COURSE TITLE:

Introduction to Non-Linear Systems

COURSE CODE:

EEE 566

Engr. Dr. Femi Onibonoje

Summary of Introductory Class

- Most real systems are nonlinear to an extent and have parameters that vary with time to a certain degree
- Linear systems obey principles of scaling and superposition, nonlinear systems do not.
- Time-invariant (autonomous) systems have constant-coefficient expressions, time-variant/varying (non-autonomous) systems
- Based on the joint criteria of linearity (or otherwise) and time variations (or otherwise), there are 4 categories of continuous-time systems i.e.

  - Linear, Time-Invariant (LTI) Systems
  - Nonlinear, Time-Invariant Systems
  - Linear, Time-Varying Systems
  - Nonlinear, Time-Varying Systems

Summary of Introductory Class (Con'd)

- **Nonlinear systems exhibit one, some or all of the following properties unlike their linear counterparts:**
  - Shape and course of outputs input-magnitude-dependent
  - Stability being input-magnitude-dependent
  - Generation of harmonics and sub-harmonics for periodically-excited systems
  - Multiple equilibrium points (with the concept of steady-state error needing revision, since there are more than just one equilibrium point)
  - Shape, course, and stability being dependent on initial conditions
  - Generation of limit cycles
  - Jumpwise amplitude and phase responses in regions of resonance
  - Existence of bifurcations
  - Occurrence of chaotic responses

Classification of Nonlinear Systems & List of Common Nonlinearities

Classification Criteria for Nonlinear Systems

The following are some criteria for classification of nonlinearities:

- **Criterion 1: Level of Importance of Nonlinearity to Operation of System**
- **Criterion 2: Inherence or Otherwise of Nonlinearity within a System**
- **Criterion 3: Mathematical Properties (Continuity or Otherwise of Nonlinearity)**
- **Criterion 4: Mathematical Properties ("Single-Valuedness" or otherwise of Nonlinearity)**
- **Criterion 5: Dynamic Behaviour of System**

Criterion 1: Importance of Nonlinearity Within System

- Based on the necessity or otherwise of the existence of nonlinearities within a system, they can be classified as
  - **Essential/Deliberate/Intentional Nonlinearities**:

    These nonlinearities are absolutely important for the basic operation of the system. Proper operation is simply impossible without them Examples are relay nonlinearity, granularity nonlinearity and nonlinear compensation.

  - **Spurious/Undesirable/Parasitic/Unintentional Nonlinearities**:

    These nonlinearities are unwanted in a system. Most nonlinearities belong in this class. Examples are hysteresis nonlinearity, saturation nonlinearity, dead-zone nonlinearity, etc.

Criterion 2: Inherence or Otherwise of Nonlinearity Within System

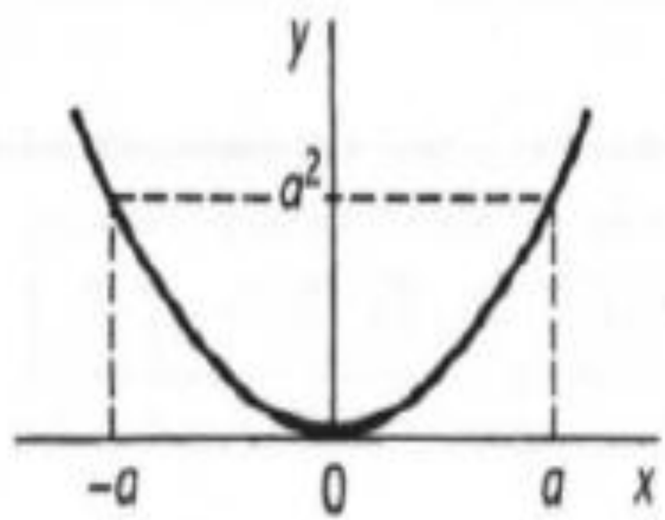
- Based on whether nonlinearities are inherent in a system or are introduced artificially, nonlinearities can be classified as
  - **Inherent/Natural Nonlinearities:**
    - These nonlinearities come naturally with the system hardware and motion.
    - They may have desirable or undesirable effects.
    - Most nonlinearities are inherent .
    - (e.g. variable-gain nonlinearity, saturation nonlinearity, hysteresis nonlinearity, etc.)
  - **Artificial Nonlinearities**:
    - These nonlinearities are artificially introduced by the Control Engineer to address natural nonlinearities within a system.
    - Examples are nonlinear compensation, on-off control laws, adaptive control laws, etc

Criterion 3: Mathematical Properties 1 (Continuity or Otherwise)…….Cont’d

A nonlinearity $y = f(u)$ is continuous if, for all values of u, the limit of $f(u)$ as u approaches each value from the left is the same as the limit as u approaches the value from the right i.e. for any value $u = c$

