter, and
λ is the filter coefficient.

7.4.3. Bag filters

Bag filters have now been almost entirely superseded for liquid filtration by other types of filter, although one of the few remaining types is the Taylor bag filter which has been widely used in the sugar industry. A number of long thin bags are attached to a horizontal feed tray and the liquid flows under the action of gravity so that the rate of filtration per unit area is very low. It is possible, however, to arrange a large filtering area in the plant of up to about 700 m^2. The filter is usually arranged in two sections so that each may be inspected separately without interrupting the operation.

7.4.4. The filter press

The filter press is one of two main types, the *plate and frame press* and the *recessed plate* or *chamber press*.

The plate and frame press

This type of filter consists of plates and frames arranged alternately and supported on a pair of rails as shown in Figure 7.6. The plates have a ribbed surface and the edges stand slightly proud and are carefully machined. The hollow frame is separated from the plate by the filter cloth, and the press is closed either by means of a hand screw or hydraulically, using the minimum pressure in order to reduce wear on the cloths. A chamber is therefore formed between each pair of successive plates as shown in Figure 7.7. The slurry is introduced through a port in each frame and the filtrate passes through the cloth on each side so that two cakes are formed simultaneously in each chamber, and these join when the frame is full. The frames are usually square and may be 100 mm–2.5 m across and 10 mm–75 mm thick.

The slurry may be fed to the press through the continuous channel formed by the holes in the corners of the plates and frames, in which case it is necessary to cut corresponding holes in the cloths which themselves act as gaskets. Cutting of the cloth can be avoided by feeding through a channel at the side although rubber bushes must then be fitted so that a leak-tight joint is formed.

The filtrate runs down the ribbed surface of the plates and is then discharged through a cock into an open launder so that the filtrate from each plate may be inspected and any plate can be isolated if it is not giving a clear filtrate. In some cases the filtrate is removed through a closed channel although it is not then possible to observe the discharge from each plate separately.

In many filter presses, provision is made for steam heating so that the viscosity of the filtrate is reduced and a higher rate of filtration obtained. Materials, such as waxes, that solidify at normal temperatures may also be filtered in steam-heated presses. Steam heating also facilitates the production of a dry cake.

Optimum time cycle. The optimum thickness of cake to be formed in a filter press depends on the resistance offered by the filter cake and on the time taken to dismantle

PROBLEM 7.3

A plate and frame press gave a total of 8 m^3 of filtrate in 1800 s and 11.3 m^3 in 3600 s when filtration was stopped. Estimate the washing time if 3 m^3 of wash water is used. The resistance of the cloth may be neglected and a constant pressure is used throughout.

Solution

For constant pressure filtration with no cloth resistance:

$$t = \frac{\mathbf{r}\mu v}{2A^2(-\Delta P)}V^2 \qquad \text{(equation 7.1)}$$

At $t_1 = 1800$ s, $V_1 = 8$ m^3, and when $t_2 = 3600$ s, $V_2 = 11$ m^3

Thus:

$$(3600 - 1800) = \frac{\mathbf{r}\mu v}{2A^2(-\Delta P)}(11^2 - 8^2)$$

$$\frac{\mathbf{r}\mu v}{2A^2(-\Delta P)} = 316$$

Since:

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{\mathbf{r}\mu v V}$$

$$= \frac{1}{(2 \times 31.6V)} = \frac{0.0158}{V}$$

The final rate of filtration $= (0.0158/11) = 1.44 \times 10^{-3}$ m^3/s.

For thorough washing in a plate and frame filter, the wash water has twice the thickness of cake to penetrate and half the area for flow that is available to the filtrate. Thus the flow of wash water at the same pressure will be one-quarter of the filtration rate.

Hence: rate of washing $= (1.44 \times 10^{-3})/4 = 3.6 \times 10^{-4}$ m^3/s

and: time of washing $= 3/(3.6 \times 10^{-4}) = \underline{\underline{8400 \text{ s}}}$ (2.3 h)

PROBLEM 7.4

In the filtration of a sludge, the initial period is effected at a constant rate with the feed pump at full capacity, until the pressure differences reaches 400 kN/m^2. The pressure is then maintained at this value for a remainder of the filtration. The constant rate operation requires 900 s and one-third of the total filtrate is obtained during this period.

Neglecting the resistance of the filter medium, determine (a) the total filtration time and (b) the filtration cycle with the existing pump for a maximum daily capacity, if the time for removing the cake and reassembling the press is 1200 s. The cake is not washed.

Solution

For a filtration carried out at a constant filtration rate for time t_1 in which time a volume V_1 is collected and followed by a constant pressure period such that the total filtration time is t and the total volume of filtrate is V, then:

$$V^2 - V_1^2 = \frac{2A^2(-\Delta P)}{\mathbf{r}\mu v}(t - t_1) \qquad \text{(equation 7.13)}$$

Assuming no cloth resistance, then:

for the constant rate period: $$t_1 = \frac{\mathbf{r}\mu v}{A^2(-\Delta P)} V_1^2 \qquad \text{(equation 7.10)}$$

Using the data given: $t_1 = 900$ s, volume $= V_1$

Thus: $$\frac{\mathbf{r}\mu v}{A^2(-\Delta P)} = \frac{900}{V_1^2}$$

(a) *For the constant pressure period*: $V = 3V_1$ and $(t - t_1) = t_p$

Thus: $$8V_1^2 = \frac{2V_1^2}{900} t_p$$

$$t_p = 3600 \text{ s}$$

Thus: total filtration time $= (900 + 3600) = \underline{\underline{4500 \text{ s}}}$

and: total cycle time $= (4500 + 1200) = \underline{\underline{5700 \text{ s}}}$

(b) *For the constant rate period*:

$$t_1 = \frac{\mathbf{r}\mu v}{A^2(-\Delta P)} V_1^2 = \frac{V_1^2}{K}$$

For the constant pressure period:

$$t - t_1 = \frac{\mathbf{r}\mu v}{2A^2(-\Delta P)}(V^2 - V_1^2) = \frac{V^2 - V_1^2}{2K}$$

Total filtration time, $$t = \frac{1}{K}\left(V_1^2 + \frac{V^2 - V_1^2}{2}\right) = \frac{(V^2 + V_1^2)}{2K}$$

Rate of filtration $$= \frac{V}{t + t_d} \text{ where } t_d \text{ is the downtime}$$

$$= \frac{2KV}{V^2 + V_1^2 + 2Kt_d}$$

For the rate to be a maximum,

$$\frac{\mathrm{d(rate)}}{\mathrm{d}V} = 0 \quad \text{or} \quad V_1^2 - V^2 + 2Kt_d = 0$$

Thus: $$t_d = \frac{1}{2K}(V^2 - V_1^2) = (t - t_1)$$

But: $$t_d = 1200 = (t - 900) \quad \text{and} \quad t = 2100 \text{ s}$$

Thus: $$\text{total cycle time} = (2100 + 1200) = \underline{\underline{3300 \text{ s}}}$$