$$\lim_{u \to c^+} f(u) = \lim_{u \to c^-} f(u) = f(c)$$

The nonlinearity $y = x^2$ below is continuous

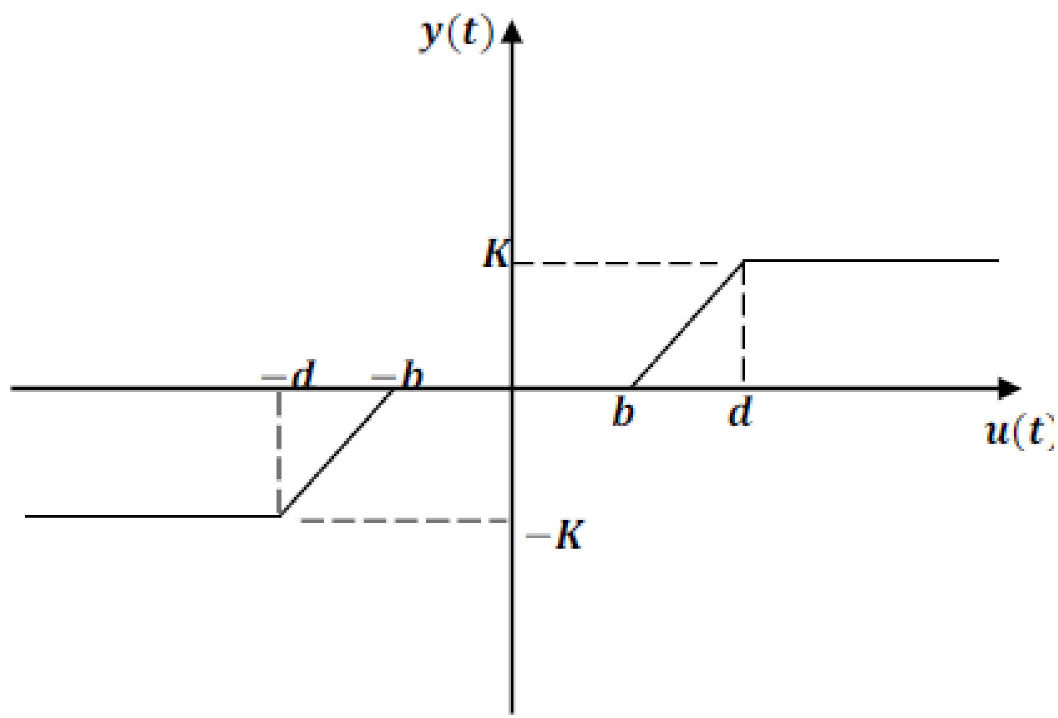


Criterion 3: Mathematical Properties 1 (Continuity or Otherwise)…….Cont'd

The nonlinearity is still continuous if it comprises two different functions $\boldsymbol{f}_1(\boldsymbol{u})$ and $\boldsymbol{f}_2(\boldsymbol{u})$ that intersect at the point $\boldsymbol{u} = \boldsymbol{c}$, then

$$\lim_{u \to c^+} f(u) = \lim_{u \to c^-} f(u) = f_1(c) = f_2(c)$$

The Saturation-with-Dead-Band Nonlinearity below is continuous in spite of being a composite of 5 different functions.



Criterion 3: Mathematical Properties 1 (Continuity or Otherwise)…….Cont'd

- "Continuity" and "Differentiability" have two different meanings altogether

- A nonlinearity is differentiable "everywhere" (or analytic) if it is differentiable at every point, with the limit

$$\lim_{h\to 0}\frac{f(c+h)-f(c)}{h} = f'(c)$$

remaining unchanged as $h \to 0$ through both positive and negative values.

- A nonlinearity cannot therefore be differentiable everywhere if it is a composite of more than a single function (like the saturation-with-dead-band nonlinearity above)

Criterion 3: Mathematical Properties 1 (Continuity or Otherwise)…….Cont'd

- Clearly, therefore, it is possible to have continuity "everywhere" but have non-differentiability at one or more points.

- Indeed, theoretically, it is possible to have functions that are "continuous everywhere" but are "differentiable nowhere".

- Functions that are "continuous everywhere" but "differentiable nowhere" are called "Weierstrass Functions", named after Karl Weierstrass who gave the first example of such a function in 1872 to challenge the notion that continuity translated to differentiability except at isolated points.

Criterion 3: Mathematical Properties 1 (Continuity or Otherwise)…….Cont'd

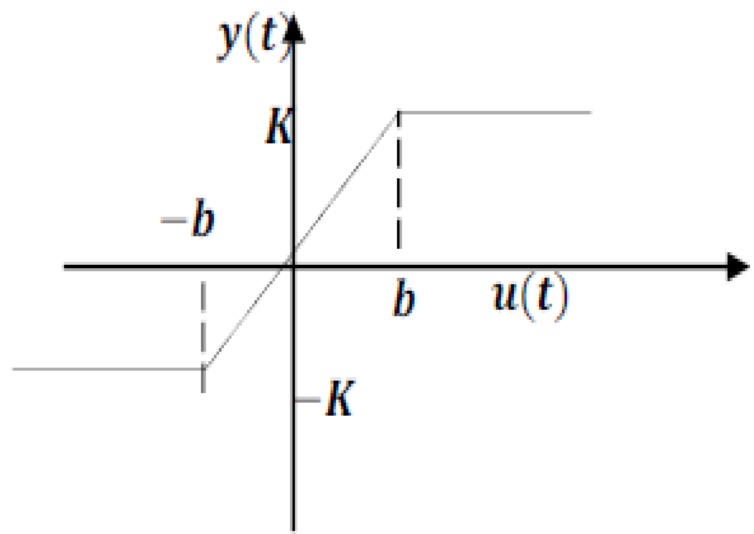
- We can therefore sub-divide continuous nonlinearities into "**Everywhere-Differentiable Continuous Nonlinearities**" and "**Continuous Nonlinearities with Non-Differentiability at one or more points**"

- Examples of "Everywhere-Differentiable Continuous Nonlinearities" are analytic nonlinearities like square nonlinearities ($y = u^2$), exponential nonlinearities ($y = e^u$), etc.

- Examples of "Continuous Nonlinearities with Non-Differentiability at One or More Points" are Saturation Nonlinearity, Saturation-with-Dead-Band Nonlinearity, Threshold I Nonlinearity, Variable-Gain Nonlinearity, etc.

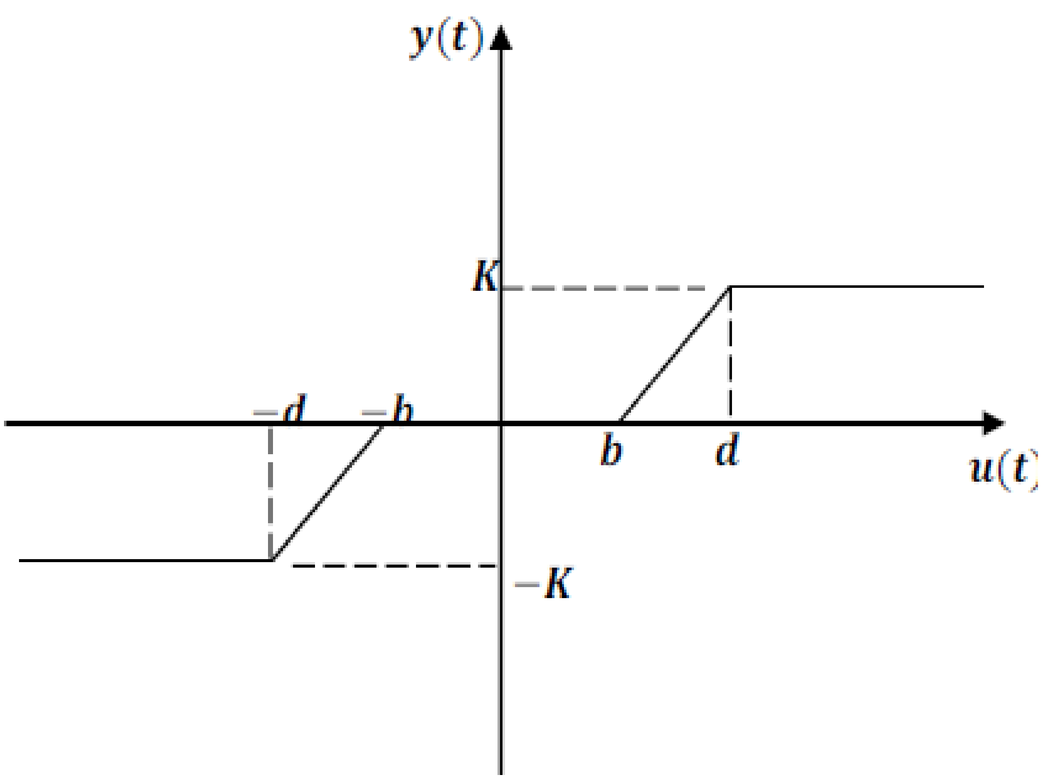
Criterion 3: Mathematical Properties 1 (Continuity or Otherwise)…….Cont'd

- Taking the Saturation Nonlinearity below as an example, the function is continuous all through but is non-differentiable at 2 points i.e. points where $\boldsymbol{u} = -\boldsymbol{b}$ and where $\boldsymbol{u} = \boldsymbol{b}$



Criterion 3: Mathematical Properties 1 (Continuity or Otherwise)…….Cont'd

- Taking the Saturation-with-Deadband Nonlinearity below as another example, the function is continuous "everywhere" but is non-differentiable at 4 points i.e. points where $\boldsymbol{u = -d}$, $\boldsymbol{u = -b}$, $\boldsymbol{u = b}$ and $\boldsymbol{u = d}$



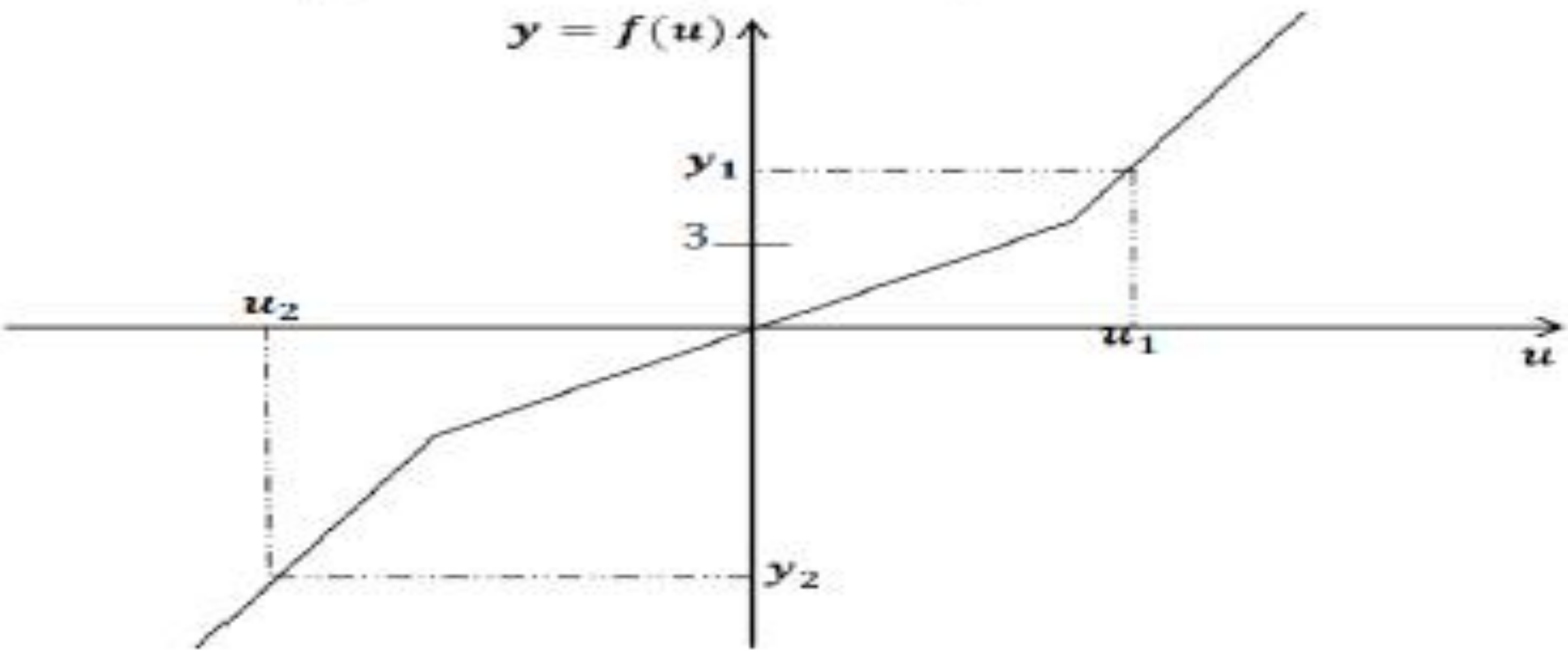
Criterion 4: Mathematical Properties 2 ("Single-Valuedness" or Otherwise)

- Based on whether nonlinearities are single-valued or otherwise, nonlinearities can be classified as:
  - **Single-Valued Nonlinearities**
  - **Multi-Valued Nonlinearities**

Criterion 4: Mathematical Properties 2 ("Single-Valuedness" or Otherwise) ..Cont'd

- A "single-valued" nonlinearity is one that has *a unique value of the dependent variable for every value of the independent variable* and also has *a unique value of the independent variable for every value of the dependent variable.*

- The variable-gain nonlinearity is an example.

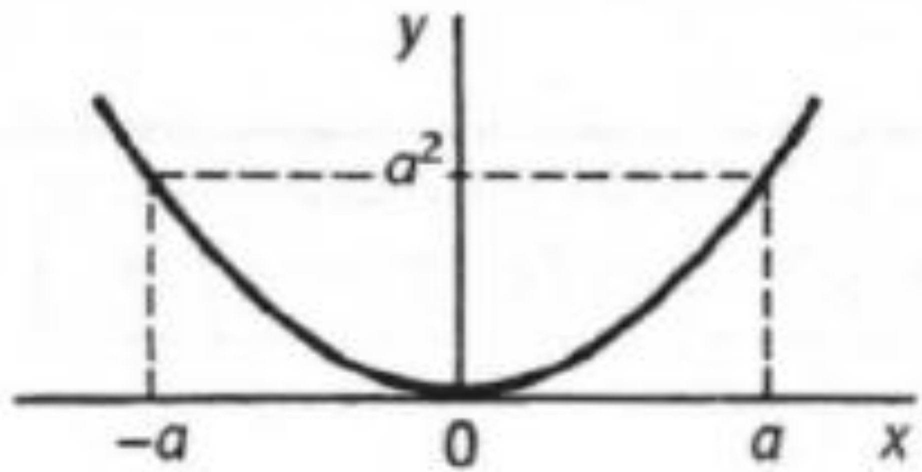


- u_1 and u_2 have unique corresponding values of y_1 and y_2 respectively and vice-versa.

Criterion 4: Mathematical Properties 2 ("Single-Valuedness" or Otherwise) ..Cont'd

- On the other hand, a "multi-valued" nonlinearity is one that has *more than one value of some or all of the dependent variable for every value of the independent variable* and/or also *more than one value of the independent variable for every value of the dependent variable.*

- The a square nonlinearity $y = x^2$ is multi-valued



- A single value of y (in this case, a^2) corresponds to two different values of x ($-a$ and a)

Criterion 5: Dynamic Characteristics

- Based on the comparison between the speed of variation of the nonlinearity and the speed of the signals in the system, nonlinearities can be classified into:
  - **Slowly-Changing Nonlinearities**
  - **Rapidly-Changing Nonlinearities**

Criterion 5: Dynamic Characteristics....Cont'd

- A slowly-changing nonlinearity is one in which the nonlinear characteristic or function is changing at a much slower rate in comparison with the signals within the system.
- Examples are component ageing, spring fatigue, catalyzer activity reduction, etc.
- A rapidly-changing nonlinearity is one in which the rate of variation of the function or characteristic of the system is of an order in the magnitude comparable to that of the signals within the system.
- Example is the heat-transfer properties of a system in comparison with the liquid flow velocity of a temperature detector in a temperature-control system.

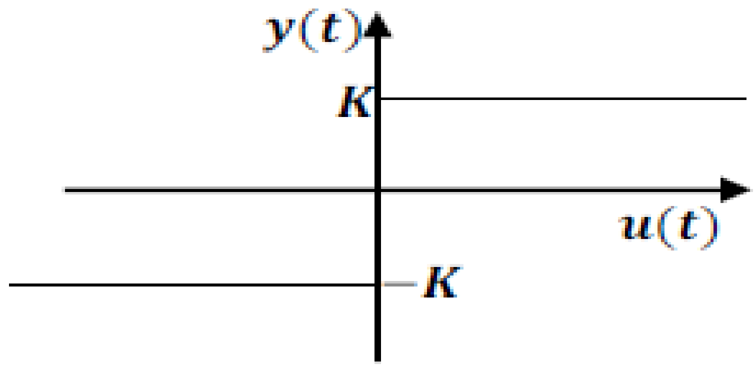
Common Nonlinearities

Based on static characteristics, these are the most common “non-analytic” nonlinearities:

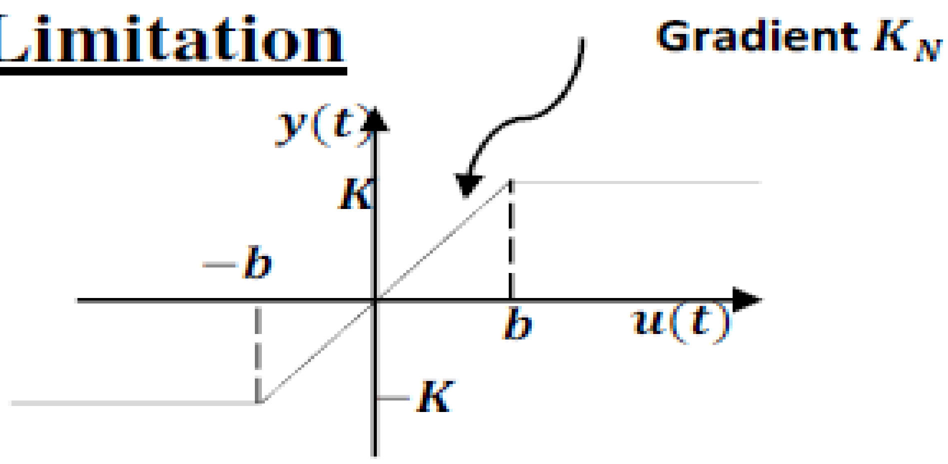
- Ideal Relay Nonlinearity
- Saturation/Limitation Nonlinearity
- Three-Position-Relay-With-DeadBand Nonlinearity
- Saturation-with-DeadBand Nonlinearity
- Dead-Band I/Dead-Zone I/Threshold I Nonlinearity
- Dead-Band II/Dead-Zone II/Threshold II Nonlinearity
- Negative-Deficiency Nonlinearity
- Granularity Nonlinearity
- Variable-Gain Nonlinearity
- Two-Position-Relay-with-Hysteresis Nonlinearity
- Hysteresis/Backlash Nonlinearity
- Three-Position-Relay-with-Hysteresis Nonlinearity
- Compound-Negative-Deficiency Nonlinearity

Common Nonlinearities…Cont’d

1. **Ideal Relay Nonlinearity**

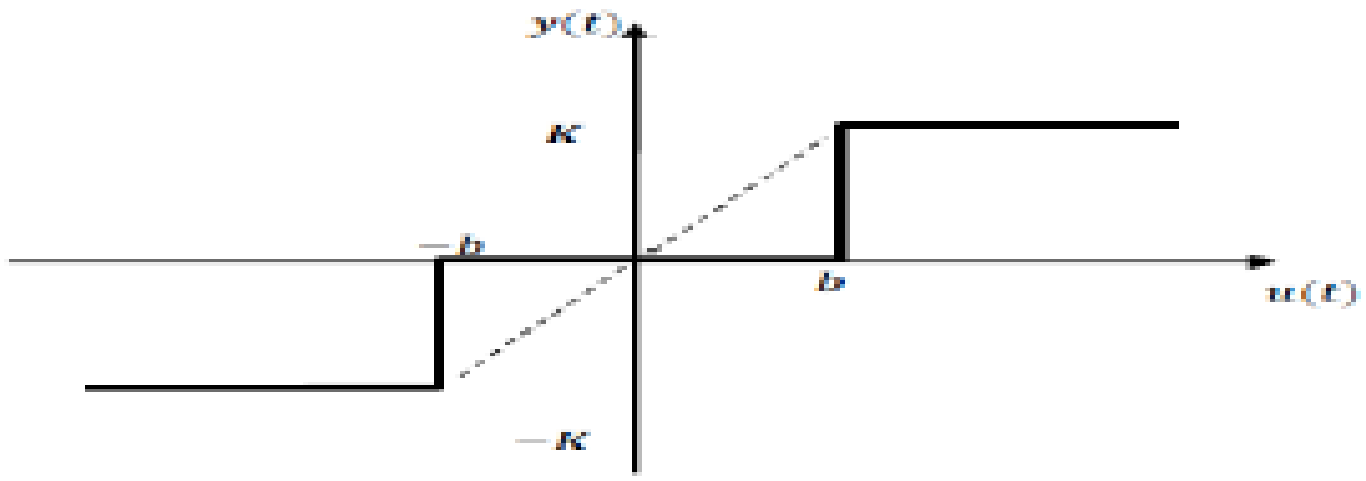


2. **Saturation/Limitation**

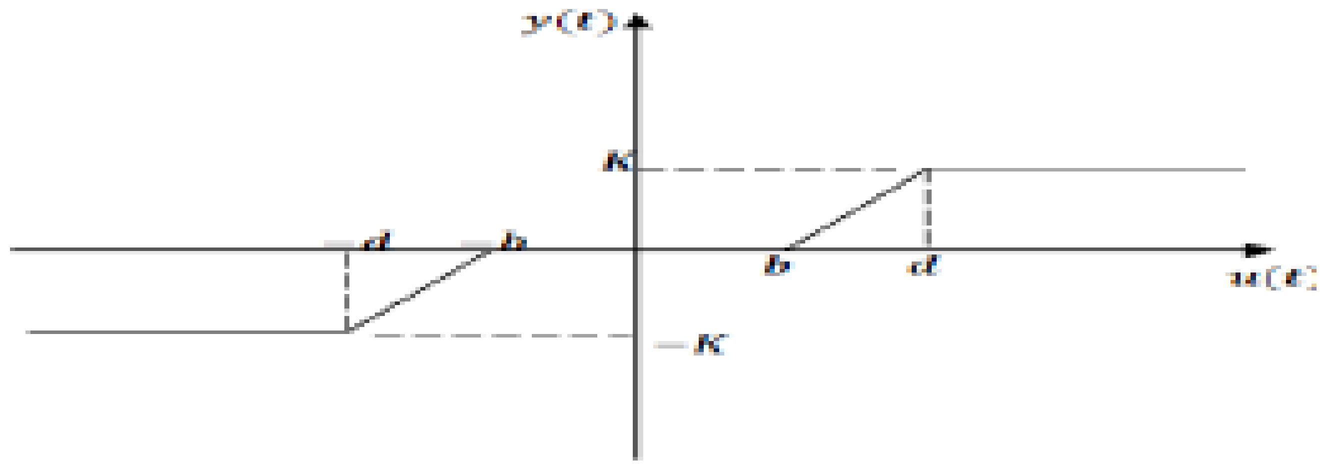


Common Nonlinearities…Cont’d

3. Three-Position Relay with Dead-Band

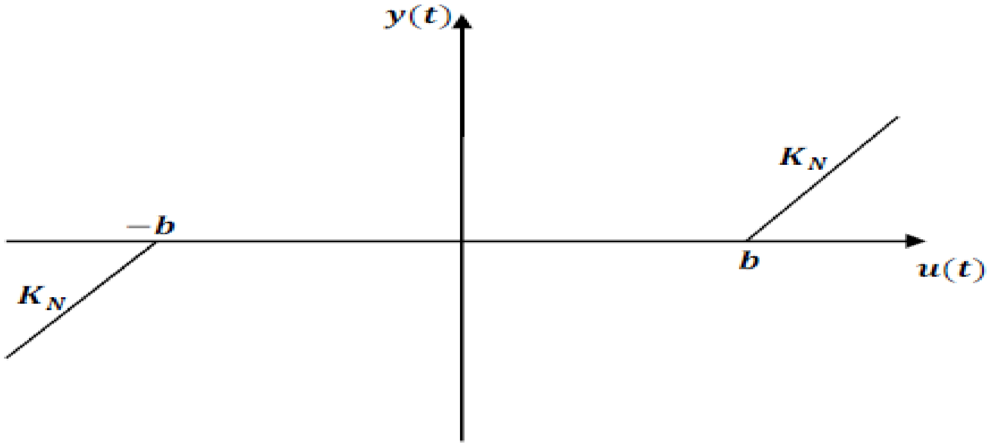


4. Saturation with Dead-Band

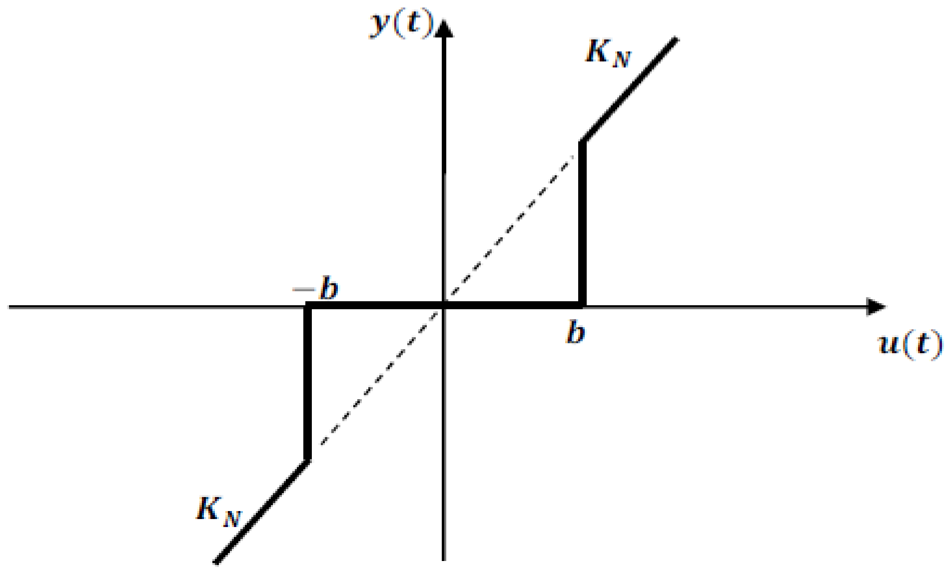


Common Nonlinearities…Cont’d

5. **Dead-Band I/ Dead-Zone I/ Threshold I**

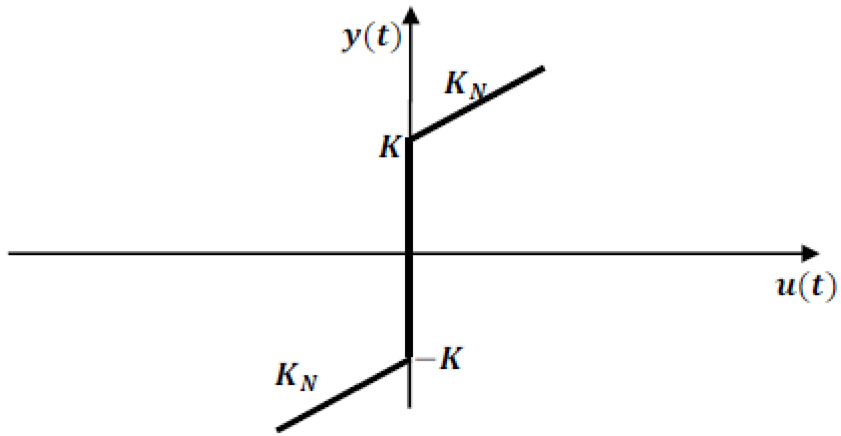


6. **Dead-Band II/ Dead-Zone II/ Threshold II**

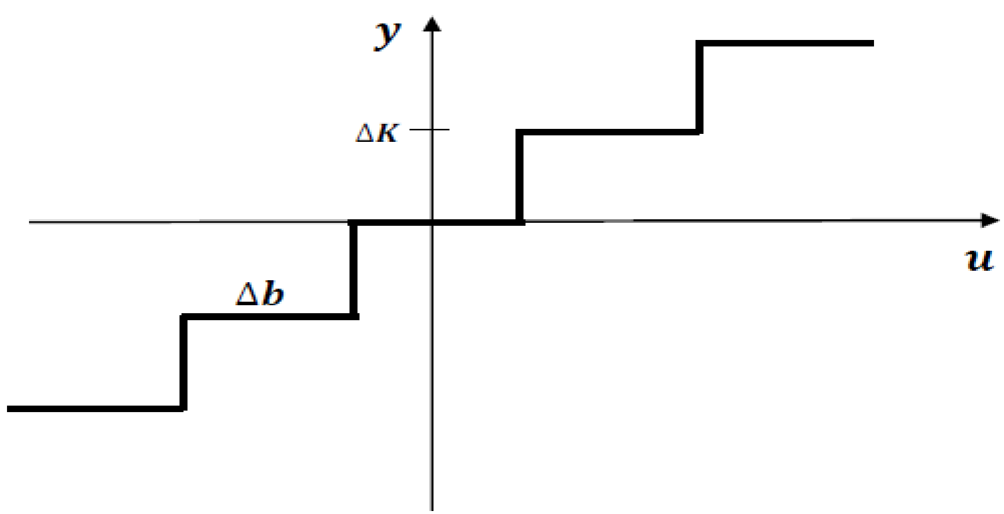


Common Nonlinearities…Cont’d

7. Negative Deficiency

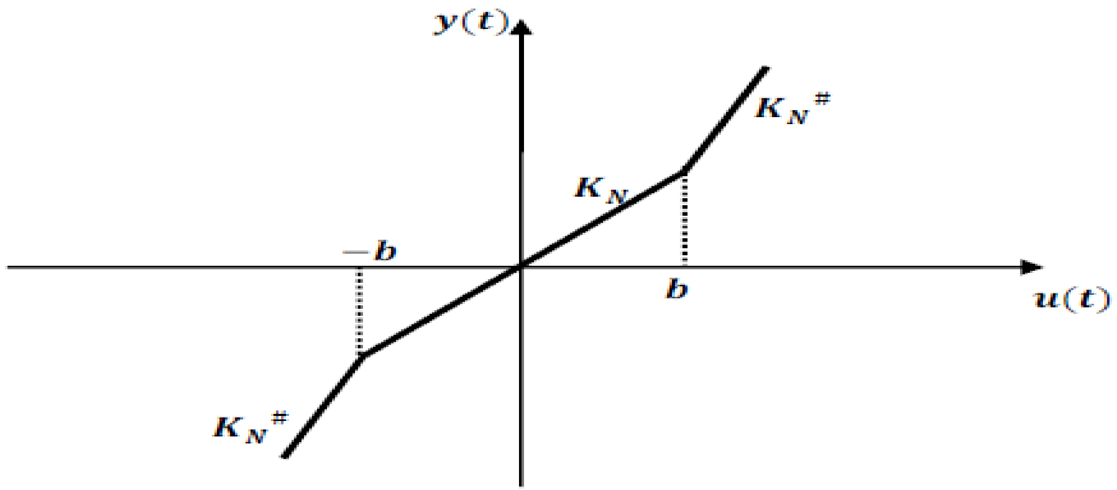


8. Granularity

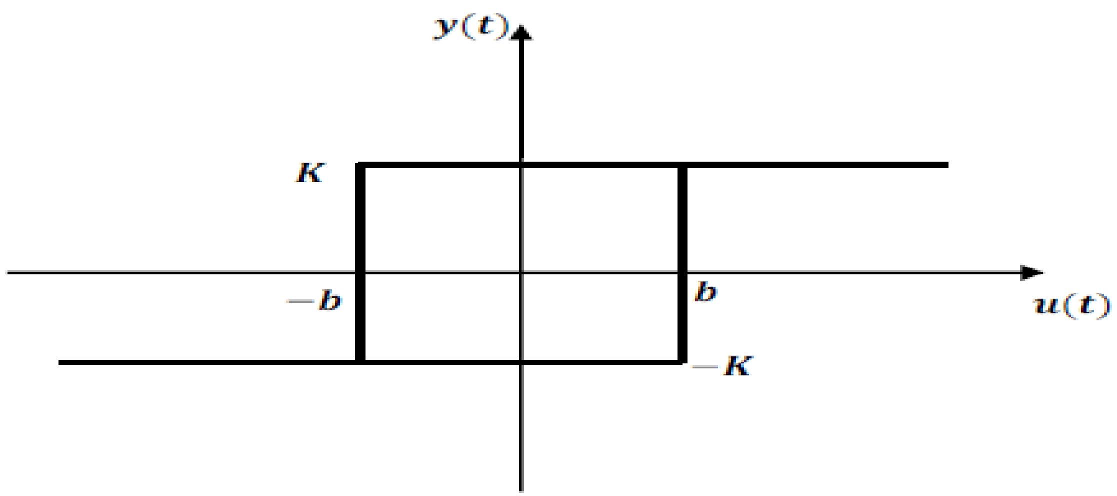


Common Nonlinearities…Cont'd

9. Variable Gain

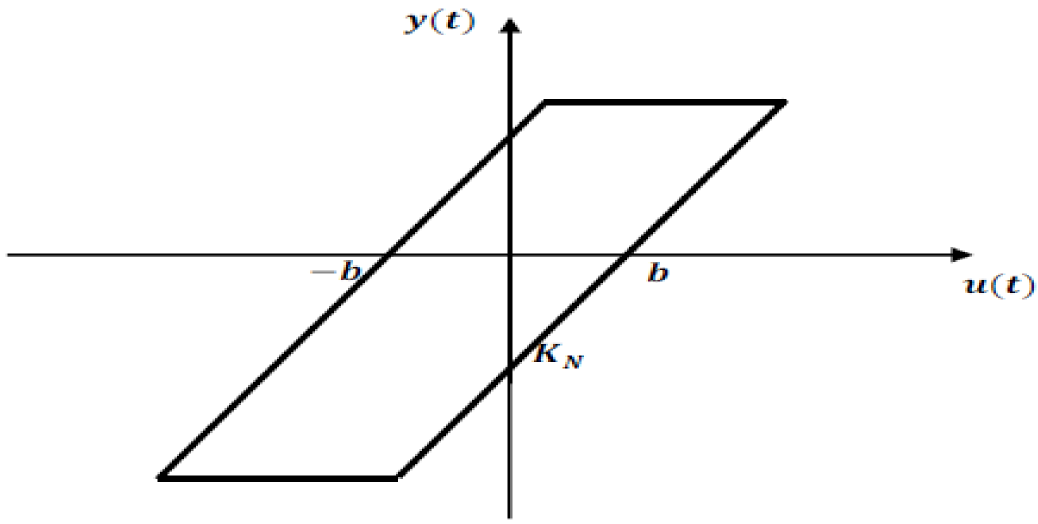


10. Two-Position Relay with Hysteresis

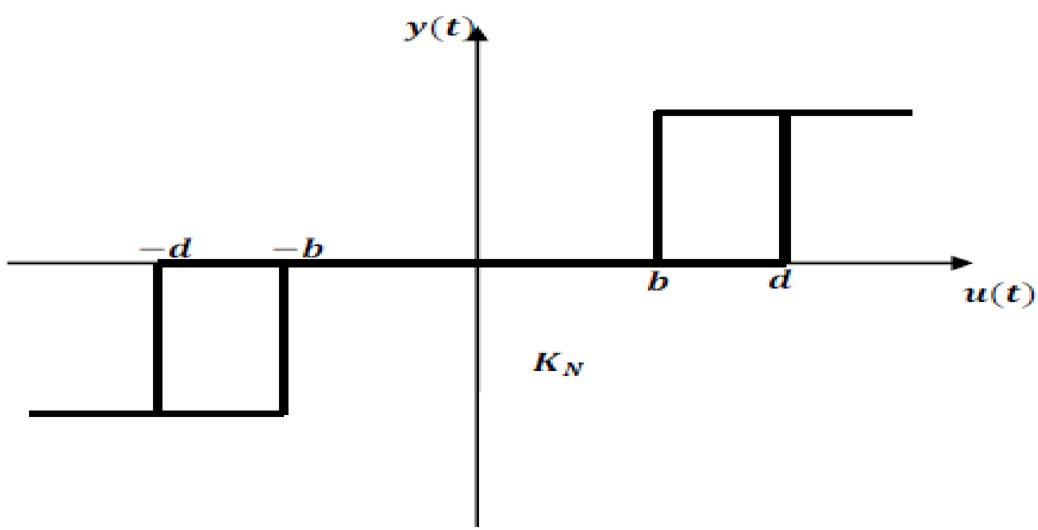


Common Nonlinearities…Cont'd

11. Hysteresis/Backlash

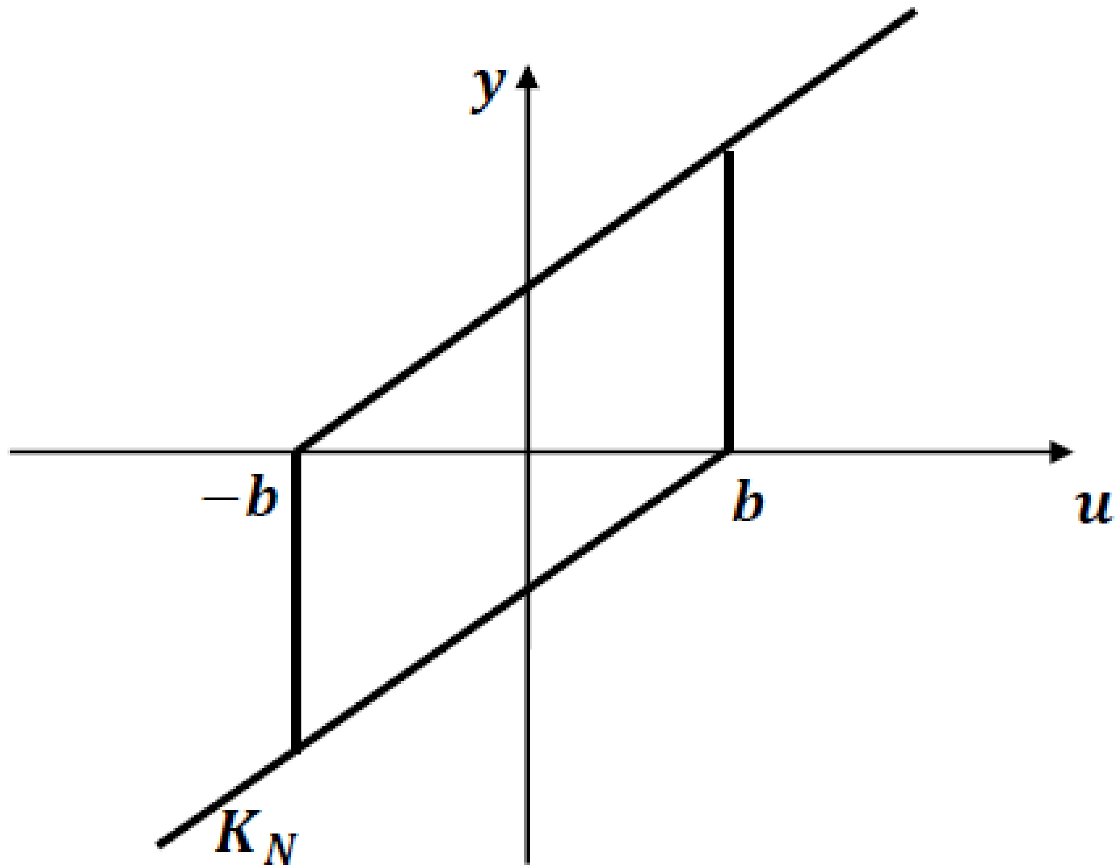


12. Three-Position Relay with Hysteresis



Common Nonlinearities…Cont’d

13. <u>Compound Negative Deficiency</u>



Summary of Lecture

- Nonlinear systems can be classified based on a number of criteria.
- Five of these criteria are listed i.e. level of importance of nonlinearity to operation of system, inherence or otherwise of nonlinearity in a system, mathematical properties of nonlinearity (continuity or otherwise), mathematical properties of nonlinearity (single-valuednessor otherwise), and the creation of nonlinearity due to dynamic behaviour of system.
- There are several common nonlinearities.
- Thirteen common nonlinearities are listed i.e. Ideal Relay, Saturation/Limitation, Three-Position Relay with Dead-Band, Saturation with Dead-Band, Dead-Band I/Dead-Zone I/Threshold I, Dead-Band II/Dead-Zone II/Threshold II, Negative Deficiency, Granularity, Variable Gain, Two-Position Relay with Hysteresis, Hysteresis/Backlash, Three-Position Relay with Hysteresis and Compound Negative Deficiency.